Efficient Multi-Carrier Communication over Doubly Spread Channels

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June 2009

(Joint work with Mr. Sungjun Hwang and Dr. Sib Das)

Outline:

This talk focuses on multicarrier communication over doubly spread channels:

- Modulation/demodulation for ICI shaping:
  - motivation,
  - max-SINR design,
  - performance.
- Receiver architectures for doubly spread channels:
  - turbo reception,
  - noncoherent equalization,
  - tree search,
  - sparsity.

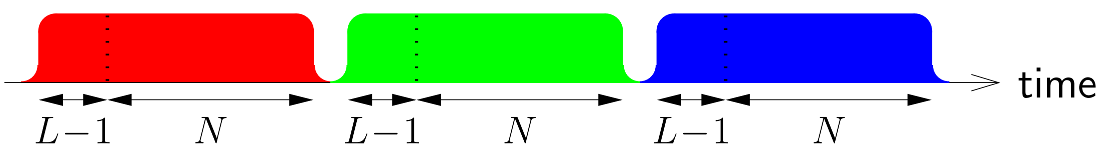
CP-OFDM:

Principal advantage:

- Low-complexity demod with delay-spreading (i.e., freq selective) chans.

Some disadvantages:

- Sensitive to Doppler-spreading (i.e., time selective) channels.
- Loss of spectral efficiency due to the insertion of guards.
  What if we increased N relative to L (i.e., $P \triangleq \frac{N}{L} \gg 4$)?
  - Complexity increases to $1 + \log_2 P + \log_2 L \ \ \frac{\text{mults}}{\text{QAM symbol}}$...not bad.
  - Reduced subcarrier spacing $\Rightarrow$ more sensitive to Doppler spread!
- Slow spectral roll-off causes interference to adjacent-band systems.
  Improves with raised-cosine pulse, but at further loss in efficiency:



- High peak-to-average power ratio (PAPR).

Question:

Can we fix CP-OFDM's

- *sensitivity to Doppler spread*
- *loss in spectral efficiency*, and
- *slow spectral roll-off*,

without spoiling its $\mathcal{O}(\log_2 L)\ \frac{\text{mults}}{\text{QAM symbol}}$ complexity scaling?

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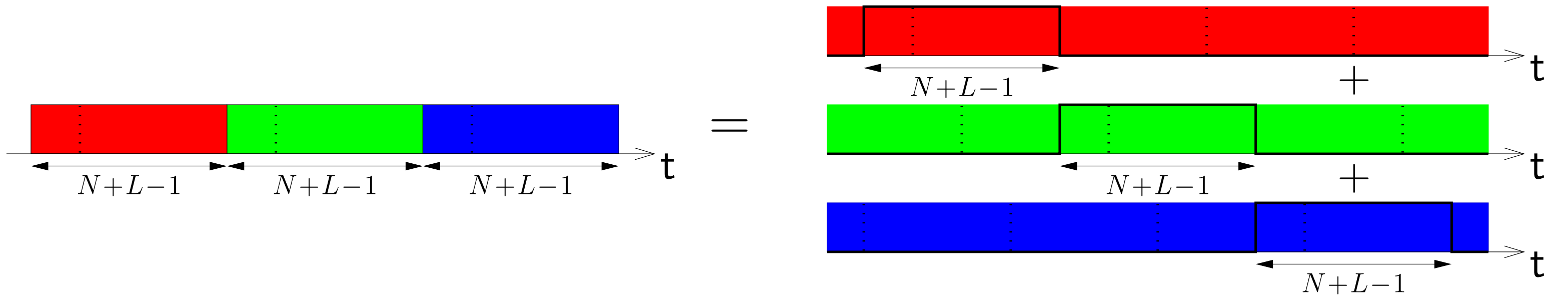
Yes!

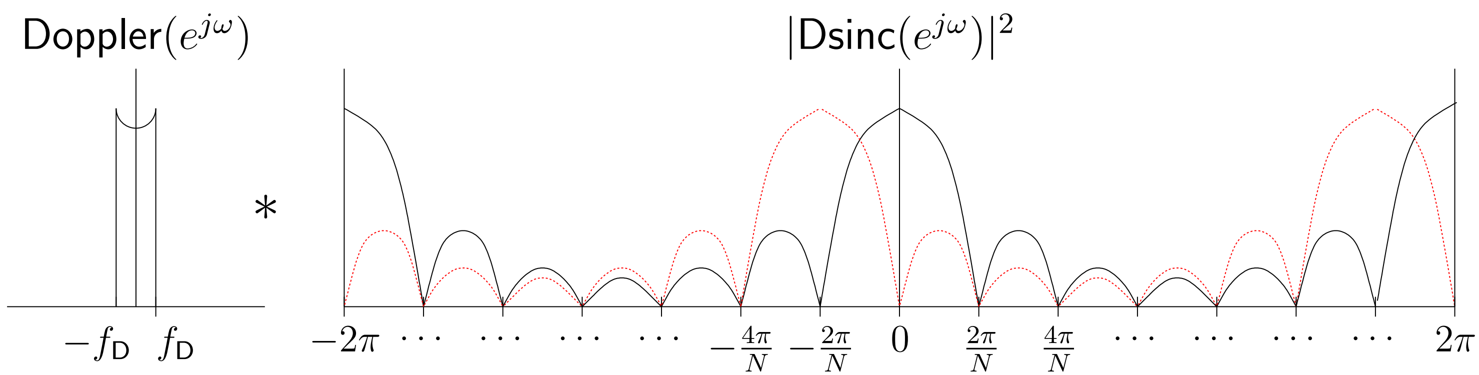
Re-think the role of "pulse shaping" in multi-carrier modulation...

Rectangular Pulses:

A standard CP-OFDM symbol can be recognized as a sum of N infinite-length complex exponentials windowed by a rectangular pulse of width $N+L-1$.

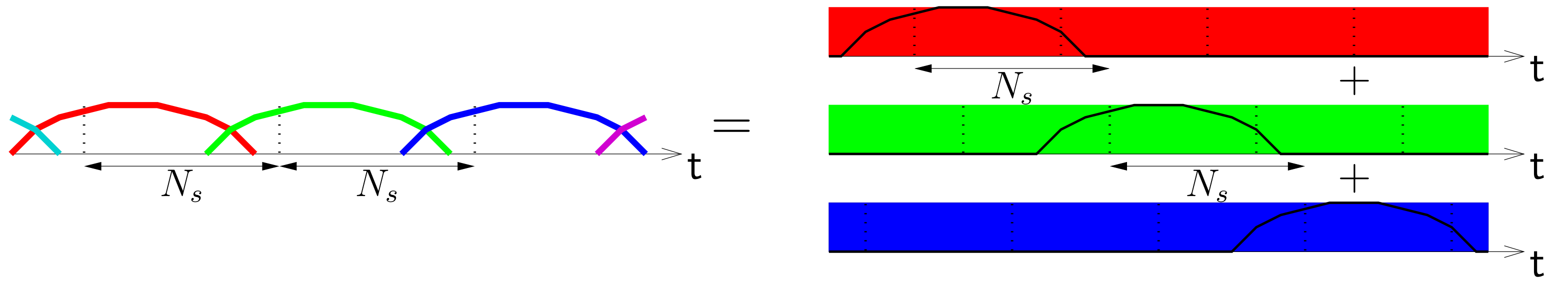


$\Rightarrow$ *Dirichlet sinc* in DTFT domain, whose slow side-lobe decay causes high sensitivity to Doppler spreading:



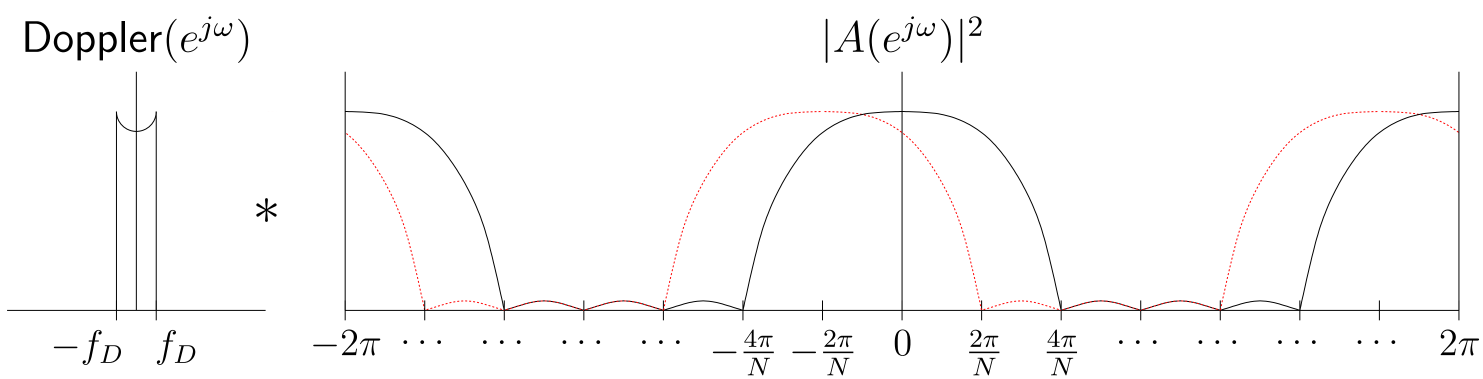
Smooth Overlapping Pulses:

What if we applied a *smooth* window instead?



The main-lobe may be *wider* but the sidelobes *decay more quickly*.

Thus, possibly stronger interference from adjacent subcarriers, but much less interference from all other subcarriers, even under large Doppler spreads:



Non-(Bi)Orthogonal FDM:

The benefits of (Bi)Orthogonal FDM are realized only when

- the channel varys slowly enough, and
- spectral efficiency is appropriately reduced.

With a properly-designed *Non-Orthogonal FDM*, we can

- tolerate large delay and Doppler spreads, and
- communicate at Nyquist rate (or above),

by *allowing*

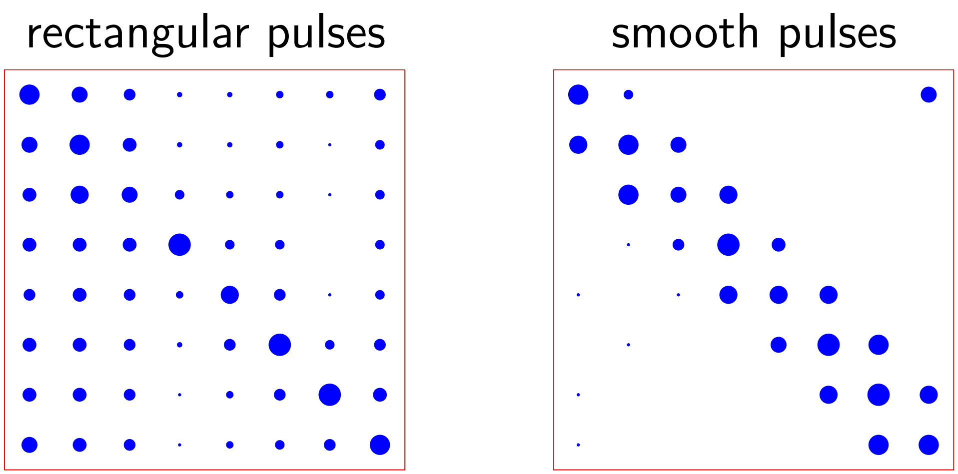
- a short span of ISI/ICI,

which can be handled by near-optimal, yet low-complexity, equalization.

Thus, we advocate *ISI/ICI shaping rather than ISI/ICI suppression*.

Visualizing the Frequency-Domain Channel Matrix:

A toy example under large Doppler spread:



Dot size proportional to log-magnitude of ICI coefficient.

Smooth Overlapping Pulses:

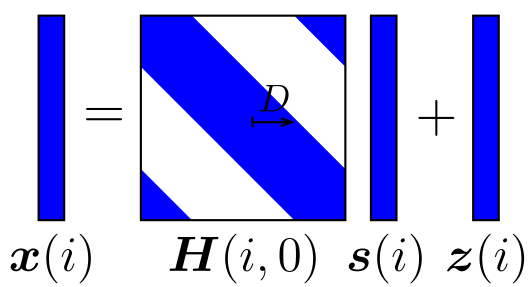
Challenge: The use of smooth overlapping pulses potentially causes both *inter-carrier interference* (ICI) and *inter-symbol interference* (ISI):

$$\boldsymbol{x}(i) = \sum_{q=-\infty}^{\infty} \underbrace{\boldsymbol{H}(i,q)}_{\text{subcarrier coupling matrix}} \boldsymbol{s}(i-q) + \boldsymbol{z}(i).$$

Difficult to equalize!

One solution: Design the pulse shapes with the goal of...

1. Completely suppressing ISI: $\boldsymbol{H}(i,q)\big|_{q\neq 0} = \boldsymbol{0}$.
2. Allowing ICI only within a radius of $D \ll N$ subcarriers. (Often $D = 1$.)



Not difficult to equalize.

Receiver Pulse-Shaping:

Though so far we've considered a non-rectangular *transmission pulse* $\{a_n\}$,

$$t_n = \sum_{i=-\infty}^{\infty} a_{n-iN_s} \sum_{k=0}^{N-1} s_k(i) e^{j\frac{2\pi}{N}kn}, \qquad n = -\infty \ldots \infty,$$

we can use, in addition, a non-rectangular *reception pulse* $\{b_n\}$:

$$x_k(i) = \sum_{n=-\infty}^{\infty} r_{n-iN_s}\, b_n\, e^{-j\frac{2\pi}{N}kn}, \qquad k = 0 \ldots N-1.$$

Above, N_s specifies the OFDM symbol period.

- Modulation efficiency $\eta \triangleq \frac{N}{N_s} \frac{\text{QAM symbols}}{\text{sec Hz}}$
- For OFDM, $N_s = N+L-1$, but now there is *no constraint* on N_s!

We focus on $N_s = N$ $\Leftrightarrow$ *no guard interval* $\Leftrightarrow$ $\eta = 1$.

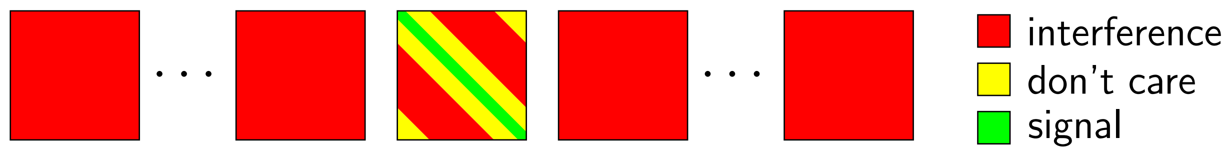
Pulse Design to Maximize SINR:

Writing the received signal energy components due to

$$\mathcal{E}_s = \sum_{(q,k,l)\in {\color{green}\blacksquare}} \mathrm{E}\{|H_{k,l}(\cdot,q)|^2\} \quad \text{“signal” and}$$

$$\mathcal{E}_i = \sum_{(q,k,l)\in {\color{red}\blacksquare}} \mathrm{E}\{|H_{k,l}(\cdot,q)|^2\} \quad \text{“interference” (ISI+ICI)}$$

where $\{\boldsymbol{H}(\cdot,q)\} =$



we can write $\quad \text{SINR} = \dfrac{\mathcal{E}_s}{\mathcal{E}_i + \mathcal{E}_n} = \dfrac{\boldsymbol{a}^H \boldsymbol{P}_1(\boldsymbol{b})\boldsymbol{a}}{\boldsymbol{a}^H \boldsymbol{P}_2(\boldsymbol{b})\boldsymbol{a}} = \dfrac{\boldsymbol{b}^H \boldsymbol{P}_3(\boldsymbol{a})\boldsymbol{b}}{\boldsymbol{b}^H \boldsymbol{P}_4(\boldsymbol{a})\boldsymbol{b}}$

where

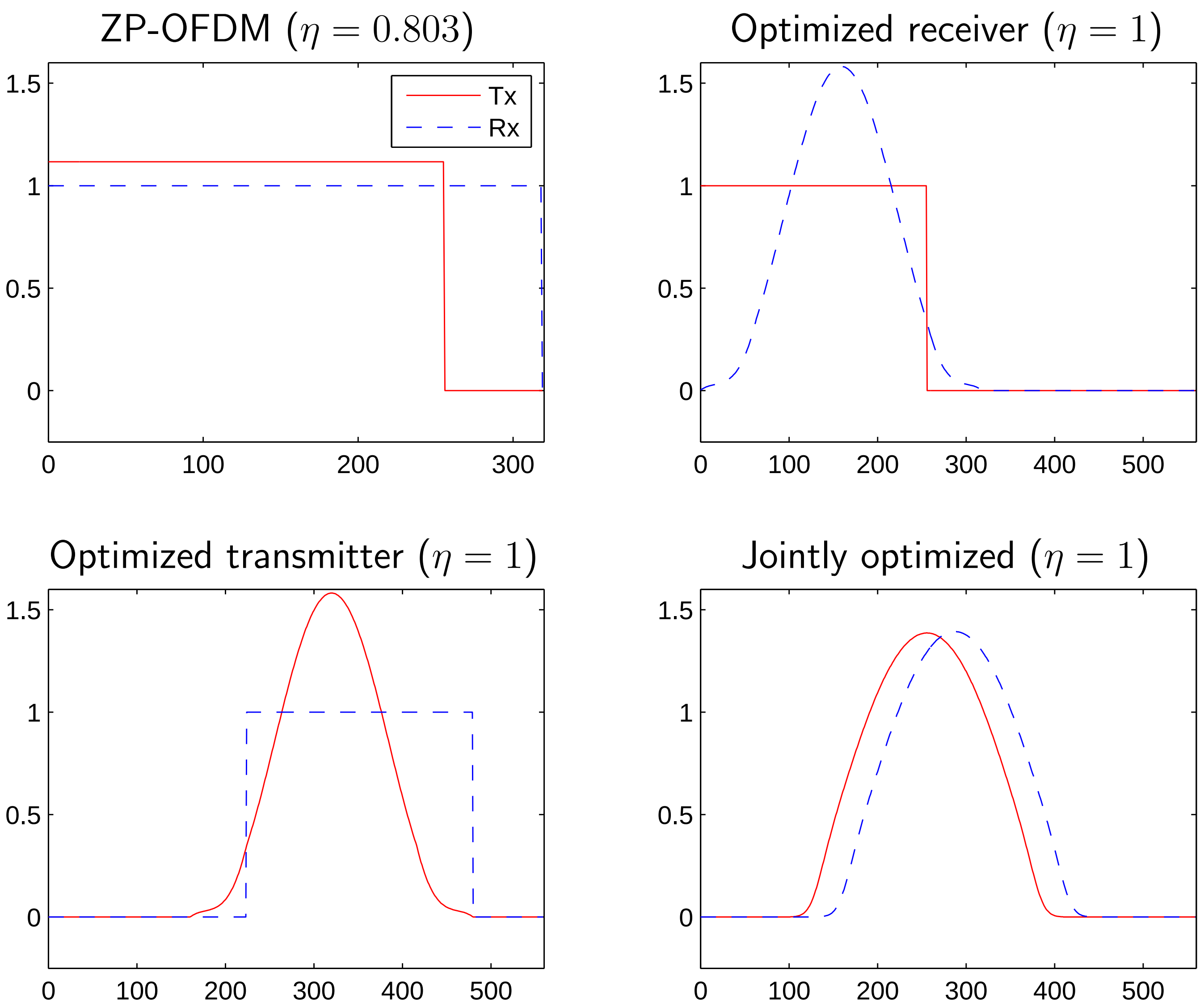
$\boldsymbol{a} =$ transmission pulse coefficients

$\boldsymbol{b} =$ reception pulse coefficients

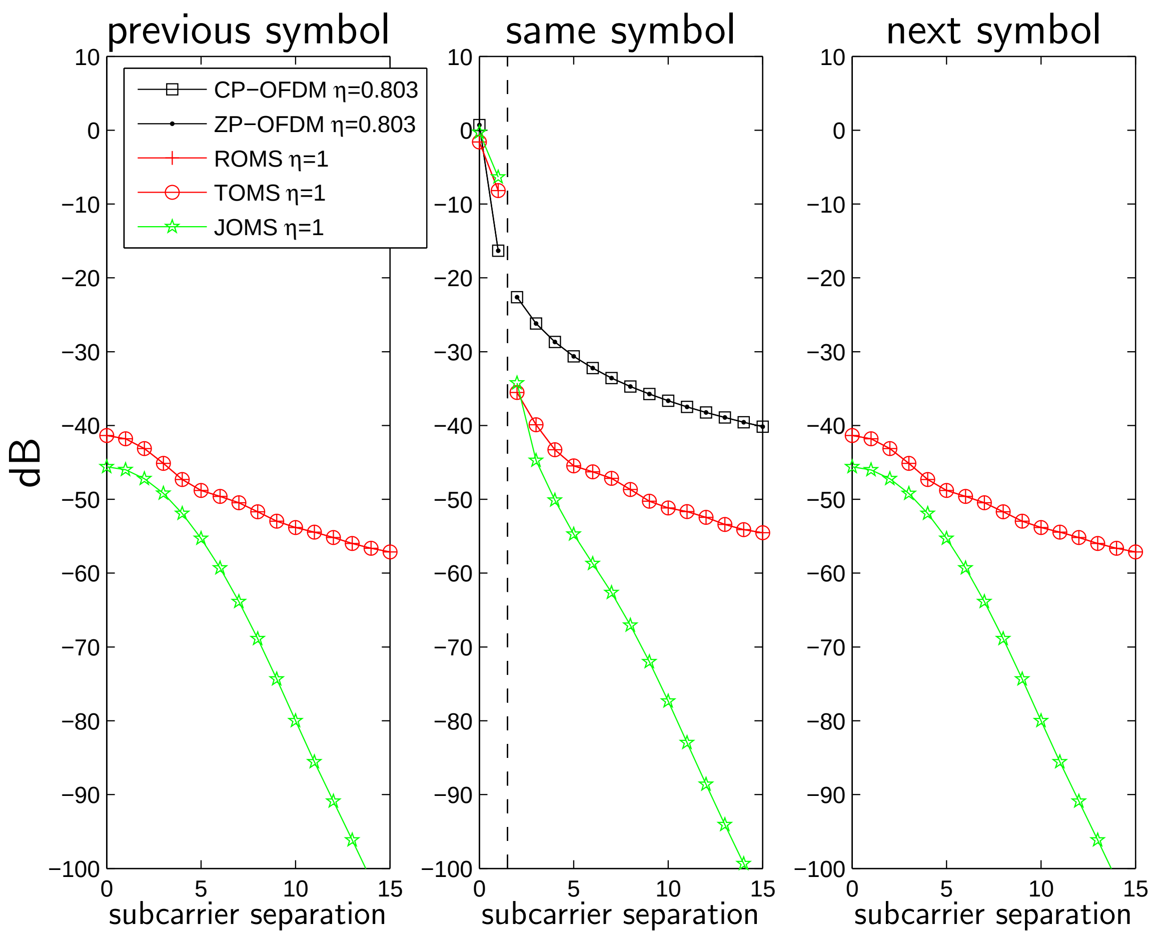
$\boldsymbol{P}_1(\cdot),\boldsymbol{P}_2(\cdot),\boldsymbol{P}_3(\cdot),\boldsymbol{P}_4(\cdot) =$ matrices dependent on scattering fxns & SNR.

$\Rightarrow$ *SINR-maximizing pulses are generalized eigenvectors.*

Max-SINR Pulse Examples:



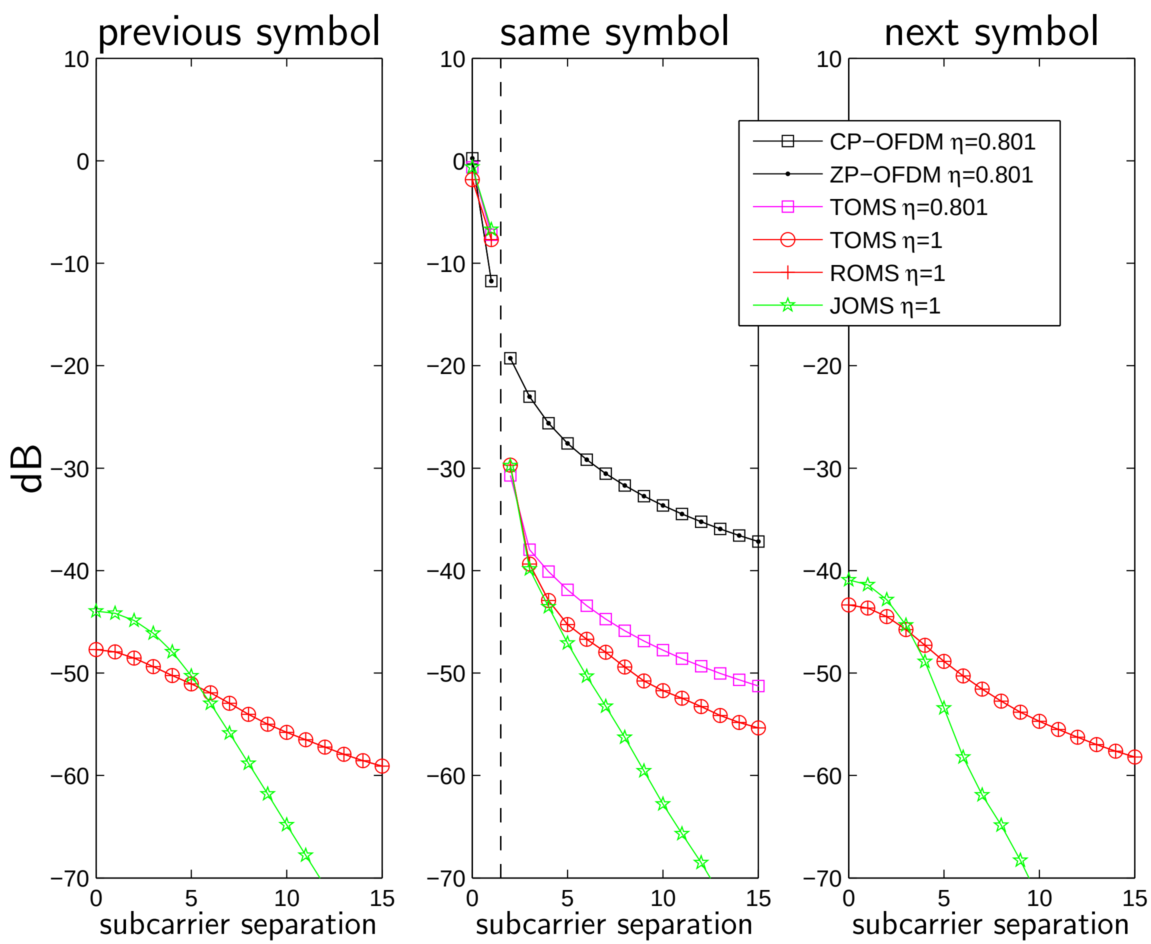
ISI/ICI Energy Profiles:



$D = 1$, SNR $= 15$dB, $L = 64$, $f_{\mathsf{D}} T_c = 7.6 \times 10^{-4}$, $N = 256$, Jakes.

(For example, RF: $f_c = 20$GHz, BW=3MHz, $T_h = 5.4\mu$s, $v = 120$km/hr.)

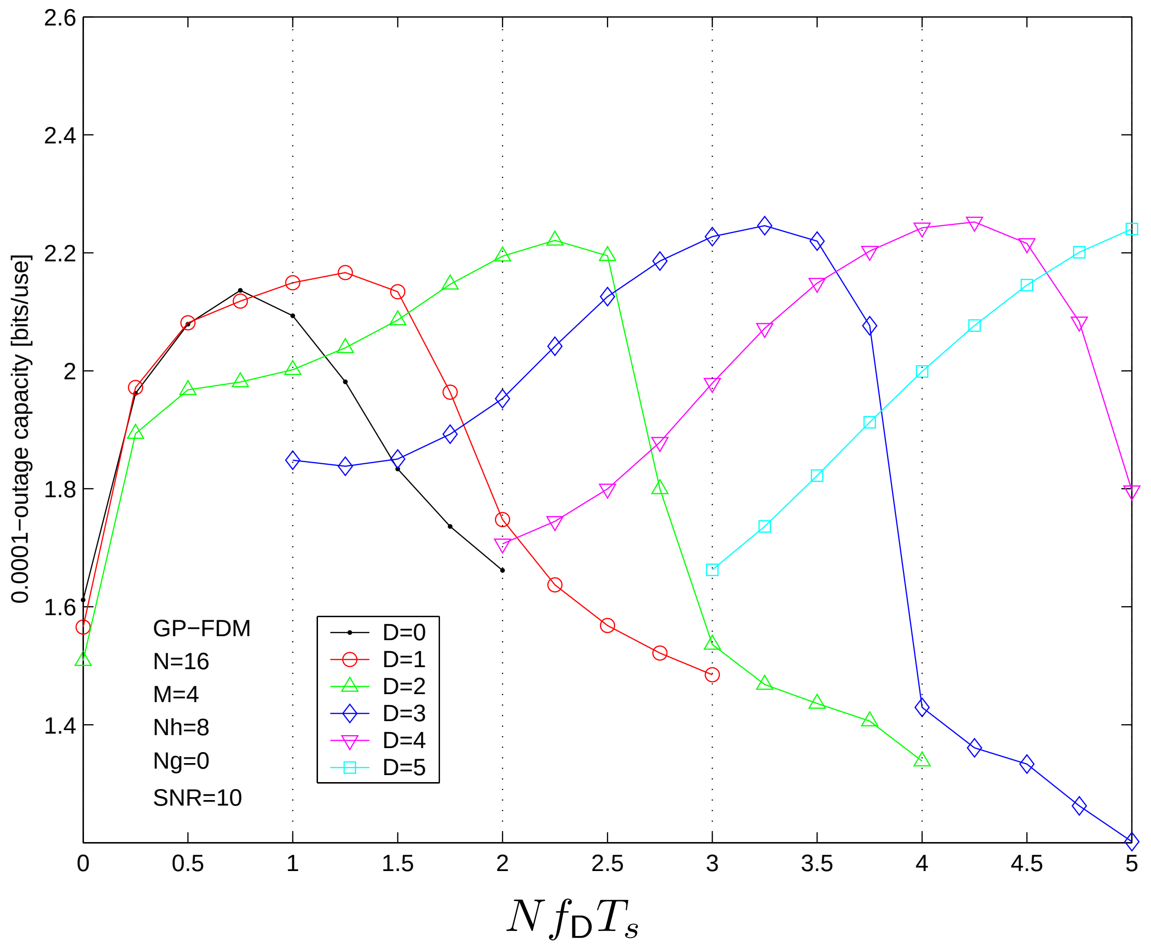
ISI/ICI Energy Profiles:



$D = 1$, SNR $= 30$dB, $L = 128$, $f_{\mathsf{D}} T_c = 7.6 \times 10^{-4}$, $N = 512$, Jakes.

(For example, UW: $f_c = 13$kHz, BW$=10$kHz, $T_h = 7$ms, $f_d = 15$Hz.)

Outage Capacity vs $N f_D T_s$ for various ICI-radii D:



- The outage-capacity optimal D obeys $D \approx \lfloor N f_D T_s \rceil$!
- ICI shaping is better than ICI suppression when $2 f_D T_s \geq \frac{1}{N}$.

Equalization of ICI:

Coherent approaches (i.e., known channel): $\frac{\text{mults}}{\text{QAM symbol}}$

1. Viterbi MLSE [Matheus/Kammeyer GLOBE 97] $\mathcal{O}(|\mathcal{S}|^D D)$
2. Soft Iterative [Das/Schniter Asilomar 04] $\mathcal{O}(D^2)$
3. Linear MMSE [Rugini/Banelli/Leus SPL 05] $\mathcal{O}(D^2)$
4. MMSE DFE [Rugini/Banelli/Leus SPAWC 05] $\mathcal{O}(D^2)$
5. Tree Search MLSD [Hwang/Schniter SPAWC 06] $\mathcal{O}(D^2)$

Non-coherent approaches (i.e., unknown channel):

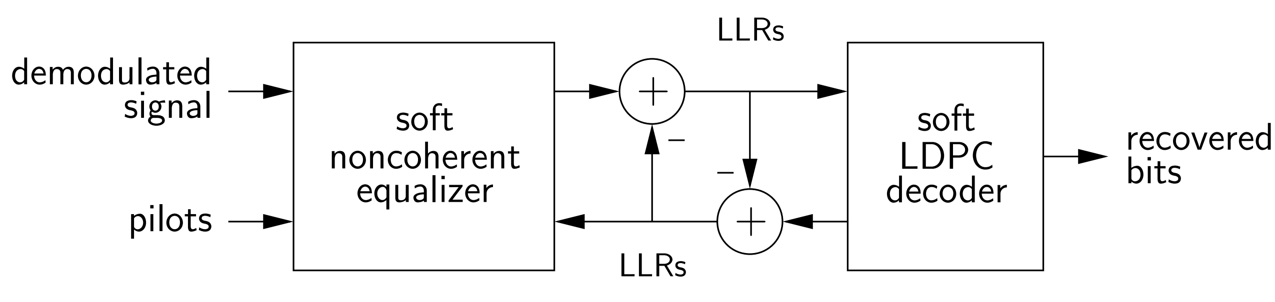
1. MLSD [Hwang/Schniter WUWNet 07] $\mathcal{O}(D^2 L^2)$
2. Soft MAP-Inspired [Hwang/Schniter Asilomar 07] $\mathcal{O}(D^2 L^2)$
3. Soft EM-Inspired [Hwang/Schniter SPAWC 09] $\mathcal{O}(D \log L)$

Noncoherent Turbo Equalization:

- Large performance gains are possible through the use of sophisticated coding schemes (e.g., LDPC).
- For complexity reasons, noncoherent decoding is split into
  1. *noncoherent equalization*, which leverages channel structure,
  2. *decoding*, which leverages the code structure.
- By *iterating* the two steps ("turbo equalization"), we hope to get

  near-optimal noncoherent decoding with practical complexity. ☺

  Note: Doing so requires *soft* equalization (and *soft* decoding).



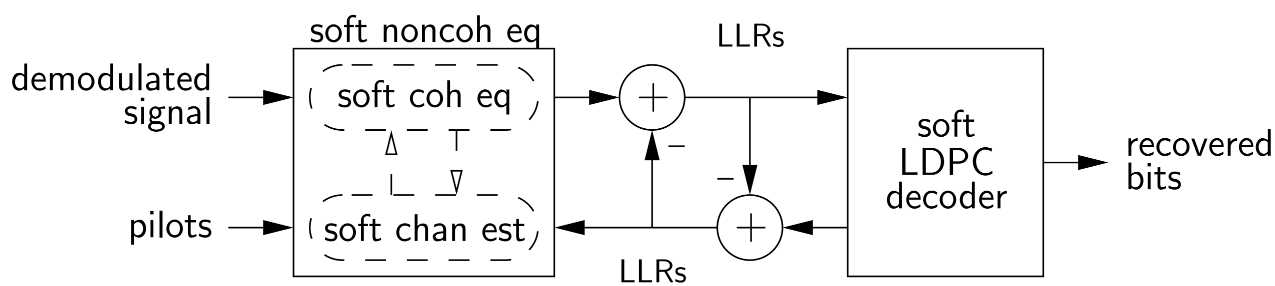
Soft Noncoherent Equalization:

By "soft noncoherent equalization" we mean

computing coded-bit LLRs in the presence of an unknown channel.

Several approaches:

1. Joint equalization/chan-est (MAP inspired)
2. Iterative equalization & chan-est (EM inspired)
3. Iterative equalization & chan-est (ad hoc)
4. Non-iterative equalization (with pilot-aided channel estimation)



1) MAP-Inspired Soft Noncoherent Equalization:

The soft equalizer needs to calculate

$$L_e(b_k|\boldsymbol{x}) = \ln \frac{\sum_{\boldsymbol{b}:\, b_k=1} \exp \mu_{\text{MAP}}(\boldsymbol{b})}{\sum_{\boldsymbol{b}:\, b_k=0} \exp \mu_{\text{MAP}}(\boldsymbol{b})} - L_a(b_k) \qquad \text{"extrinsic LLR"}$$

$$\approx \max_{\boldsymbol{b}\in\mathcal{B}|b_k=1} \mu_{\text{MAP}}(\boldsymbol{b}) - \max_{\boldsymbol{b}\in\mathcal{B}|b_k=0} \mu_{\text{MAP}}(\boldsymbol{b}) - L_a(b_k) \qquad \text{"max-log"}$$

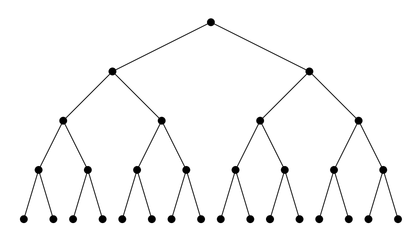
with $\mathcal{B} \triangleq$ set of M most probable coded-bit vectors $\boldsymbol{b}$.

Can find $\mathcal{B}$ via tree search, by recursively updating MAP metric $\mu_{\text{MAP}}(\boldsymbol{b})$:

$$-\mu_{\text{MAP}}(\boldsymbol{b}) = \tfrac{1}{\sigma^2}\|\boldsymbol{x} - \boldsymbol{B}\boldsymbol{F}\hat{\boldsymbol{\theta}}_{\boldsymbol{b}}\|^2 + \hat{\boldsymbol{\theta}}_{\boldsymbol{b}}^H \boldsymbol{R}_\theta^{-1}\hat{\boldsymbol{\theta}}_{\boldsymbol{b}} + \ln(\pi^N|\boldsymbol{\Phi}_{\boldsymbol{b}}|) - \sum_{k:\, b_k=1} L_a(b_k)$$

where $\hat{\boldsymbol{\theta}}_{\boldsymbol{b}} \triangleq$ per-survivor MMSE estimate of basis expansion coefficients $\boldsymbol{\theta}$.

Basis expansion $\boldsymbol{F}$ constructed to exploit sparsity.



Complexity $= 2NN_\theta^2|\mathcal{S}|M$ mults per OFDM symbol.

Sparsity Tracking & Pilots:

Pilots are used for

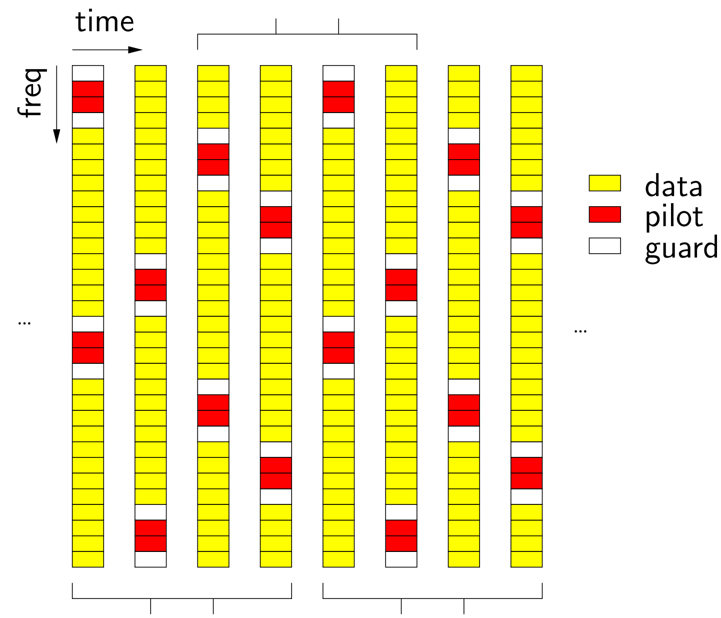
1. channel estimation (along with surviving/soft symbol hypotheses)
2. tracking the sparse delay-power profile (DPP).

MMSE channel estimation:

- uses pilots from P MCM symbols

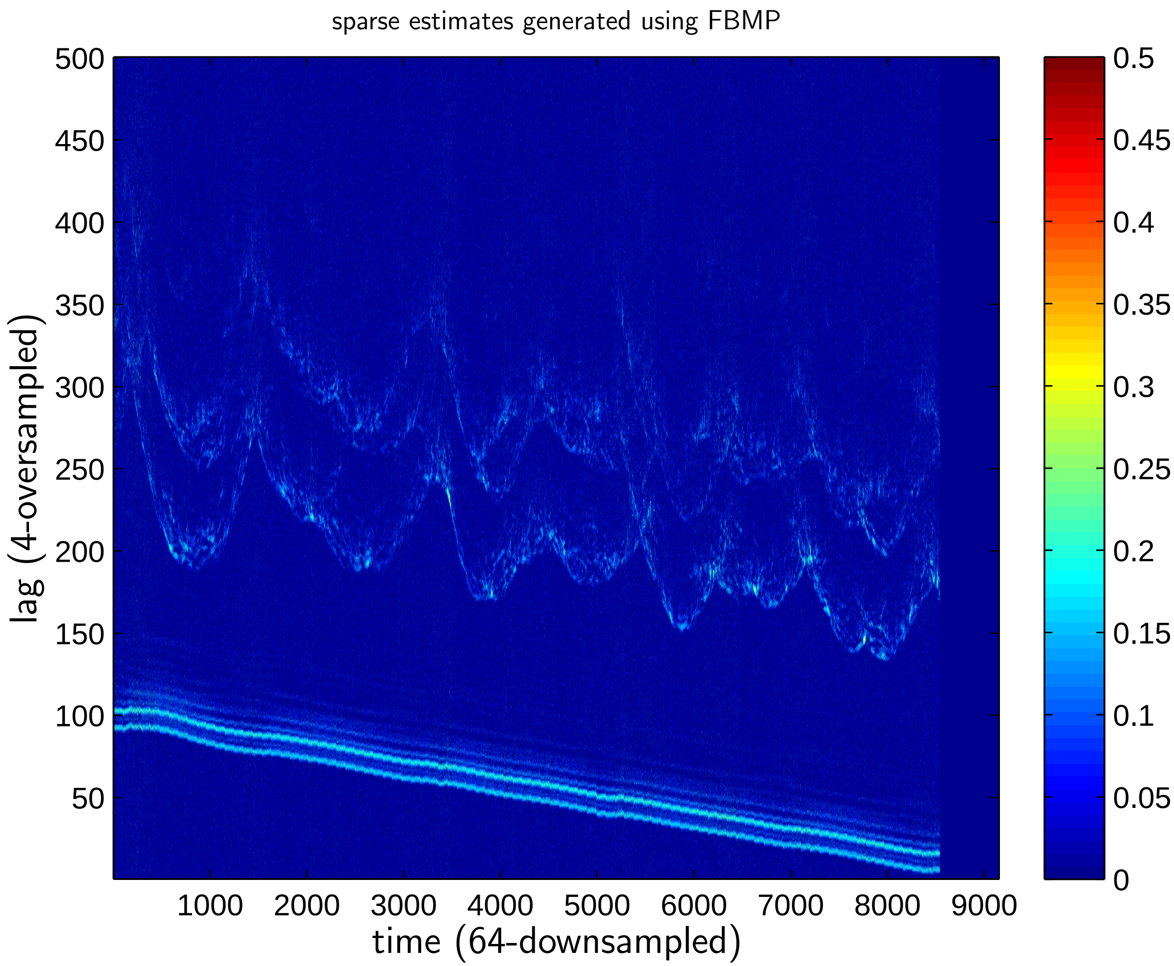
DPP tracking:

- simple: threshold MMSE chan-est
- better: sparse reconstruction



$P = 4,\ K = 2$

Underwater Acoustic Channel — Impulse Response:



<u>wideband scale:</u>

a_{sft}: .00004

a_{spd}: .0013

<u>Doppler [Hz]:</u>

f_{sft}: .35 to .78

f_{spd}: 10 to 23

<u>Delay [ms]:</u>

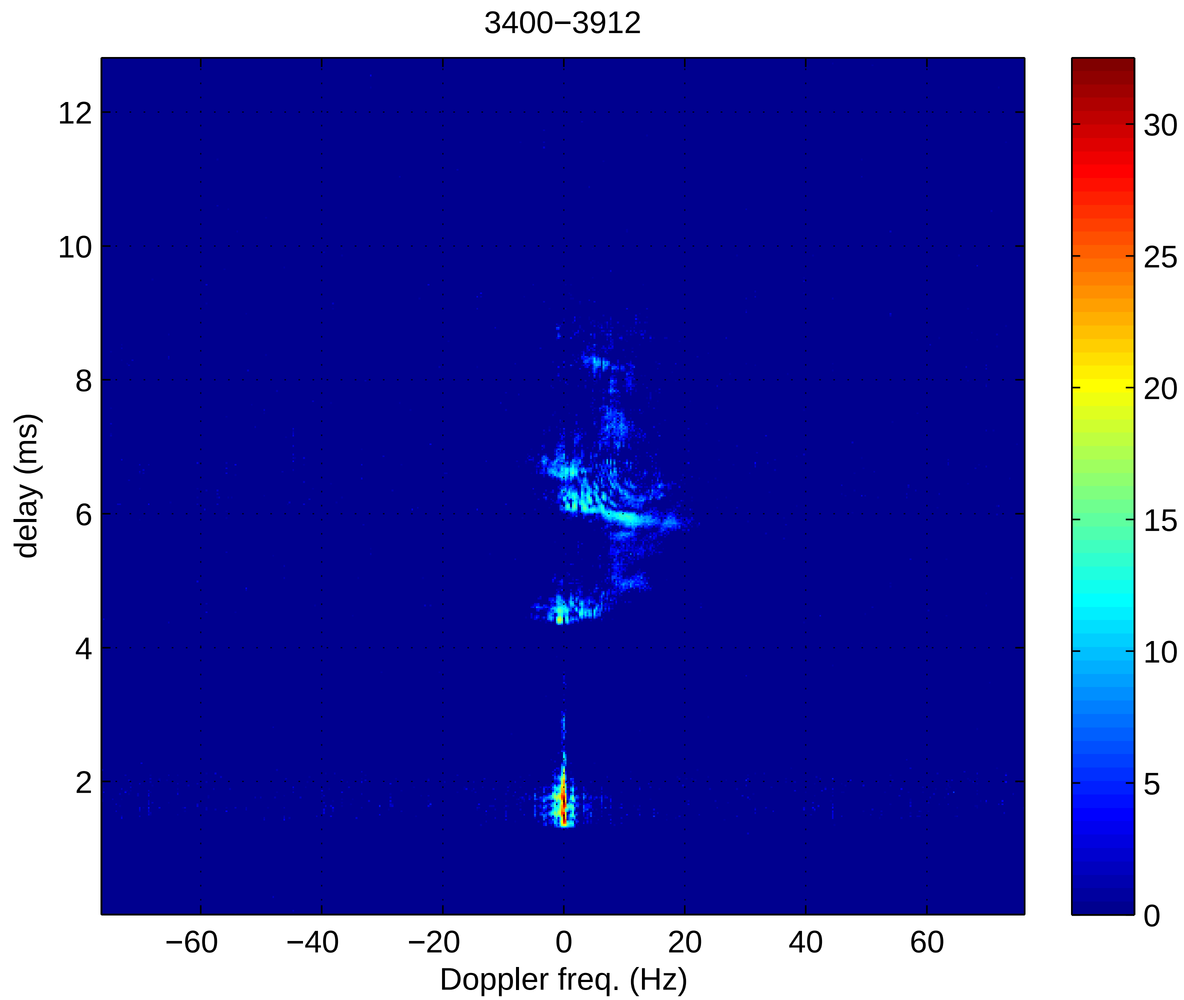
T_h: 8

<u>Product:</u>

$2f_{\mathsf{spd}}T_h$: .16 to .32

SPACE-08 experiment, 60m, 8kHz-18kHz

Underwater Acoustic Channel — Scattering Function:



Doppler spread positive for this time span (but not in general).

Numerical Simulations:

Motivated by "surf zone" channel from

J. C. Preisig and G. Deane, "Surface wave focusing and acoustic communications in the surf zone," *J. Acoust. Soc. Am.*, vol. 116, pp. 2067-2080, Oct. 2004

- delay/Doppler spread: 7ms/30Hz (N_h =50, $f_\text{D}T_c$=0.002)
- 5 large taps, 45 small taps (with 2% of total energy)

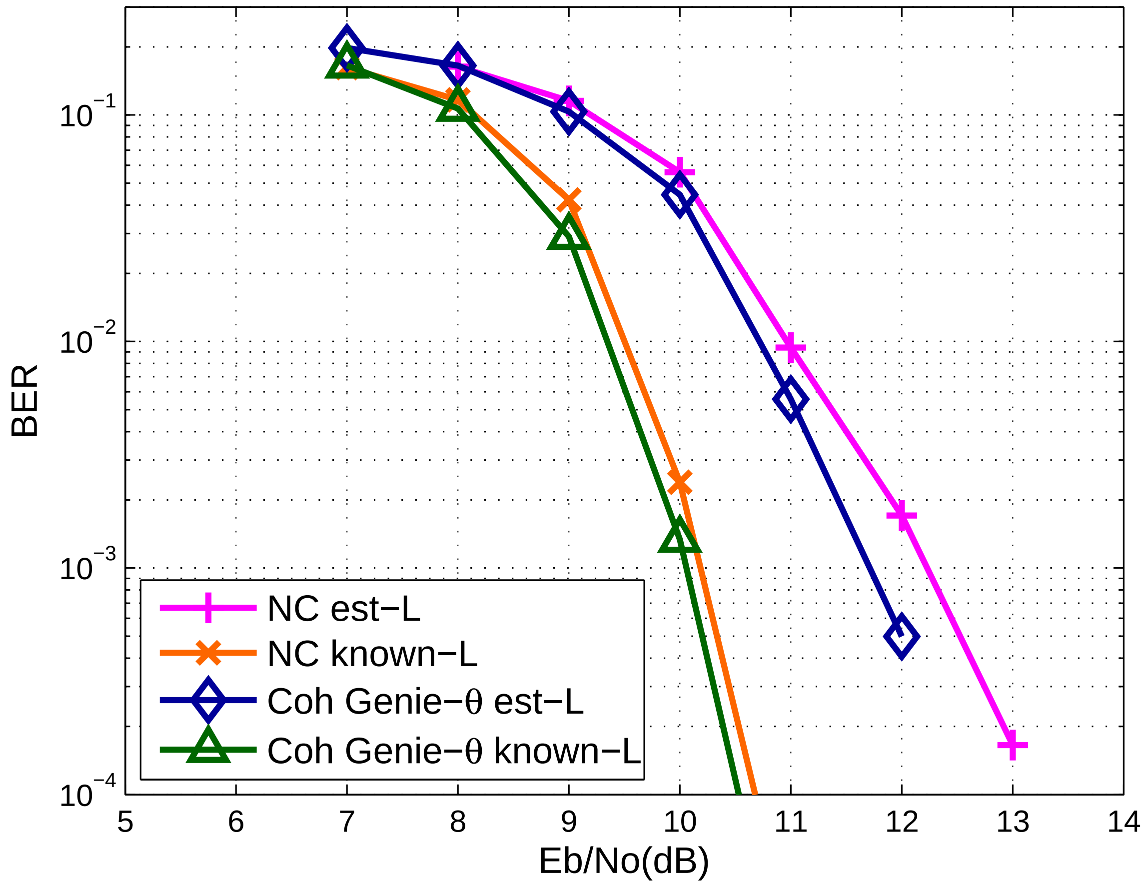
Transmitter:

- $(12288, 6144)$ LDPC code (rate-$\frac{1}{2}$)
- 128 QPSK subcarriers over 7.5 kHz BW
- 25% pilots (P=4 and K =1)

Receiver:

- assumes 3-tap ICI channel & 10 sparse delay taps (out of 50)
- MAP-inspired, breadth-first tree-search with M =32

Numerical Simulations:



"NC" = MAP-inspired noncoherent scheme

"Coh Genie" = uses 100% pilots

"est-L" = estimated sparse locs

"known-L" = known sparse locs

2) EM-Inspired Soft Noncoherent Equalization:

The i^{th} iteration of the Bayesian EM alg specifies

$$\hat{\boldsymbol{\theta}}[i+1] \;=\; \arg\min_{\hat{\boldsymbol{\theta}}} \mathrm{E}\left\{ \ln p(\boldsymbol{y}, \boldsymbol{s}|\hat{\boldsymbol{\theta}}) \,\middle|\, \boldsymbol{y}, \hat{\boldsymbol{\theta}}[i] \right\} + \ln p(\hat{\boldsymbol{\theta}})$$

Under Gaussian $\boldsymbol{\theta}$, this reduces to

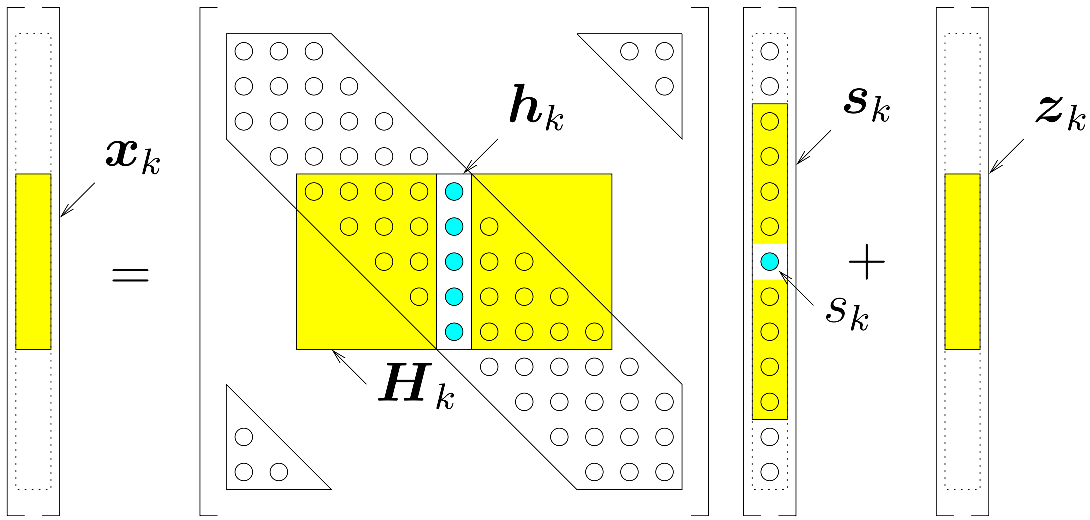
> *MMSE estimation of $\boldsymbol{\theta}$ using the soft symbol estimates computed via the previous channel estimate $\hat{\boldsymbol{\theta}}[i]$.*

Thus we iterate the following two steps:

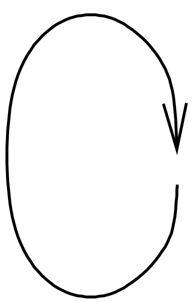
1. Soft channel estimation (using pilots & symbol means/variances)
2. Soft coherent equalization:
   - Compute LLRs using channel estimate $\hat{\boldsymbol{\theta}}[i]$,
   - Compute symbol means/variances from bit LLRs.

Using conjugate gradient for matrix inversion, complexity $= 2DN\log_2 N$.

2a) Iterative Soft ICI Cancellation:

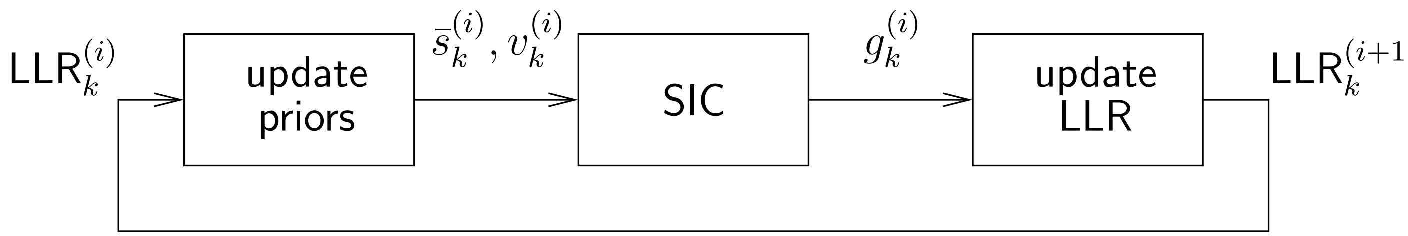


$$\boldsymbol{x}_k = \boldsymbol{H}_k \boldsymbol{s}_k + \boldsymbol{z}_k$$



- Soft interference cancellation using mean of $\boldsymbol{s}_k$.
- Assuming Gaussian residual interference and using the covariance of $\boldsymbol{s}_k$, compute LLRs(s_k).
- Using LLRs(s_k), update mean/covariance of s_k.
- $k \rightarrow \langle k+1 \rangle_N$.

Iterative Soft ICI Cancellation (BPSK example):



$$\bar{s}_k^{(i)} \triangleq \widehat{\mathrm{E}\{s_k|\hat{s}_k\}} = \tanh(\mathsf{LLR}_k^{(i)}/2)$$

$$v_k^{(i)} \triangleq \widehat{\mathrm{var}(s_k|\hat{s}_k)} = 1 - (\bar{s}_k^{(i)})^2$$

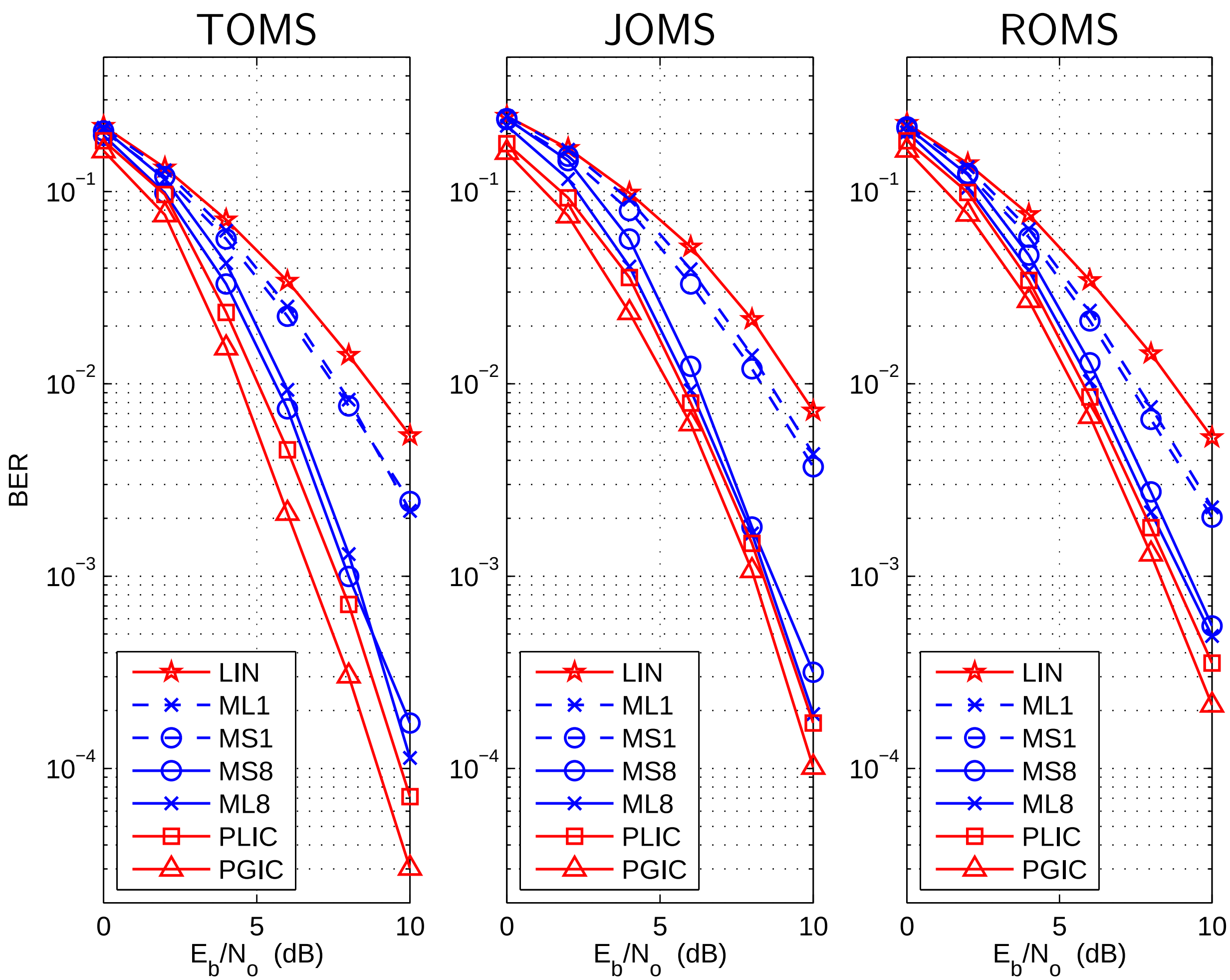
$$\boldsymbol{y}_k^{(i)} = \boldsymbol{x}_k - \boldsymbol{H}_k \bar{\boldsymbol{s}}_k^{(i)}$$

$$g_k^{(i)} = \boldsymbol{y}_k^{(i)H} \left(\boldsymbol{R}_z + \boldsymbol{H}_k \mathcal{D}(\boldsymbol{v}_k^{(i)}) \boldsymbol{H}_k^H\right)^{-1} \boldsymbol{h}_k$$

$$\mathsf{LLR}_k^{(i+1)} = \mathsf{LLR}_k^{(i)} + 2\,\mathrm{Re}(g_k^{(i)})$$

Complexity: $\underset{\textit{iters}}{M} \times \underset{\textit{mtx inv}}{\mathcal{O}(D^2)}$ per BPSK symbol.

Coherent Turbo Iterative Soft-ICI Cancellation:



rate-$\frac{1}{2}$ conv code

QPSK

$N = 64$

$D = 2$

$L = 16$

$f_{\mathsf{D}}T_c = 0.003$

for example:

$f_c = 20$GHz

BW$= 3$MHz

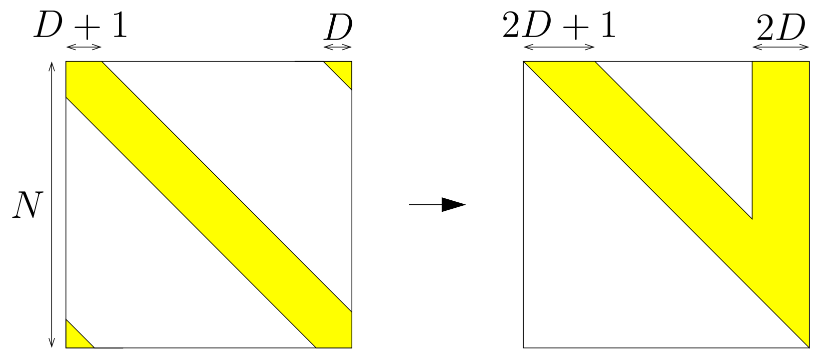
$T_h = 5.4\mu$s

$v = 3\times 160$km/hr

2b) Coherent Tree Search:

Two-step procedure:

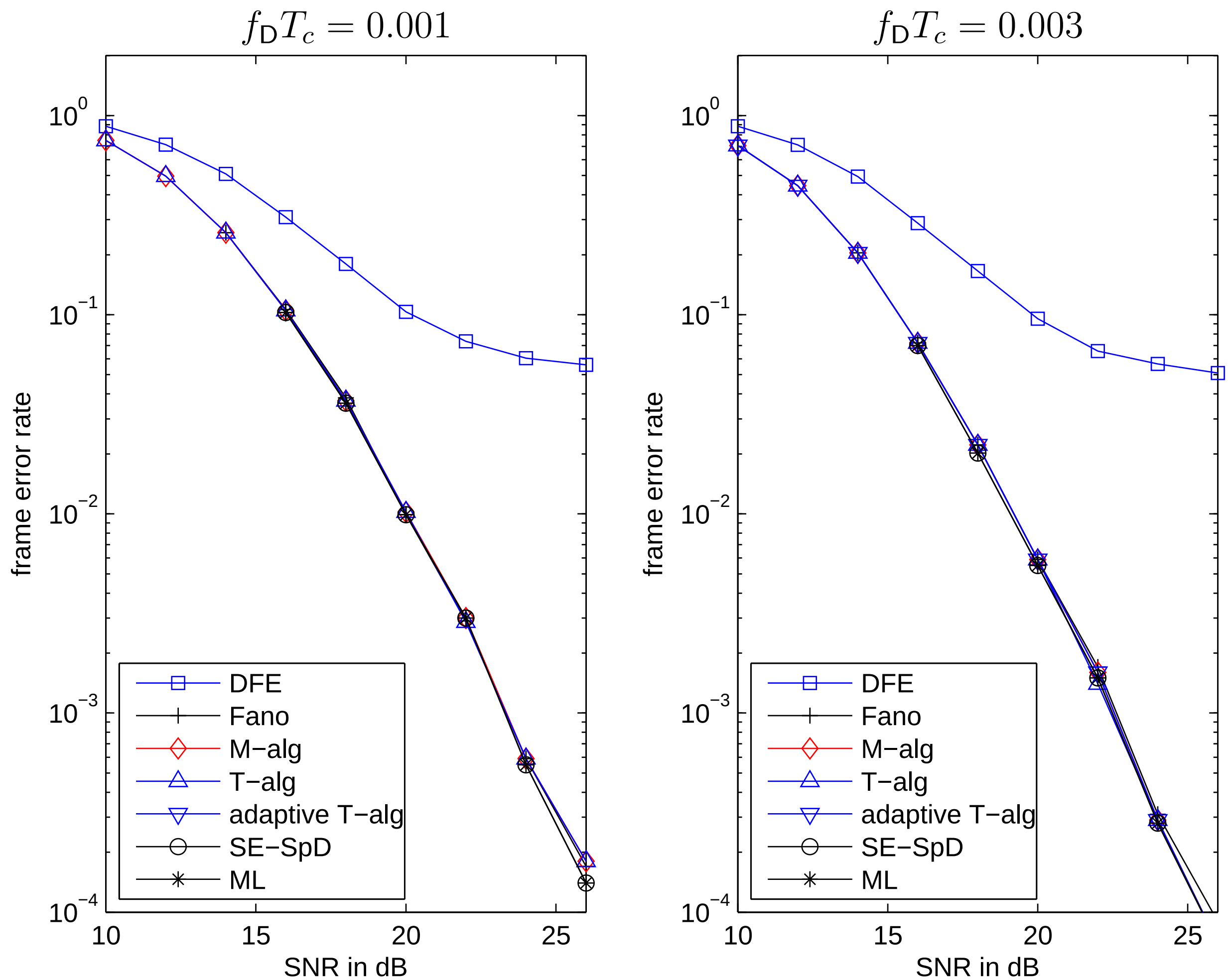
1. MMSE-GDFE pre-processing [Damen/ElGamal/Caire TIT 03]:



$\rightsquigarrow \mathcal{O}(D^2N)$ algorithm [Hwang/Schniter SPAWC 06].

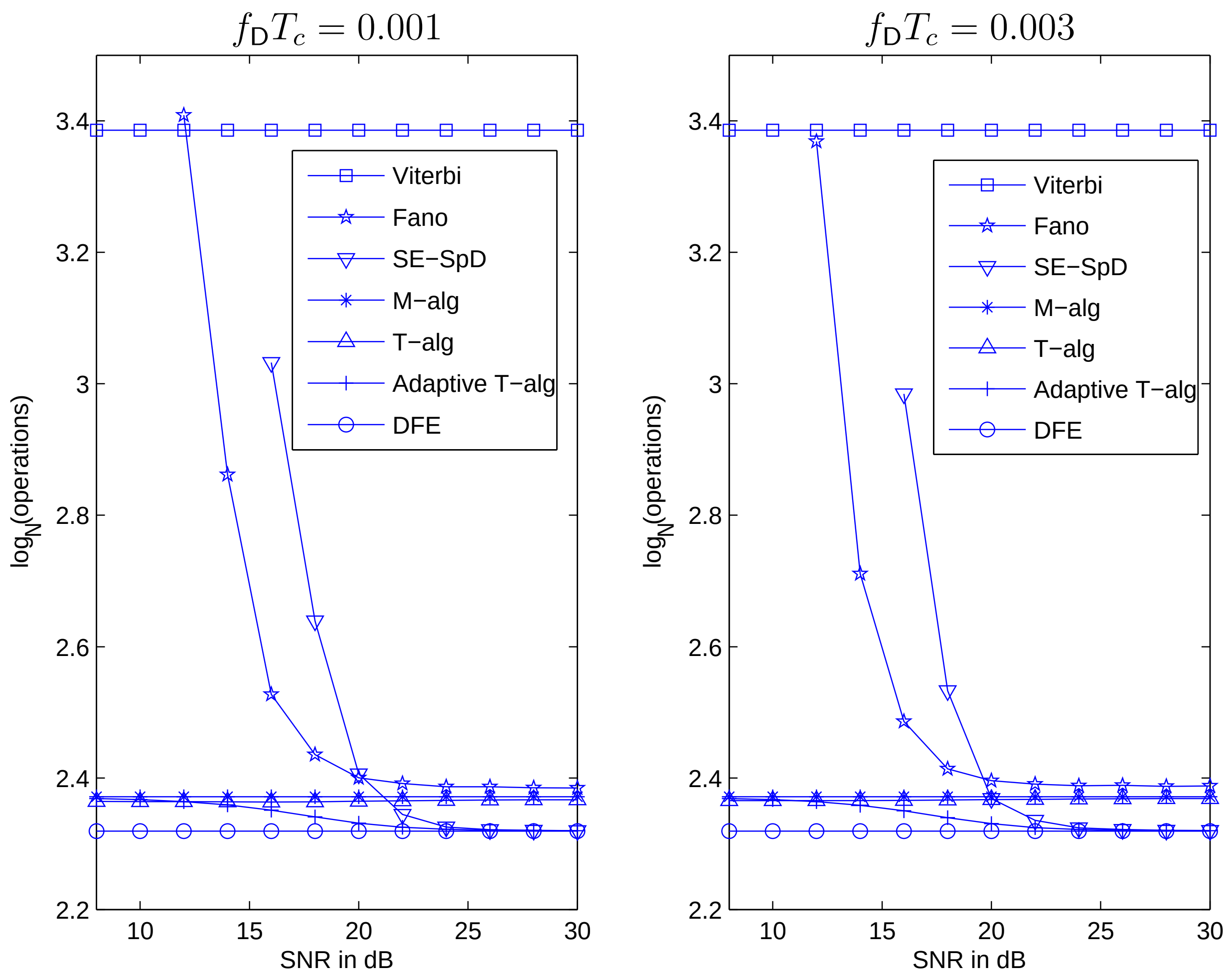
2. Near-optimal yet efficient tree search. Options include:
   - Depth-first search (e.g., Schnor-Euchner sphere decoder),
   - Best-first search (e.g., Fano alg, stack alg),
   - Breadth-first search (e.g., M-alg, T-alg, Pohst sphere decoder).

Coherent Tree-Search (Hard) Performance:



Suboptimal tree search is almost indistinguishable from MLSD!

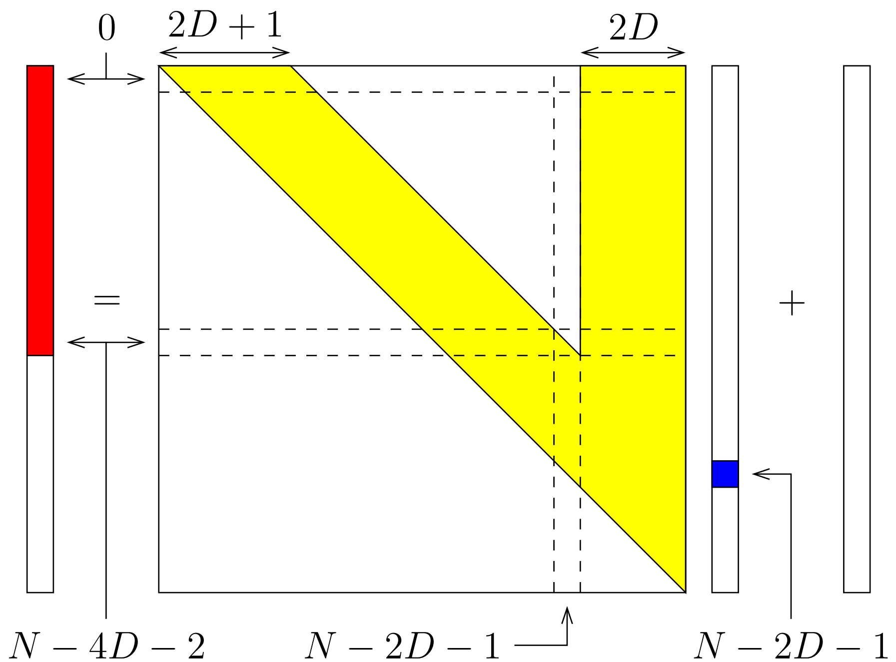
Average Complexity (MACs/frame):



Breadth-first & DFE stay cheap, while depth-first & Fano explode!

Error Masking due to V-shaped Channel Matrix:

After MMSE-GDFE pre-processing, we get the following system:



Key point: The blue symbol does not affect any of the red observations.

Error-masking explains the complexity explosion of the depth-first and Fano searches!

Related Work:

- Single-carrier frequency-domain equalization.
- Pilot pattern designs:
  - MMSE optimal (under CE-BEM assumption).
  - Achievable-rate optimal (under CE-BEM assumption).
- Theoretical analysis of pulse-shaped multicarrier modulation:
  - Achievable-rate characterized.
- Theoretical analysis of doubly selective channel:
  - Noncoherent capacity characterized (under CE-BEM assumption).

(See `http://www.ece.osu.edu/~schniter/pubs_by_topic.html`)

Conclusions:

(Bi)Orthogonal FDM:

- $\mathcal{O}(\log_2 L)\ \frac{\text{mults}}{\text{QAM symbol}}$ equalization of delay-spread channels.
- Loss in spectral efficiency due to guard interval.
- Sensitive to Doppler spread.

Non-(Bi)Orthogonal FDM:

- No need for a guard interval; high spectral efficiency.
- Large simultaneous delay & Doppler spreads $\Rightarrow$ no ISI and short ICI.

Conclusions:

Equalization of Short ICI Span:

- *Uncoded Coherent*:
  Tree search gives ML-like performance with DFE-like complexity.
- *Turbo Coherent*:
  Iterative soft ICI cancellation in turbo configuration performs close to perfect-interference-cancellation bound.
- *Uncoded Non-Coherent*:
  Tree-search with fast metric update gives ML-like performance with $\mathcal{O}(D^2L^2)$ complexity.
- *Turbo Non-Coherent*:
  Tree-search with fast metric update performs close to genie-aided bound with $\mathcal{O}(D^2L^2)$ complexity, but EM may do almost as well with $\mathcal{O}(D \log L)$ complexity.

Thanks for listening!