On VSB Modulation

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VSB modulation still has wide applications in analog communication, but not much any more for data communication. —*Gitlin, Hayes, and Weinstein, 1992.*

1 Introduction

The ATSC's recent adoption of the VSB modulation method for digital cable television transmission [2] motivates the importance of a good VSB-transmission model. An understanding of the model should provide answers to the following questions:

- What is meant by the equivalence between SQAM and VSB?
- Does there exist a valid system model with transmitted sequence of the form $\{\text{Re}, \text{Im}, \text{Re}, \text{Im}, \cdots\}$?

2 What is VSB?

Vestigial sideband modulation (VSB) is a modulation method which attempts to eliminate the spectral redundancy of pulse amplitude modulated (PAM) signals. It is well known that modulating a real data sequence by a cosine carrier results in a symmetric double-sided passband spectrum. The symmetry implies that one of the sidebands is redundant, and thus removing one sideband with an ideal brickwall filter should preserve the ability for perfect demodulation. As brickwall filters with zero transition bands cannot be physically realized, the filtering actually implemented in attempting such a scheme leaves a vestige of the redundant sideband, hence the name "VSB".

In understanding VSB, consider the use of a Nyquist filter, i.e., a filter which satisfies the Nyquist criterion. Such filters satisfy the time and frequency domain properties

$$g(t) \quad : \quad \begin{cases} g(t) = 1 & t = 0, \\ g(t) = 0 & t = nT, n \in \mathbb{Z}\backslash 0 \end{cases} \tag{1}$$

$$1 \quad = \quad \sum_{n=-\infty}^{\infty} G\left(j(\omega - \frac{2\pi}{T}n)\right). \tag{2}$$

The raised-cosine filters, a common family of Nyquist filters, have real positive frequency responses locally symmetric about $\omega = \frac{\pi}{T}$ and are easy to visualize.

For PAM using data sequence $\{a_n\}$, the following use of the pulse shape: $s(t) = \sum_n a_n g(t - nT)$, guarantees no inter-symbol interference when sampling the transmitted sequence $s(t)$ at the proper instants: $s(nT) = a_n$. Here T denotes the baud interval. In the frequency domain, implementing the Nyquist filtering with a raised cosine filter results in a double-sideband spectrum with lowpass cutoff at $|\omega| = \frac{\pi}{T}$.

Next we describe how the raised cosine filter can be to used to create a VSB-modulated signal. Here we have to be careful in defining the baud interval T. Consider the real-valued data sequence $\{a_k\}$ for which we desire to transmit one symbol every T_r seconds. We just determined that a PAM modulation of $\{a_k\}$ would have two sidebands, each of width $\frac{\pi}{T_r}$. With VSB modulation, we intend to transmit only one sideband and thus expect a total passband of width $\frac{\pi}{T_r}$. Considering the baseband data spectrum, the lower VSB sideband may be isolated by modulating its center up to DC and then lowpass filtering the result. (If desired, the remaining sideband may be moved back to its original position. For our discussion, however, we do not consider this as important.) The raised cosine filter which meets our spectral requirements has cutoff at $|\omega| = \frac{\pi}{2T_r}$ and thus has the time domain property $g(n2T_r) = 0$ for $n \neq 0$. The process we have just described results in the (complex-baseband) signal

$$s_{bb}(t) \quad = \quad \sum_k a_k e^{jk\pi/2} g(t - kT_r). \tag{3}$$

Note that a real-valued pulse-shape results in frequency-symmetric lowpass filtering.

Though other VSB modulation procedures exist, they differ only in small details. One such procedure is the subject of the next section.

3 A Frequency Domain Understanding of the SQAM-VSB Relationship

Staggered QAM (SQAM) [1] is a method of modulating a real-valued data sequence $\{a_k\}$ in which the even and odd subsequences, $\{a_{2k}\}$ and $\{a_{2k+1}\}$, are quadrature modulated using differently delayed versions of a real-valued pulse shaping filter $g(t)$. Formally, the SQAM-modulated signal of baud interval T_c is defined by:

$$\begin{aligned} s(t) &= \sum_k a_{2k} g(t - kT_c) cos(\omega_c t) + \sum_k a_{2k+1} g(t - kT_c - \tau_o) sin(\omega_c t) \qquad (4) \\ &= \mathrm{Re}\left\{ e^{-j\omega_c t} \left(\sum_k a_{2k} g(t - kT_c) + j \sum_k a_{2k+1} g(t - kT_c - \tau_o) \right) \right\} \end{aligned}$$

Note that both a_{2k} and a_{2k+1} are transmitted in the k^{th} baud interval. Comparing to the previous section, $T_c = 2T_r$.

It has been claimed that when the "staggering" delay τ_o is chosen as $\tau_o = T_c/2$, SQAM results in a form of VSB. We confirm this by looking at the spectrum of the transmitted signal. For convenience, we focus our attention on the complex baseband representation:

$$s_{bb}(t) \quad = \quad \sum_k a_{2k} g(t - kT_c) + j \sum_k a_{2k+1} g(t - kT_c - T_c/2) \tag{5}$$

Computing the Fourier transform,

$$\begin{aligned}
S_{bb}(j\omega) &= \int_{-\infty}^{\infty} s_{bb}(t)e^{-j\omega t}dt \\
&= \int_{-\infty}^{\infty}\left(\sum_k a_{2k}g(t-kT_c) + j\sum_k a_{2k+1}g(t-kT_c-T_c/2)\right)e^{-j\omega t}dt \\
&= \sum_k a_{2k}\int_{-\infty}^{\infty} g(t-kT_c)e^{-j\omega t}dt + j\sum_k a_{2k+1}\int_{-\infty}^{\infty} g(t-kT_c-T_c/2)e^{-j\omega t}dt \\
&= \sum_k a_{2k}e^{-j\omega kT_c}\int_{-\infty}^{\infty} g(\tau)e^{-j\omega\tau}d\tau + je^{-j\omega T_c/2}\sum_k a_{2k+1}e^{-j\omega kT_c}\int_{-\infty}^{\infty} g(\tau)e^{-j\omega\tau}d\tau,
\end{aligned}$$

where we have used the substitution $\tau = t - kT_c$. Using the facts that

$$\frac{1+e^{j\pi m}}{2} = \begin{cases} 1 & m \text{ even} \\ 0 & m \text{ odd} \end{cases}, \qquad \frac{1-e^{j\pi m}}{2} = \begin{cases} 0 & m \text{ even} \\ 1 & m \text{ odd} \end{cases},$$

and the notation

$$G(j\omega) = \int_{-\infty}^{\infty} g(\tau)e^{-j\omega\tau}d\tau,$$

we can rewrite the summations to yield

$$\begin{aligned}
S_{bb}(j\omega) &= G(j\omega)\left(\sum_m a_m\left(\frac{1+e^{j\pi m}}{2}\right)e^{-j\omega mT_c/2} + je^{-j\omega T_c/2}\sum_m a_m\left(\frac{1-e^{j\pi m}}{2}\right)e^{-j\omega(m-1)T_c/2}\right) \\
&= G(j\omega)\sum_m a_m\left(\frac{1+e^{j\pi m}}{2} + j\frac{1-e^{j\pi m}}{2}\right)e^{-j\omega mT_c/2} \\
&= G(j\omega)\sum_m a_m\big(cos(m\pi/2)+sin(m\pi/2)\big)e^{-j(\omega-\pi/T_c)mT_c/2}.
\end{aligned}$$

Noting that $cos(m\pi/2) + sin(m\pi/2) = \{1, 1, -1, -1, 1, 1, -1, -1, \dots\}$, it is convenient to define the sign-altered data sequence

$$\hat{a}_m = a_m\big(cos(m\pi/2)+sin(m\pi/2)\big) \quad \Leftrightarrow \quad \begin{cases} \hat{a}_{2m} = a_{2m}(-1)^m \\ \hat{a}_{2m+1} = a_{2m+1}(-1)^m \end{cases}.$$

Then, using the DTFT notation

$$\hat{A}(\omega) = \sum_m \hat{a}_m e^{-j\omega m},$$

we obtain a simplified form for the SQAM spectrum:

$$S_{bb}(j\omega) = G(j\omega)\hat{A}\left(\frac{T_c}{2}\left(\omega - \frac{\pi}{T_c}\right)\right). \tag{6}$$

Equation (6) indicates that the lower sideband of the symmetric spectrum $\hat{A}(\omega\frac{T_c}{2})$ is modulated up to DC and then filtered by $G(j\omega)$. So far we have not imposed any constraints on the pulse shape $g(t)$. Consider now the case when $G(j\omega)$ is a lowpass filter with cutoff $\omega = \frac{\pi}{2T_r}$.

Then, recalling $T_c = 2T_r$, $S_{bb}(j\omega)$ is **equivalent** to a (DC-centered) VSB-modulated spectrum of data sequence $\{\hat{a}_m\}$ with identical lower- and upper-sideband filtering. It follows that $s(t)$ can written directly in terms of $\hat{a}_m$:

$$s_{bb}(t) \;=\; \sum_m \hat{a}_m e^{jm\pi/2} g(t - mT_r), \tag{7}$$

which can be compared to (3).

4 The Structure of the VSB Data Sequence

The previous section established that a $T/2$-staggered SQAM modulation of a given data sequence is equivalent to (spectrally-centered) VSB modulation of a sign-altered version of the data sequence. In other words, these two approaches are closely related. In this section, we seek to dispell the rumor that the VSB (or SQAM) source sequence is structured in such a way that alternating elements are strictly real, strictly imaginary.

In both (3) and (7), we notice that the modulated data sequence $\hat{a}_m e^{jm\pi/2}$ has the form $\{\mathrm{Re}, \mathrm{Im}, \mathrm{Re}, \mathrm{Im}, \dots\}$, since a_m and $\hat{a}_m$ are strictly real and $e^{jm\pi/2} = \{1, j, -1, -j, \dots\}$. The properties of the pulse-shaping filter, however, prevent $s_m = s_{bb}(mT_c)$ from having this property. Recall from 2 that a $G(j\omega)$ with cutoff at $|\omega| = \frac{\pi}{2T_r} = \frac{\pi}{T_c}$ has the property

$$g(t) \;:\; \begin{cases} g(t) = 1 & t = 0, \\ g(t) = 0 & t = m2T_r, m \in \mathbb{Z}\backslash 0 \end{cases}$$

though, in general, $g(mT_r) \neq 0$. Even with a rectangular pulse shape, $g(mT_r) \neq 0$ for $m = \pm 1$.

To summarize, the lowpass filtering required to restrict VSB to bandwidth $\frac{\pi}{T_r} = \frac{2\pi}{T_c}$ prevents the transmitted data sequence from having an alternating real/imaginary structure (even with rectangular pulse shaping). It makes sense that transmission of real data in half the bandwidth requires full (nontrivial) use of the diversity provided by the real and quadrature "channels".

When elements of the real sequence $\{a_m\}$ are independent, we expect the VSB symbol sequence $\{s_k\} = \{s_{bb}(kT_c)\}$ to have independent real and imaginary elements, thus satisfying the circular symmetry property required of globally convergent CMA [3, 4, 5]: $\mathrm{E}\{s_k^2\} = 0$.

References

[1] R. D. Gitlin and Y. Ho, "The Performance of Staggered Quadrature Amplitude Modulation in the Presence of Phase Jitter," *IEEE Transactions on Communications*, vol. COM-23, no. 3, pp. 348-352, Mar. 1975.

[2] Advanced Television Systems Committee, *ATSC Television Standard*, Document A/53, Sep. 16, 1995.

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[5] Ye Li and Zhi Ding, "Global convergence of fractionally spaced Godard (CMA) adaptive equalizers," *IEEE Transactions on Signal Processing*, vol. 44, no.4, pp. 818-26, April 1996.