Correspondence

On the Optimality of the ARQ-DDF Protocol

Kambiz Azarian, Hesham El Gamal, *Senior Member, IEEE*, and Philip Schniter, *Senior Member, IEEE*

***Abstract*—In this correspondence, the performance of the automatic repeat request-dynamic decode and forward (ARQ-DDF) cooperation protocol is analyzed in two distinct scenarios. The first scenario is the multiple access relay channel where a single relay is dedicated to simultaneously help two multiple access users. For this setup, it is shown that the ARQ-DDF protocol achieves the channel's optimal diversity multiplexing tradeoff (DMT). The second scenario is the cooperative vector multiple access channel where two users cooperate in delivering their messages to a destination equipped with two receiving antennas. For this setup, a new variant of the ARQ-DDF protocol is developed where the two users are purposefully instructed not to cooperate in the first round of transmission. Lower and upper bounds on the achievable DMT are then derived. These bounds are shown to converge to the optimal tradeoff as the number of transmission rounds increases.**

***Index Terms*—Automatic repeat request (ARQ), cooperative diversity, cooperative vector multiple-access (CVMA) channel, diversity-multiplexing tradeoff (DMT), dynamic decode and forward (DDF), half-duplex node, multiple-access relay (MAR) channel.**

I. Background

The dynamic decode and forward (DDF) protocol was proposed in [1] as an efficient method to exploit cooperative diversity in the half-duplex relay channel (the same protocol was independently devised for other scenarios in [2] and [3]). In this paper, the DDF protocol is combined with the automatic repeat request (ARQ) mechanism to derive new variants that are matched to the multiple access relay (MAR) and cooperative vector multiple access (CVMA) channels. These variants, some of them presented in [11]–[13], are shown to achieve the optimal tradeoff between throughput and reliability, in the high signal-to-noise ratio (SNR) regime. For simplicity of presentation, in this correspondence, the number of users is restricted to two.

Throughout this correspondence, all channels are assumed to be flat Rayleigh-fading and quasi-static. The quasi-static assumption implies that the channel gains remain fixed over a coherence interval, but change independently from one coherence interval to the next. In order to highlight the benefits of cooperation and ARQ, as opposed to temporal interleaving, the long-term static channel model of [6] is adopted where all ARQ rounds corresponding to a message take place over the same coherence interval. The channel gains are assumed to be mutually independent and of unit variance. The additive Gaussian noise processes are mutually independent, circularly symmetric, white, and of variance σ^2. All nodes operate synchronously and are subject to a short-term power constraint. This constraint ensures that the average energy available to a symbol for transmission E is fixed. Under these assumptions, the average SNR of a link ρ is defined as

$$\rho \triangleq \frac{E}{\sigma^2}. \tag{1}$$

Also, $f(\rho)$ is said to be exponentially equal to ρ^b, denoted by $f(\rho) \doteq \rho^b$, when

$$\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = b. \tag{2}$$

In (2), b is called the exponential order of $f(\rho)$. $\dot{\leq}$ and $\dot{\geq}$ are defined similarly.

Except for Section III, where the destination is equipped with two receiving antennas, all nodes are assumed to have a single antenna. It is also assumed that the nodes operate in the half-duplex mode, i.e., at any point in time, a node can either transmit or receive, but not both. This constraint is motivated by the typically large difference between the incoming and outgoing signal power levels. In this correspondence, the coherent transmission paradigm is adopted where only the receiving end of any link knows the channel gain, i.e., except for the ACK/NACK feedback bits, no other channel state information (CSI) is available to the transmitting nodes. The maximum allowable number of ARQ rounds is denoted by L, where each round consists of T consecutive symbol intervals ($L = 1$ corresponds to the non-ARQ scenario). The first-round rate of transmission (at the destination) is denoted by R_1, while the average throughput is denoted by η. These two quantities are related through [6]

$$\eta = \frac{R_1}{1 + \sum_{\ell=1}^{L-1} p(\ell)} \tag{3}$$

where $p(\ell)$ denotes the probability that the destination requests for the $(\ell + 1)$th round of transmission.

Throughout this correspondence, random Gaussian codebooks with asymptotically large block lengths are used to derive information theoretic bounds on the achievable performance. Results related to the design of practical coding/decoding schemes that approach the fundamental limits established here are reported in [4]. More specifically, our analysis tool in this correspondence is the diversity multiplexing tradeoff (DMT) introduced by Zheng and Tse in [5]. To define DMT for a non-ARQ (i.e., $L = 1$) symmetric (i.e., the users transmit at the same rate and power) multiple access channel with two users, a family of codes $C(\rho) = \{C_1(\rho), C_2(\rho)\}$ is considered such that the code $C_j(\rho)$ (used by user $j \in \{1, 2\}$) has a rate $R_1(\rho)/2$ bits per channel use (bpcu) and a maximum-likelihood (ML) error probability $P_{E_j}(\rho)$. For this family, the multiplexing gain r_1 and the diversity gain d are defined as

$$r_1 \triangleq \lim_{\rho \to \infty} \frac{R_1(\rho)}{\log \rho}$$
$$d \triangleq \min_{j \in \{1,2\}} \left\{ -\lim_{\rho \to \infty} \frac{\log P_{E_j}(\rho)}{\log \rho} \right\}. \tag{4}$$

Manuscript received January 21, 2006; revised October 10, 2007. This work was supported in part by the National Science Foundation. Parts of the material in this paper were presented at the 42nd Allerton Conference on Communications, Control and Computing, Monticello, IL, September 2004, the IEEE 6th Workshop on Signal Processing Advances in Wireless Communications, New York City, NY, June 2005, and the 2006 IEEE International Zurich Seminar on Communications. This work was performed in part while H. El Gamal was visiting Nile University, Cairo, Egypt.

K. Azarian was with the Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH 43210 USA and also with the Qualcomm Incorporated, San Diego, CA 92121 USA (e-mail: azariany@ece.osu.edu).

H. El Gamal and P. Schniter are with the Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH 43210 USA (e-mail: helgamal@ece.osu.edu; schniter@ece.osu.edu).

Communicated by M. Médard, Associate Editor for Communication.

Color versions of Figures 1 and 2 in this correspondence are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TIT.2008.917722

Now, the DMT of $C(\rho)$ is a characterization of diversity gain d, as a function of multiplexing gain r_1, i.e., $d = f(r_1)$. For ARQ scenarios (i.e., $L > 1$), r_1 is replaced with the effective multiplexing gain r_e [6]

$$r_e \triangleq \lim_{\rho \to \infty} \frac{\eta}{\log \rho} \tag{5}$$

to capture the variable-rate nature of such schemes. It is noteworthy that while establishing the asymptotic equality of the effective rate η, and the first-round rate R_1, i.e., $\lim_{\rho\to\infty} \eta = R_1$, requires $\lim_{\rho\to\infty} p(\ell) = 0$, for $L-1 \geq \ell \geq 1$ [refer to (3)], this is not the case for establishing $r_e = r_1$. This is because (3) results in

$$R_1 \geq \eta \geq \frac{R_1}{L}$$

which in turn asserts that

$$r_e = r_1. \tag{6}$$

The rest of this correspondence is organized as follows. Section II is devoted to the MAR channel whereas the CVMA channel is discussed in Section III. This correspondence ends with a few concluding remarks in Section IV. To enhance the flow of presentation, only those parts of the proofs that are novel and do not follow from the techniques outlined in [1] are reported here. These parts mainly include the techniques that are necessary to deal with the ARQ or multiuser aspects of the protocols. The interested reader, however, is referred to [7] for complete and detailed proofs.

II. The Multiple Access Relay Channel

In the two-user MAR channel, a relay node is assigned to assist the two multiple access users. The users are not allowed to help each other (due to practical limitations, for example). The relay node is constrained by the half-duplex assumption. We proceed toward our main result in this section via a step-by-step approach. First, we prove the optimality of the ARQ-DDF protocol in the relay channel (i.e., a MAR channel with a single user). The proof for this result introduces the machinery necessary to handle the ARQ mechanism. In Lemma 2, we analyze the DDF protocol (without ARQ) in the two user MAR channel. This step elucidates the multiuser aspects of the problem. Finally, we combine these ideas in Theorem 3 to prove the optimality of the DDF-ARQ protocol in the two-user MAR channel.

Lemma: The optimal DMT for the relay channel with $L \geq 2$ ARQ rounds is given by

$$d_R(r_e, L) = 2\left(1 - \frac{r_e}{L}\right), \qquad \text{for } 1 > r_e \geq 0. \tag{7}$$

Furthermore, this optimal tradeoff is achieved by the proposed ARQ-DDF protocol.

Proof: A simple min-cut max-flow examination reveals that the optimal diversity gain for this channel is upper bounded by those of the 2×1 (*perfect* cooperation between the source and the relay) and 1×2 (*perfect* cooperation between the relay and the destination) ARQ multiple-input–multiple-output (MIMO) channels [6], thus

$$d_{\mathrm{R}}(r_e, L) \leq 2\left(1 - \frac{r_e}{L}\right), \qquad \text{for } 1 > r_e \geq 0. \tag{8}$$

Next, we prove the achievability of this upper bound, by characterizing the DMT for the proposed ARQ-DDF relay protocol. Toward this end, we first construct an ensemble of random Gaussian codes and characterize its average error probability P_E. We then argue that there are codes in the ensemble that perform at least as well as the average, therefore achieving P_E.

Let $C(\rho) = \{C_s(\rho), C_r(\rho)\}$ denote the random codes used by the source and the relay, respectively. These are codes of length LT symbols, rate R_1/L bpcu and generated by an independent identically distributed (i.i.d.) complex Gaussian random process of mean zero and variance E. Let us denote the message to be sent by m_0. This message consists of $R_1 T$ information bits. We denote the source and relay codewords corresponding to m_0 by $\mathbf{x}_s(m_0)$ and $\mathbf{x}_r(m_0)$, respectively. We also denote the signature of message m_0 at the destination by $\mathbf{s}(m_0)$, i.e.,

$$\mathbf{y} = \mathbf{s}(m_0) + \mathbf{n} \tag{9}$$

where $\mathbf{y}$ and $\mathbf{n}$ denote the destination received signal and additive noise, respectively. It is important to realize that $\mathbf{s}(m_0)$ not only depends on the message m_0, but also on the channel realization and the relay noise. Also, notice that $\mathbf{x}_r(m_0)$ is only partially transmitted. This is because the half-duplex relay, itself, needs first to listen to the source to be able to decode the message. Finally, we use superscript ℓ to denote the portion of the signal that corresponds to the first ℓ rounds of transmission. The decoder $\{\varphi, \psi\}$ consists of two functions $\varphi = \{\varphi^\ell\}_{\ell=1}^{L}$ and $\psi = \{\psi^\ell\}_{\ell=1}^{L-1}$.

- At round $\ell (L \geq \ell \geq 1), \varphi^\ell$ outputs the message that minimizes $|\mathbf{y}^\ell - \mathbf{s}^\ell|^2$, i.e.,

$$\varphi^\ell(\mathbf{y}^\ell) = \arg\min_m |\mathbf{y}^\ell - \mathbf{s}^\ell(m)|^2, \qquad \text{for } L \geq \ell \geq 1. \tag{10}$$

  We denote the event that $\varphi^\ell(\mathbf{y}^\ell)$ differs from m_0, with m_0 denoting the transmitted message, by E^ℓ.
- At round $\ell (L-1 \geq \ell \geq 1), \psi^\ell$ outputs a one, if $m = \varphi^\ell(\mathbf{y}^\ell)$ [given by (10)] is the *unique* message for which

$$|\mathbf{y}^\ell - \mathbf{s}^\ell(m)|^2 \leq \ell T(1+\delta)\sigma^2 \tag{11}$$

  where σ^2 denotes the destination noise variance and δ is a small positive value. In any other case, ψ^ℓ outputs a zero. We denote the event that ψ^ℓ outputs a one, by A^ℓ.

The decoder uses φ and ψ to decode the message as follows.

1) At the end of round ℓ, $(L-1 \geq \ell \geq 1)$, the decoder computes both $\varphi^\ell(\mathbf{y}^\ell)$ and $\psi^\ell(\mathbf{y}^\ell)$. If $\psi^\ell(\mathbf{y}^\ell) = 1$, then the decoder declares $\varphi^\ell(\mathbf{y}^\ell)$ as the received message and sends back an ACK. Otherwise, it requests for another round of transmission by sending back a NACK signal.
2) At the end of the Lth round, the decoder outputs $\varphi^L(\mathbf{y}^L)$ as the received message.

To characterize the average error probability P_E, we first use the Bayes' rule to write

$$P_E \leq P_{E|\overline{E}_r} + P_{E_r}$$

where E_r and $\overline{E}_r$ denote the events that the relay makes an error in decoding the message, and its complement, respectively. Since the relay starts transmission only after the mutual information between its received signal and the source signal exceeds $R_1 T$, we have (e.g., refer to [10, Th. 10.1.1])

$$P_{E_r} \leq \epsilon, \qquad \text{for any } \epsilon > 0.$$

This means that

$$P_E \dot{\leq} P_{E|\overline{E}_r}.$$

For the sake of notational simplicity, in the sequel, we denote $P_{E|\overline{E}_r}$ by P_E. In characterizing P_E, we take the approach of El Gamal *et al.* in [6], i.e., we upper bound P_E by

$$P_E \leq \sum_{\ell=1}^{L-1} P_{E^\ell, A^\ell} + P_{E^L}. \tag{12}$$

Notice that P_{E^ℓ, A^ℓ} upper bounds the probability of *undetected* errors with $\ell < L$ rounds of transmission, while P_{E^L} upper bounds the probability of *decoding* errors at the end of L transmission rounds. The next step in characterizing P_E is to show that, for the decoder of interest, the undetected errors do not dominate the overall error event, i.e.,

$$P_E \dot{\leq} P_{E^L}. \tag{13}$$

Toward this end, we note that

$$P_{E^\ell, A^\ell} \leq \Pr\{|\mathbf{n}^\ell|^2 > \ell T(1+\delta)\sigma^2\}. \tag{14}$$

To understand (14), let us assume that $|\mathbf{n}^\ell|^2 \leq \ell T(1+\delta)\sigma^2$. We can then use (9) to conclude that $|\mathbf{y}^\ell - \mathbf{s}^\ell(m_0)|^2 \leq \ell T(1+\delta)\sigma^2$, where m_0 denotes the transmitted message. This, however, is in contradiction with $E^\ell \cap A^\ell$. This is because the latter event implies that some message m_1, other than m_0, is the *unique* message for which $|\mathbf{y}^\ell - \mathbf{s}^\ell(m_1)|^2 \leq \ell T(1+\delta)\sigma^2$. Thus, $E^\ell \cap A^\ell \subseteq \{|\mathbf{n}^\ell|^2 > \ell T(1+\delta)\sigma^2\}$, which means that (14) is indeed true. Now, $|\mathbf{n}^\ell|^2$ has a central chi-squared distribution with $2\ell T$ degrees of freedom. One can use the Chernoff bound to upper bound the tail of this distribution to get

$$\Pr\{|\mathbf{n}^\ell|^2 > \ell T(1+\delta)\sigma^2\} \leq (1+\delta)^{\ell T} e^{-\ell T \delta}. \tag{15}$$

This, however, in conjunction with (14) means that, for any $\delta > 0$, it is possible to choose T large enough, such that

$$P_{E^\ell, A^\ell} \leq \epsilon, \qquad \text{for any } \epsilon > 0. \tag{16}$$

Now, (13) follows from (16), together with (12). Examination of E^L reveals that P_{E^L} is the probability of error for the DDF relay protocol at a multiplexing gain of r_1/L, i.e.,

$$P_{E^L} \doteq \rho^{-d_{\mathrm{DDF-R}}\left(\frac{r_1}{L}\right)} \tag{17}$$

where $d_{\mathrm{DDF-R}}(\cdot)$ denotes the diversity gain achieved by the DDF relay protocol [1]. Using (13), we conclude

$$P_E \dot{\leq} \rho^{-d_{\mathrm{DDF-R}}\left(\frac{r_1}{L}\right)}. \tag{18}$$

Now, (18), together with (6), and the fact that for $1 > r_e \geq 0, d_{\mathrm{DDF-R}}(\frac{r_e}{l}) = 2(1 - \frac{r_e}{L})$, give

$$P_E \dot{\leq} \rho^{-2\left(1-\frac{r_e}{L}\right)}, \qquad \text{for } 1 > r_e \geq 0. \tag{19}$$

Note that P_E, as given by (19), only characterizes the *average* error probability. To complete the proof, we argue that there exists a code in the ensemble that performs at least as well as the average, therefore achieving P_E.

The next scenario to be considered is the non-ARQ MAR channel. In our DDF protocol for this channel, the two sources transmit their individual messages during every symbol interval in the codeword, while the relay listens to the sources until it collects sufficient energy to decode *both* of them error free. After decoding, the relay uses an *independent* code book to encode the two messages *jointly*. The encoded symbols are then transmitted for the rest of the codeword.

Lemma: The optimal diversity gain for the symmetric two-user MAR channel is upper bounded by

$$d_{\mathrm{MAR}}(r) \leq \begin{cases} 2-r, & \text{if } \frac{1}{2} \geq r \geq 0 \\ 3(1-r), & \text{if } 1 \geq r \geq \frac{1}{2}. \end{cases} \tag{20}$$

Furthermore, the DMT achieved by the DDF protocol is lower bounded by

$$d_{\mathrm{DDF-MAR}}(r) \geq \begin{cases} 2-r & \text{if } \frac{1}{2} \geq r \geq 0 \\ 3(1-r) & \text{if } \frac{2}{3} \geq r \geq \frac{1}{2} \\ 2\frac{1-r}{r} & \text{if } 1 \geq r \geq \frac{2}{3}. \end{cases} \tag{21}$$

Proof: A simple min-cut max-flow examination reveals that the optimal diversity gain for this channel is upper bounded by

$$d_{\mathrm{MAR}}(r) \leq \min\left\{d_{3\times1}(r), d_{2\times2}(r), d_{2\times1}\left(\frac{r}{2}\right), d_{1\times2}\left(\frac{r}{2}\right)\right\} \tag{22}$$

where $d_{m\times n}(\cdot)$ denotes the optimal diversity gain for an $m \times n$ MIMO channel. In (22), e.g., $d_{3\times1}(r)$ corresponds to the case where the relay is in *perfect* cooperation with the two sources, while $d_{1\times2}(r/2)$ represents the scenario where one of the sources is decoded error free and the relay is *perfectly* cooperating with the destination to decode the other source. Now, (22) results in (20) and the proof of the converse part is complete.

In order to derive a lower bound on the diversity gain achieved by the DDF MAR protocol, we upper bound the *source-specific* ML error probabilities, with that of the *joint* ML decoder. Furthermore, instead of characterizing the latter probability for specific codes, in the sequel, we characterize its average P_E over the ensemble of random Gaussian codes. It is then straightforward to see that there exists a code in the ensemble, whose error probability is better than P_E. To characterize P_E, we use the Bayes' rule to derive the following upper bound:

$$P_E \leq P_{E|\overline{E}_r} + P_{E_r}$$

where E_r and $\overline{E}_r$ denote the events that the relay makes errors in decoding the messages, and its complement, respectively. Next, we note that if we denote the signals transmitted by the two sources and the relay by $\{x_{j,k}\}_{k=1}^T$ and $\{x_{r,k}\}_{k=T'+1}^T$, respectively, and the signals received by the relay and the destination by $\{y_{r,k}\}_{k=1}^{T'}$ and $\{y_k\}_{k=1}^T$, then the number of symbol intervals T' that the relay waits before decoding the messages satisfies

$$\frac{TR}{2} \leq I\left(\{x_{1,k}\}_{k=1}^{T'}; \{y_{r,k}\}_{k=1}^{T'} | \{x_{2,k}\}_{k=1}^{T'}\right) \tag{23}$$

$$\frac{TR}{2} \leq I\left(\{x_{2,k}\}_{k=1}^{T'}; \{y_{r,k}\}_{k=1}^{T'} | \{x_{1,k}\}_{k=1}^{T'}\right) \tag{24}$$

$$TR \leq I\left(\{x_{1,k}\}_{k=1}^{T'}, \{x_{2,k}\}_{k=1}^{T'}; \{y_{r,k}\}_{k=1}^{T'}\right). \tag{25}$$

In these expressions, R is the *total* data rate (in bpcu) at the destination and $I(.;.)$ denotes the mutual information function. Now, observing that (23)–(25) guarantee that (e.g., refer to [10, Sec. 14.3.1])

$$P_{E_r} \leq \epsilon, \qquad \text{for any } \epsilon > 0$$

we conclude

$$P_E \dot{\leq} P_{E|\overline{E}_r}.$$

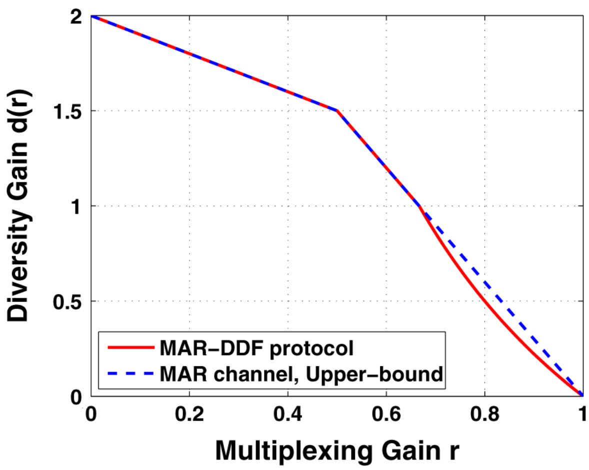

Fig. 1. DMT achieved by the DDF protocol in the MAR channel, along with an upper bound on the achievable DMT ($L = 1$).

For the sake of notational simplicity, in the sequel, $P_{E|\overline{E}_r}$ is denoted by P_E. In characterizing P_E, we follow the approach of Tse *et al.* [9], by partitioning the error event E into the set of partial error events E_I, i.e.,

$$E = \bigcup_I E_I$$

where I denotes any *nonempty* subset of $\{1, 2\}$ and E_I (referred to as type-I error) is the event that the joint ML decoder incorrectly decodes the messages from sources whose indices belong to I while correctly decoding all other messages. Because the partial error events are mutually exclusive, we have

$$P_E = \sum_I P_{E_I}. \tag{26}$$

Characterization of P_{E_I} is straightforward and follows the techniques outlined in [1], therefore, we omit the derivations and only report the final results (please refer to [7] for the complete derivations), i.e.,

$$P_{E_I} \dot{\leq} \rho^{-d_I(r)} \tag{27}$$

where

$$d_{\{1\}}(r) = d_{\{2\}}(r) = \begin{cases} 2 - r, & \frac{1}{2} > r \geq 0 \\ \frac{4 - 5r}{2(1 - r)}, & \frac{2}{3} \geq r \geq \frac{1}{2} \\ \frac{2 - r}{2r}, & 1 \geq r \geq c\frac{2}{3} \end{cases} \tag{28}$$

and

$$d_{\{1,2\}}(r) = \begin{cases} 3(1 - r), & \frac{2}{3} > r \geq 0 \\ 2\frac{1 - r}{r}, & 1 \geq r \geq \frac{2}{3}. \end{cases} \tag{29}$$

Now, (28) and (29), together with (27) and (26), result in (21) and thus complete the proof of the achievability part.

Fig. 1 compares the upper and lower bounds in Lemma 2 where the optimality of the DDF protocol for $2/3 \geq r \geq 0$ is evident. This observation is the key to establishing the following result.

Theorem 3: The optimal DMT for the symmetric two-user MAR channel with $L \geq 2$ ARQ rounds is given by

$$d_{\mathrm{MAR}}(r_e, L) = 2 - \frac{r_e}{L}, \qquad \text{for } 1 > r_e \geq 0. \tag{30}$$

Furthermore, this optimal tradeoff is achieved by the proposed ARQ-DDF protocol.

Proof: A simple min-cut max-flow examination reveals that the optimal diversity gain for this channel is upper bounded by [compare to (22)]

$$d_{\mathrm{MAR}}(r_e, L) \leq \min\left\{ d_{3\times 1}(r_e, L), d_{2\times 2}(r_e, L) \times d_{2\times 1}\left(\frac{r_e}{2}, L\right), d_{1\times 2}\left(\frac{r_e}{2}, L\right) \right\} \tag{31}$$

where $d_{m\times n}(\cdot, \cdot)$ denotes the optimal diversity gain for an $m \times n$ ARQ MIMO channel (refer to [6]). Now, [6] results in (30) and the proof of the converse is complete.

Next, we prove that the proposed protocol achieves this upper bound. To do this, we only need to describe the encoder and the decoder. The rest of the proof then follows that of Lemma 1, line by line. Toward this end, let $\mathcal{C}(\rho) = \{\mathcal{C}_1(\rho), \mathcal{C}_2(\rho), \mathcal{C}_r(\rho)\}$ denote the random codes used by the two sources and the relay, respectively. These are codes of length LT, generated by an i.i.d complex Gaussian random process of mean zero and variance E. The rates of these codes are different, though. While $\mathcal{C}_1(\rho)$ and $\mathcal{C}_2(\rho)$ are of rate $R_1/2L$ bpcu, $\mathcal{C}_r(\rho)$ is of rate R_1/L bpcu. In other words, the relay code has twice the rate of the source codes. This means that, corresponding to each pair of source codewords $(\mathbf{x}_1(m_1), \mathbf{x_2}(m_2)) \in \mathcal{C}_1(\rho) \times \mathcal{C}_2(\rho)$, there exists a codeword $\mathbf{x}_r(\mathbf{m}) \in \mathcal{C}_r(\rho)$, where $\mathbf{m} \triangleq (m_1, m_2)$. We call $\mathbf{m}$ the joint message. Note that since each of the two source messages, i.e., m_1 and m_2, consists of $R_1T/2$ information bits, the joint message $\mathbf{m}$ has a total of R_1T information bits in it. As before, we denote the destination signature corresponding to the joint message $\mathbf{m}$, by $\mathbf{s}(\mathbf{m})$, i.e.,

$$\mathbf{y} = \mathbf{s}(\mathbf{m}) + \mathbf{n}.$$

In order to decode the source messages and produce the ACK/NACK signals, the destination uses a *joint* bounded distance decoder. This decoder is identical to the one devised for the ARQ-DDF relay protocol (refer to Lemma 1), with the only modification that the joint message $\mathbf{m}$ takes the role of m everywhere, e.g., $\varphi^\ell(\cdot)$ is now defined as [compare to (10)]

$$\varphi^\ell(\mathbf{y}^\ell) \triangleq \arg\min_{\mathbf{m}} |\mathbf{y}^\ell - \mathbf{s}^\ell(\mathbf{m})|^2, \qquad \text{for } L \geq \ell \geq 1.$$

In the proposed decoder, the destination provides a total of one bit of feedback, for the two sources, per transmission round. Therefore, there is no need for defining *source-specific* $\varphi(\cdot)$ and $\psi(\cdot)$ functions. With the encoder and decoder defined, one can now follow the same steps taken in the proof of Lemma 1 to show that [compare to (18)]

$$P_E \dot{\leq} \rho^{-d_{\mathrm{DDF-MAR}}\left(\frac{r_1}{L}\right)}. \tag{32}$$

Now, (32) and (6), together with the fact that for $1 > r_e \geq 0, d_{\mathrm{DDF-MAR}}(\frac{r_e}{L}) = 2 - \frac{r_e}{L}$ [refer to (21)], result in [compare to (19)]

$$P_E \dot{\leq} \rho^{-\left(2 - \frac{r_e}{L}\right)}, \qquad \text{for } 1 > r_e \geq 0. \tag{33}$$

It is then straightforward to argue that there exists at least one code in the ensemble that achieves (33). This proves the achievability of (30) and thus, completes the proof.

Overall, the main conclusion in this section is establishing the fact that a *single* relay can be efficiently shared by *multiple* users such that it enhances the diversity gain achieved by *all* of them.

III. Cooperative Vector Multiple-Access Channel

In the CVMA channel, the two single-antenna users are allowed to assist each other, as long as they do not violate the half duplex constraint. The challenge in this scenario stems from the availability of two receiving antennas at the destination, which increases the channel's degrees of freedom to two. Loosely speaking, in order to exploit these two degrees of freedom, the two users need to transmit *new* independent symbols continuously, which prevents them from cooperation under the half duplex constraint (non-ARQ case). More rigorously, it is straightforward to see that with $L = 1$, any half-duplex cooperation protocol that achieves full diversity (i.e., $d(0) = 3$) falls short of achieving full rate (i.e., $d(r) > 0$, for all $r < 2$). To get around this problem, in the case of $L \geq 2$, we purposefully instruct the users *not* to cooperate in the first round of transmission. In fact, a user continues transmitting its message while it receives NACK signals. Only when a user receives an ACK signal, it starts listening to the other user (assuming the other user has not been successfully decoded yet). Once the cooperating user decodes the message of its partner, it starts helping (i.e., the typical DDF protocol). The following result establishes lower and upper bounds on the DMT achieved by this protocol.

Theorem 4: The optimal diversity gain for the symmetric two-user CVMA channel with L ARQ rounds is upper bounded by

$$d_{\text{CVMA}}(r_e, L) \leq \min\left\{3\left(1 - \frac{r_e}{2L}\right), 4 - \frac{3r_e}{L}\right\}, \quad \text{for } 2 > r_e \geq 0. \tag{34}$$

For $L = 2$, the diversity gain achieved by the proposed ARQ-DDF protocol satisfies

$$d_{\text{DDF}-\text{CVMA}}(r_e, 2) \geq \begin{cases} 3 - r_e, & 1 > r_e \geq 0 \\ 4 - 2r_e, & \frac{4}{3} > r_e \geq 1 \\ 2 - \frac{r_e}{2}, & 2 > r_e \geq \frac{4}{3}. \end{cases} \tag{35}$$

Furthermore, as L increases, the ARQ-DDF diversity gain converges to the optimal value, i.e.,

$$\lim_{L \to \infty} d_{\text{DDF}-\text{CVMA}}(r_e, L) = 3, \qquad \text{for } 2 > r_e \geq 0. \tag{36}$$

Proof: A simple min-cut max-flow examination reveals that the optimal diversity gain for this channel is upper bounded by

$$d_{\text{CVMA}}(r_e, L) \leq \min\left\{d_{2\times 2}(r_e, L), d_{1\times 3}\left(\frac{r_e}{2}, L\right)\right\} \tag{37}$$

where $d_{m\times n}(\cdot,\cdot)$ denotes the optimal diversity gain for an $m \times n$ ARQ MIMO channel. Now, (37) results in (34) and the proof of the converse part is complete.

To prove the achievability part, let $C(\rho) = \{C_1(\rho), C_2(\rho)\}$ denote the random codes used by the two sources. These are codes of length $2T$ symbols, rate $R_1/4$ bpcu, and generated by an i.i.d complex Gaussian random process of mean zero and variance E. Let us also denote the two messages to be sent by m_1 and m_2. Note that each message consists of $R_1T/2$ information bits, such that the joint message $\mathbf{m} \triangleq (m_1, m_2)$ consists of a total of R_1T bits. We denote the codewords corresponding to m_1 and m_2 by $\mathbf{x}_1(m_1)$ and $\mathbf{x}_2(m_2)$. As before, the destination signatures of m_1, m_2, and $\mathbf{m}$ are denoted by $\mathbf{S}(m_1), \mathbf{S}(m_2)$, and $\mathbf{S}(\mathbf{m})$, respectively. Thus

$$\mathbf{Y} = \mathbf{S}(\mathbf{m}) + \mathbf{N}$$

or

$$\mathbf{Y} = \mathbf{S}(m_1) + \mathbf{S}(m_2) + \mathbf{N}$$

where $\mathbf{Y} \in \mathbb{C}^{2\times 2T}$ and $\mathbf{N} \in \mathbb{C}^{2\times 2T}$ represent the destination received signal and additive noise, respectively. We denote the signal received through antenna $j \in \{1, 2\}$ by $\mathbf{y}_j$. Similarly, the contribution of message m_i $i \in \{1, 2\}$, to the signal received through antenna j, is denoted by $\mathbf{s}_j(m_i)$. As before, we use the superscript ℓ to denote the portion of the signal that corresponds to the first ℓ rounds of transmission.

Next, we describe the decoder. Since the performance analysis for the optimal decoder seems intractable, in the sequel, we describe a *suboptimal* bounded distance decoder and analyze its performance. Obviously, this analysis provides a *lower bound* on the diversity gain achieved through the protocol. To describe the decoder, let us label the source and the receiving antenna that are connected through the channel with the highest signal to interference (due to the other source) and noise ratio by s (s stands for superior), while labeling the remaining source and receiving antenna by i (i stands for inferior). This means that

$$\frac{|g_{ss}|^2\rho}{|g_{is}|^2\rho + \sigma^2} \geq \max\left\{\frac{|g_{si}|^2\rho}{|g_{ii}|^2\rho + \sigma^2}, \frac{|g_{is}|^2\rho}{|g_{ss}|^2\rho + \sigma^2}, \frac{|g_{ii}|^2\rho}{|g_{si}|^2\rho + \sigma^2}\right\} \tag{38}$$

where, e.g., g_{si} denotes the gain of the channel connecting source s to receive antenna i. Now, the decoder $\{\varphi, \psi\}$ uses the two sets of functions $\varphi = \{\varphi_j^1, \varphi_s^1, \varphi_j^2, \varphi_i^2\}$ and $\psi = \{\psi_j^1, \psi_s^1\}$ to decode the messages and produce the ACK/NACK feedback bits, as follows.

1) At the end of the first round, the decoder uses φ_j^1 to *jointly* decode the two messages, i.e.,

$$\varphi_j^1(\mathbf{Y}^1) \triangleq \arg\min_{\mathbf{m}} \|\mathbf{Y}^1 - \mathbf{S}^1(\mathbf{m})\|^2. \tag{39}$$

We denote the event that $\varphi_j^1(\mathbf{Y}^1)$ is different from the actual joint message sent by E_j^1.

2) To decide whether it has correctly decoded the two messages or not, the decoder uses ψ_j^1, where ψ_j^1 outputs a one, if $\mathbf{m} = \varphi_j^1(\mathbf{Y}^1)$ [given by (39)] is the *unique* joint message satisfying

$$\|\mathbf{Y}^1 - \mathbf{S}^1(\mathbf{m})\|^2 \leq 2T(1+\delta)\sigma^2. \tag{40}$$

In (40), δ is some positive value. In any other case, ψ_j^1 outputs a zero. Now, if $\psi_j^1(\mathbf{Y}^1) = 1$, then the decoder sends back ACK signals to *both* of the users, declaring $\varphi_j^1(\mathbf{Y}^1)$ as the decoded joint message. This causes the two sources to start transmission of their next messages. Otherwise, it proceeds to the next step as described below. We denote the event $\psi_j^1(\mathbf{Y}^1) = 1$ by A_j^1.

3) At the end of the first round and in the event of failure in jointly decoding the two messages, i.e., $\overline{A_j^1}$, the decoder uses φ_s^1 to decode the superior message, treating the inferior source's contribution as interference. In doing so, the decoder only utilizes the signal it has received through its s antenna, i.e.,

$$\varphi_s^1(\mathbf{y}_s^1) \triangleq \arg\min_{m_s} \left|\mathbf{y}_s^1 - \mathbf{s}_s^1(m_s)\right|^2. \tag{41}$$

We denote the event that $\varphi_s^1(\mathbf{y}_s^1)$ is different from the actual superior message sent by E_s^1.

4) To decide whether it has correctly decoded the superior message or not, the decoder uses ψ_s^1, where ψ_s^1 outputs a one, if m_s is the *unique* superior message satisfying

$$\left|\mathbf{y}_s^1 - \mathbf{s}_s^1(m_s)\right|^2 \leq T(1+\delta)(|g_{is}|^2\rho + \sigma^2). \tag{42}$$

In (42), δ is some positive value. In any other case, ψ_s^1 outputs a zero. Now, if $\psi_s^1(\mathbf{y}_s^1) = 0$, the decoder requests for a second round of transmission by sending back NACK signals to *both* of the sources. However, if $\psi_s^1(\mathbf{y}_s^1) = 1$, then the decoder sends back an ACK signal to the superior source, declaring $\varphi_s^1(\mathbf{y}_s^1)$ as the decoded superior message, while requesting a second round of transmission for the inferior message by sending back a NACK signal to the inferior source. We denote the event $\psi_s^1(\mathbf{y}_s^1) = 1$ by A_s^1.

5) At the end of the second round and conditioned on successful decoding of the superior message in the first round, i.e., $A_s^1 \cap \overline{A_j^1}$, the decoder declares $\varphi_i^2(\mathbf{Y}^2, \varphi_s^1(\mathbf{y}_s^1))$ as the decoded inferior message where

$$\varphi_i^2\left(\mathbf{Y}^2, \varphi_s^1\left(\mathbf{y}_s^1\right)\right) \triangleq \arg\min_{m_i} \left\|\mathbf{Y}^2 - \mathbf{S}^2\left(\varphi_s^1\left(\mathbf{y}_s^1\right)\right) - \mathbf{S}^2(m_i)\right\|^2. \quad (43)$$

In (43), $\varphi_s^1(\mathbf{y}_s^1)$ is the decoded superior message as given by (41). We denote the event that $\varphi_i^2(\mathbf{Y}^2, \varphi_s^1(\mathbf{y}_s^1))$ is different from the actual inferior message sent by E_i^2.

6) Finally, in case of failure in decoding the superior message at the end of the first round, i.e., $\overline{A_s^1} \cap \overline{A_j^1}$, the decoder declares $\varphi_j^2(\mathbf{Y}^2)$ as the decoded joint message where

$$\varphi_j^2(\mathbf{Y}^2) \triangleq \arg\min_{\mathbf{m}} \|\mathbf{Y}^2 - \mathbf{S}^2(\mathbf{m})\|^2. \quad (44)$$

We denote the event that $\varphi_j^2(\mathbf{Y}^2)$ is different from the actual joint message sent by E_j^2.

Having described the decoder, we next characterize its average error probability P_E. Toward this end, we first notice that

$$P_E \leq P_{E|\overline{E_r}} + P_{E_r}$$

where E_r and $\overline{E_r}$ denote the event that the relaying source makes an error in decoding the inferior message, and its complement, respectively. Since the superior source only starts relaying after the mutual information between its received signal and inferior source's transmitted signal exceeds $R_1T/2$, we have (e.g., refer to [10, Th. 10.1.1])

$$P_{E_r} \leq \epsilon, \qquad \text{for any } \epsilon > 0.$$

This means that

$$P_E \dot{\leq} P_{E|\overline{E_r}}.$$

For the sake of notational simplicity, in the sequel, we denote $P_{E|\overline{E_r}}$ by P_E. To characterize P_E, we write

$$P_E = P_{E_j^1, A_j^1} + P_{E_s^1, A_s^1, \overline{A_j^1}} + P_{E_i^2, \overline{E_s^1}, A_s^1, \overline{A_j^1}} + P_{E_j^2, \overline{A_s^1}, \overline{A_j^1}}. \quad (45)$$

To understand (45), note that the first two terms correspond to making an *undetected* error in decoding one or both of the messages at the end of the first round, while the last two terms correspond to making a decoding error after requesting for two rounds of transmission. Next, we upper bound (45) by

$$P_E \leq P_{E_j^1, A_j^1} + P_{E_s^1, A_s^1} + P_{E_i^2, \overline{E_s^1}} + P_{E_j^2, \overline{A_s^1}}. \quad (46)$$

We start evaluating (46) by characterizing $P_{E_j^1, A_j^1}$. By examining the definitions for events E_j^1 and A_j^1 [refer to (39) and (40)], and through an argument similar to the one given for (14), we get

$$P_{E_j^1, A_j^1|g,h} \leq \Pr\{\|\mathbf{N}^1\|^2 > 2T(1+\delta)\sigma^2\}$$

which for large enough T gives [compare to (16)]

$$P_{E_j^1, A_j^1|g,h} \leq \epsilon, \qquad \text{for any } \epsilon > 0$$

or

$$P_{E_j^1, A_j^1} \leq \epsilon, \qquad \text{for any } \epsilon > 0. \quad (47)$$

Next, we characterize $P_{E_s^1, A_s^1}$. Toward this end, we first fix a channel realization. Then, through examining the definitions for events E_s^1 and A_s^1 [refer to (41) and (42)], and by pursuing the same steps which led to (14), we get

$$P_{E_s^1, A_s^1|g,h} \leq \Pr\left\{\left|\mathbf{s}_s^1(m_i) + \mathbf{n}_s^1\right|^2 > T(1+\delta)(\left|g_{is}\right|^2\rho + \sigma^2)\right\}$$

where m_i denotes the actual inferior message sent and $\mathbf{s}_s^1(m_i)$ represents its signature, at the end of the first round, at the superior antenna. Realizing that, conditioned on a certain channel realization, $|\mathbf{s}_s^1(m_i) + \mathbf{n}_s^1|^2$ has a central chi-squared distribution with $2T$ degrees of freedom, we conclude that for large enough T, we have [compare to (16)]

$$P_{E_s^1, A_s^1|g,h} \leq \epsilon, \qquad \text{for any } \epsilon > 0$$

or

$$P_{E_s^1, A_s^1} \leq \epsilon, \qquad \text{for any } \epsilon > 0. \quad (48)$$

In other words, (47) and (48) mean that, through using long enough codes, one can make the probability of making undetected errors arbitrarily small. Note that for doing so, the bounded distance decoder does not employ any kind of cyclic redundancy check (CRC) techniques. Now, using (46)–(48), we conclude

$$P_E \dot{\leq} P_{E_i^2, \overline{E_s^1}} + P_{E_j^2, \overline{A_s^1}}. \quad (49)$$

To characterize $P_{E_i^2, \overline{E_s^1}}$, we first fix a channel realization and then write

$$\begin{aligned} P_{E_i^2, \overline{E_s^1}|g,h} &= P_{\overline{E_s^1}|g,h} P_{E_i^2|\overline{E_s^1}, g,h} \\ &\leq P_{E_i^2|\overline{E_s^1}, g,h}. \end{aligned} \quad (50)$$

Now, using (43), it is straightforward to verify that (refer to [1])

$$\begin{aligned} P_{PE_i^2|\overline{E_s^1}, g,h} \leq &\left(1 + \frac{1}{2}\rho(|g_{is}|^2 + |g_{ii}|^2)\right)^{-(T+T')} \\ &\times \left(1 + \frac{1}{2}\rho(|g_{ss}|^2 + |g_{si}|^2 + |g_{is}|^2 + |g_{ii}|^2)\right. \\ &\left. + \frac{1}{4}\rho^2|g_{ss}g_{ii} - g_{si}g_{is}|^2\right)^{-(T-T')} \end{aligned} \quad (51)$$

where T' is the number of symbol intervals, in the second round, that the superior source needs to listen to the inferior one, before decoding its message, i.e.,

$$T' \triangleq \min\left\{T, \left\lceil \frac{TR_1}{2\log_2(1+|h|^2\rho)} \right\rceil\right\}. \quad (52)$$

We can further use (51) to write

$$\begin{aligned} P_{PE_i^2|\overline{E_s^1}, g,h} \leq &\left(1 + \frac{1}{2}\rho(|g_{is}|^2 + |g_{ii}|^2)\right)^{-(T+T')} \\ &\times \left(1 + \frac{1}{2}\rho(|g_{ss}|^2 + |g_{si}|^2 + |g_{is}|^2 + |g_{ii}|^2)\right)^{-(T-T')}. \end{aligned} \quad (53)$$

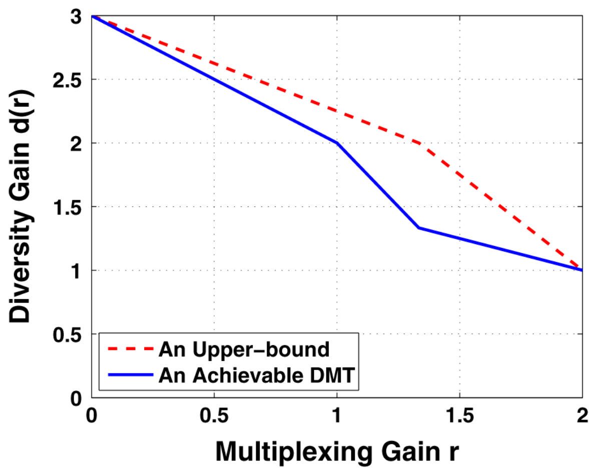

Fig. 2. DMT achieved by the ARQ-DDF protocol in the CVMA channel, along with an upper bound on the achievable DMT ($L = 2$).

We notice that in deriving (53) from (51), we have ignored the term $\frac{1}{4}\rho^2|g_{ss}g_{ii} - g_{si}g_{is}|^2$. As a consequence, the resulting upper bound may be loose, specially since the ignored term is of second order with respect to ρ. However, this is indeed necessary for the sake of analysis tractability (we notice that the technique developed in [8, Th. 1] to deal with a similar term does not apply here since g_{ss}, g_{si}, g_{is}, and g_{ii} are not Gaussian distributed). Characterization of $P_{E_i^2,\overline{E_s^1}}$ from (53) is straightforward and follows the techniques developed in [1], therefore we only report the final result here (please refer to [7] for the details)

$$P_{E_i^2,\overline{E_s^1}} \dot{\leq} \rho^{-d_i(r_1)} \tag{54}$$

where

$$d_i(r_1) = \begin{cases} 3 - r_1, & 1 > r_1 \geq 0 \\ \dfrac{2(4-r_1)}{2+r_1}, & 2 \geq r_1 \geq 1. \end{cases} \tag{55}$$

Likewise, characterization of the second term of (49), i.e., $P_{E_j^2,\overline{A_s^1}}$, follows the techniques outlined in [1] and is detailed in [7]

$$P_{E_j^2,\overline{A_s^1}} \dot{\leq} \rho^{-d_{s,j}(r_1)} \tag{56}$$

where

$$d_{s,j}(r_1) = \begin{cases} 4 - 2r_1, & \frac{4}{3} > r_1 \geq 0 \\ 2 - \dfrac{r_1}{2}, & 2 \geq r_1 \geq \frac{4}{3}. \end{cases} \tag{57}$$

Now, (49), together with (54) and (56), gives

$$d_{\mathrm{DDF-CVMA}}(r_1, 2) \geq \min\{d_i(r_1), d_{s,j}(r_1)\}$$

where $d_{\mathrm{DDF-CVMA}}(r_1, 2)$ denotes the diversity gain achieved by the protocol. Using (55), (57), and (6), we conclude

$$d_{\mathrm{DDF-CVMA}}(r_e, 2) \geq \begin{cases} 3 - r_e, & 1 > r_e \geq 0 \\ 4 - 2r_e, & \frac{4}{3} > r_e \geq 1 \\ 2 - \dfrac{r_e}{2}, & 2 > r_e \geq \frac{4}{3}. \end{cases} \tag{58}$$

The last step in proving the achievability of (35) is arguing the existence of a code in the ensemble that performs at least as well as the average, thus achieving (58). This completes the proof of the achievability part.

The key observation in proving the asymptotic optimality part (which for brevity reasons is not reported here) is that as L grows to infinity, the dominant error becomes the event when one of the users is decoded successfully, while the other one remains in error, even after L rounds of transmission. The asymptotic optimality then follows from the fact that, as L increases, the user in error gets a progressively better chance for being helped by the other one (please refer to [7] for a detailed proof).

Fig. 2 compares the upper and lower bounds in (34) and (35) for $L = 2$. Clearly, this figure shows the full diversity and full rate properties of the proposed ARQ-DDF protocol. Finally, we comment that the analysis of the ARQ-DDF protocol in Theorem 4 can be repeated for $L > 2$.

IV. Conclusion

In this correspondence, we combined the DDF protocol with the ARQ mechanism to develop efficient cooperation schemes for the MAR and CVMA channels. The proposed ARQ-DDF protocol was shown to achieve the optimal DMT for the MAR channel. In the CVMA scenario, we argued that the ARQ-DDF protocol achieves significant cooperative diversity gains while exploiting all of the channel's degrees of freedom and despite the half duplex constraint.

References

[1] K. Azarian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4152–4172, Dec. 2005.

[2] P. Mitran, H. Ochiai, and V. Tarokh, "Space-time diversity enhancements using collaborative communications," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2041–2057, Jun. 2005.

[3] M. Katz and S. Shamai, "Transmitting to colocated users in wireless ad hoc and sensory networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3540–3563, Oct. 2005.

[4] A. Murugan, K. Azarian, and H. El Gamal, "Cooperative lattice coding and decoding in half-duplex channels," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 268–279, Feb. 2007.

[5] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple antenna channels," *IEEE Trans. Inf. Theory* vol. 49, no. 5, pp. 1073–1096, May 2003.

[6] H. El Gamal, G. Caire, and M. O. Damen, "The MIMO ARQ channel: Diversity-multiplexing-delay tradeoff," *IEEE Trans. Inf. Theory*, vol. 52, no. 8, pp. 3601–3621, Aug. 2006.

[7] K. Azarian, "Outage limited cooperative channels: Protocols and analysis" Ph.D. dissertation, Dept. Electr. Comput. Eng., The Ohio State University, Columbus, OH, Aug. 2006 [Online]. Available: http://www.ohiolink.edu/etd)

[8] D. Chen, K. Azarian, and J. N. Laneman, "A case for amplify-forward relaying in the block-fading multiaccess channel," *IEEE Trans. Inf. Theory*, accepted for publication.

[9] D. N. C. Tse, P. Viswanath, and L. Zheng, "Diversity-multiplexing tradeoff in multiple access channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 9, pp. 1859–1874, Sep. 2004.

[10] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

[11] K. Azarian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half duplex cooperative channels," presented at the Allerton Conf. Commun. Control Comput., Monticello, IL, Oct. 2004.

[12] Y. Nam, K. Azarian, H. El Gamal, and P. Schniter, "Cooperation through ARQ," in *Proc. IEEE 6th Workshop Signal Process. Adv. Wireless Commun.*, New York, Jun. 5–8, 2005, pp. 1023–1027.

[13] K. Azarian and H. El Gamal, "Cooperation in outage-limited multiple-access channels," presented at the IEEE Int. Zurich Seminar Commun., Zurich, Switzerland, 2006.