ON THE ROBUSTNESS OF BLIND LINEAR RECEIVERS FOR SHORT-CODE CDMA

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Abstract: In this paper, we examine the performance of minimum output energy, subspace, and minimum entropy criteria for blind acquisition/ linear-equalization of short-code CDMA signals. Environments with significant multipath, user load, and asynchronism are considered, and it is found that a minimum entropy criterion offers significantly superior MSE performance in these situations.

1. INTRODUCTION

Direct sequence code division multiple access (DS-CDMA) systems have received considerable attention as a flexible means of communication between multiple mobile users and centralized base stations. Since CDMA users share the same time and frequency resources, demodulation of a particular user is principally concerned with suppression of interference from other users. This is a challenging problem when user transmissions are asynchronous, of relatively disparate power levels, and corrupted by background noise and the interference resulting from multipath propagation—a serious problem in so-called "wideband" CDMA schemes (e.g., IMT-2000).

We are interested in comparing the performance of blind linear symbol-estimation schemes that operate with knowledge of only the desired user's spreading sequence. In other words, all user delays, multipath channels, and interfering sequences are unknown, and no training data is available. Such schemes attempt to solve the so-called "blind acquisition and equalization" problem.

In this paper, we focus on three criteria for blind acquisition/ equalization that have been shown to exhibit performance close to the minimum mean-squared error (MMSE) linear receiver under particular, though often idealized, conditions. The approaches are based on Minimum Output Energy (MOE), Subspace Decomposition (SS), and Minimum Entropy (ME) concepts, respectively. In order to gauge the relative applicability of these methods to what might be "practical" settings, we are interested in examining performance under more general (non-ideal) operating conditions. For example, the literature is replete with performance evaluations based on single-cell systems operating at a fraction of their potential user capacity. What happens as the number of users increases? Similarly, some criteria rely on an estimate of the desired user's multipath delay spread and/or an estimate of the number of interfering signals. What happens when these estimates are incorrect?

Supported in part by NSF grants MIP-9509011, ECS-9811297, an Intel Foundation Fellowship, and Applied Signal Technology. Direct all correspondence to Phil Schniter. *tele*: (607) 254-8819, *fax*: (607) 255-9072.

Towards this goal, we provide analysis and numerical comparisons of the MSE performance of receivers designed using MOE, SS, and ME criteria.

Notation: We use bold lowercase for vectors, bold uppercase for matrices, $(\cdot)^t$ for transposition, and $(\cdot)^H$ for conjugate transposition. All vector/matrix indices start with zero (i.e., we denote the first element of $\mathbf{x}$ by x_0 rather than x_1), and $\mathbf{I}_P$ denotes the $P \times P$ identity matrix, $\lceil \cdot \rceil$ denotes ceiling, $\langle \cdot \rangle_N$ modulo-N, $\mathrm{E}\{\cdot\}$ expectation, and $\|\mathbf{x}\|_p := \sqrt[p]{\sum_m |x_m|^p}$ the ℓ_p norm of $\mathbf{x}$.

2. DS-CDMA SYSTEM MODEL

In this section we construct a discrete-time model of a CDMA system with a receiver employing multiple sensors and/or sampling at a multiple of the chip rate. Say K users communicate symbol sequences $\{s_n^{(k)}\}$ indexed by n and with symbol spacing of T seconds. For short-code CDMA transmission, each of the k^{th} user's symbols are multiplied by a fixed spreading sequence $\{c_0^{(k)}, \ldots, c_{N-1}^{(k)}\}$, where $c_i^{(k)}$ is known as a "chip" and the chip duration is $T_c = T/N$. Consider a receiver which generates P samples per chip duration. Stacking the N_r previous $P \times 1$ samples $\{\mathbf{r}_i\}$ at time $t = nT$ into the received vector $\mathbf{r}(n) := (\mathbf{r}_{nN}^t, \ldots, \mathbf{r}_{nN-N_r+1}^t)^t$, we may construct $\mathbf{r}(n)$ as

$$\mathbf{r}(n) = \sum_{k=1}^{K} \boldsymbol{\mathcal{H}}^{(k)} \mathbf{s}^{(k)}(n) + \mathbf{w}(n), \quad \text{for} \quad \boldsymbol{\mathcal{H}}^{(k)} := \mathbf{H}^{(k)} \mathbf{C}^{(k)} \qquad (1)$$

where $\mathbf{C}^{(k)}$ accounts for the spreading operation, $\mathbf{H}^{(k)}$ accounts for the vector propagation channel (which incorporates pulse-shape filtering, multipath, and path loss), and $\mathbf{s}^{(k)}(n)$ contains the contributing symbols, all for user k. $\mathbf{w}(n)$ is a $PN_r \times 1$ vector of noise samples constructed in the same manner as $\mathbf{r}(n)$. The details of constructing $\mathbf{H}^{(k)}$, $\mathbf{C}^{(k)}$ and $\mathbf{s}^{(k)}(n)$ are given below.

For simplicity, we assume linear, time-invariant, finite impulse response (vector) channels of order $L_k - 1$, specified by the set of $P \times 1$ impulse response coefficients $\{\mathbf{h}_0^{(k)}, \ldots, \mathbf{h}_{L_k-1}^{(k)}\}$, such that $\mathbf{h}_0^{(k)} \neq \mathbf{0} \neq \mathbf{h}_{L_k-1}^{(k)}$. It will be convenient to construct a spreading code matrix $\mathbf{C}^{(k)}$ that explicitly incorporates the k^{th} user's degree of synchronization with the observation interval. This can be accomplished by choosing

$$\mathbf{C}^{(k)} := \begin{pmatrix} c_{a_k}^{(k)} \cdots c_{N-1}^{(k)} & & & \\ & c_0^{(k)} \cdots c_{N-1}^{(k)} & & \\ & & \ddots & \\ & & & c_0^{(k)} \cdots c_{b_k}^{(k)} \end{pmatrix}^t, \qquad (2)$$

$$\mathbf{H}^{(k)} := \begin{pmatrix} \mathbf{h}_0^{(k)} & \dots & \mathbf{h}_{L_k-1}^{(k)} & & & \\ & \mathbf{h}_0^{(k)} & \dots & \mathbf{h}_{L_k-1}^{(k)} & & \\ & & \ddots & & \ddots & \\ & & & \mathbf{h}_0^{(k)} & \dots & \mathbf{h}_{L_k-1}^{(k)} \end{pmatrix}. \quad (3)$$

where $a_k := \langle 1 - L_k - \delta_k \rangle_N$ and $b_k := \langle N_r - 1 - \delta_k \rangle_N$ for synchronization delay δ_k in the range $0 \leq \delta_k < N$. The construction of (2) and (3) implies that for some ν, the ν^{th} column of $\boldsymbol{\mathcal{H}}^{(k)}$ has $P\delta_k$ zero entries on top. E.g., $\boldsymbol{h}_\nu^{(k)} := [\boldsymbol{\mathcal{H}}^{(k)}]_\nu =$

$$\underbrace{\begin{pmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ c_0^{(k)} \mathbf{h}_{L_k-1}^{(k)} \\ \vdots \\ c_{N-1}^{(k)} \mathbf{h}_0^{(k)} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix}}_{\boldsymbol{h}_\nu^{(k)}} = \underbrace{\begin{pmatrix} \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ c_0^{(k)} \mathbf{I}_P & & \\ \vdots & \ddots & \\ c_{N-1}^{(k)} \mathbf{I}_P & & c_0^{(k)} \mathbf{I}_P \\ & \ddots & \vdots \\ & & c_{N-1}^{(k)} \mathbf{I}_P \\ \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}}_{\mathbf{C}_{\delta_k, L_k}^{(k)}} \underbrace{\begin{pmatrix} \mathbf{h}_{L_k-1}^{(k)} \\ \vdots \\ \mathbf{h}_0^{(k)} \end{pmatrix}}_{\mathbf{g}^{(k)}} \quad (4)$$

When $\delta_k = 0$, the k^{th} user is "synchronized," and the upper zero blocks of $\boldsymbol{h}_\nu^{(k)}$ and $\mathbf{C}_{\delta_k, L_k}^{(k)}$ shown in (4) disappear. Finally, $\mathbf{s}^{(k)}(n) := (s_n^{(k)}, \dots, s_{n-M_k+1}^{(k)})^t$ is the k^{th} user's source vector, contributing $M_k := \lceil \frac{N_r + L_k - 1}{N} \rceil$ symbols to the observation $\mathbf{r}(n)$.

The expression for $\mathbf{r}(n)$ may be further consolidated through definition of the global channel matrix $\boldsymbol{\mathcal{H}}$ and the multiuser source vector $\mathbf{s}(n)$. This yields $\mathbf{r}(n) = \boldsymbol{\mathcal{H}}\mathbf{s}(n) + \mathbf{w}(n)$.

$$\boldsymbol{\mathcal{H}} := \left(\boldsymbol{\mathcal{H}}^{(1)}, \boldsymbol{\mathcal{H}}^{(2)}, \dots \boldsymbol{\mathcal{H}}^{(K)}\right), \quad (5)$$

$$\mathbf{s}(n) := \left(\mathbf{s}^{(1)t}(n), \mathbf{s}^{(2)t}(n), \dots, \mathbf{s}^{(K)t}(n)\right)^t. \quad (6)$$

3. PERFORMANCE MEASURES

In the sequel, we will consider blind determination of $PN_r \times 1$ receivers $\mathbf{f}$ generating linear estimates $y_n = \mathbf{f}^H \mathbf{r}(n)$ of $s_{n-\nu}^{(k)}$, where k is the desired user and ν is an arbitrary fixed symbol delay. Without loss of generality, assume that we desire user $k = 1$. Then, defining the multiuser system impulse response vector $\mathbf{q} := (\mathbf{f}^H \boldsymbol{\mathcal{H}})^t$ and the interference response vector $\bar{\mathbf{q}}_\nu := (q_0, \dots, q_{\nu-1}, 0, q_{\nu+1}, \dots)^t$, we can write

$$y_n = q_\nu s_{n-\nu}^{(1)} + \underbrace{\bar{\mathbf{q}}_\nu^t \mathbf{s}(n)}_{\text{MAI and ISI}} + \underbrace{\mathbf{f}^H \mathbf{w}(n)}_{\text{noise}}. \quad (7)$$

As a common performance measure, we will consider unbiased mean-squared error (UMSE) $\mathcal{E}_\nu$, defined as follows:

$$\mathcal{E}_\nu := \mathrm{E}\{|y_n/q_\nu - s_{n-\nu}^{(1)}|^2\}. \quad (8)$$

It is well known that the Wiener receiver of delay ν, given by $\mathbf{f}_{\nu|\text{mse}} = \mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1} \boldsymbol{h}_\nu^{(1)}$, minimizes UMSE. Here we have used $\mathbf{R}_{\mathbf{r},\mathbf{r}}$ to denote the received autocorrelation matrix $\mathrm{E}\{\mathbf{r}(n)\mathbf{r}^H(n)\}$, which will be invertible for noise variance $\sigma_w^2 > 0$, and $\boldsymbol{h}_\nu^{(1)}$ to denote the ν^{th} column of $\boldsymbol{\mathcal{H}}^{(1)}$. It can be shown that

$$\mathcal{E}_{\nu|\text{mse}} = \left(\boldsymbol{h}_\nu^{(1)H} \mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1} \boldsymbol{h}_\nu^{(1)}\right)^{-1} - \sigma_s^2. \quad (9)$$

4. SUMMARY OF BLIND ACQUISITION/EQUALIZATION ALGORITHMS

In this section, we summarize representative algorithms from the minimum output energy (MOE) [1, 2, 3, 4], subspace (SS) [2, 5, 6], and minimum entropy (ME) [7] families of blind CDMA acquisition/equalization techniques. The goal of these algorithms is to estimate the symbol sequence of a particular user knowing only his spreading sequence. In other words, the multipath channels, timing offsets, background noise characteristics, and spreading sequences for undesired users are all unknown.

4.1. A Minimum Output Energy Technique

The MOE receiver originally proposed by Honig, Madhow, and Verdú in [1] assumed timing synchronization with the desired user and did not explicitly incorporate the effects of multipath propagation. MOE techniques incorporating blind timing acquisition and handling limited amounts of multipath were discussed by Madhow in [2], while more sophisticated incorporation of multipath was made by Tsatsanis in [3, 4], though desired user synchronization was assumed.

Here we present an extension to [4] that uses MOE-based criteria to accomplish blind equalization *and* acquisition:

$$\{\mathbf{f}_{\text{moe}}, \delta_{\text{moe}}, \mathbf{g}_{\text{moe}}\} := \arg \max_{0 \leq \delta < N} \max_{\mathbf{g}:\, \|\mathbf{g}\|_2 = 1} \min_{\mathbf{f}:\, \mathbf{C}_{\delta,L}^{(1)H} \mathbf{f} = \mathbf{g}} \mathbf{f}^H \mathbf{R} \mathbf{f}. \quad (10)$$

Under the assumptions of perfect synchronization and channel modeling (i.e., $\delta_{\text{moe}} = \delta_1 = 0$ and $L = L_1$), Tsatsanis [4] derived expressions for the SINR of $\mathbf{f}_{\text{moe}}$ given by (10). We can apply similar techniques to quantify the UMSE in the case that the true delay δ_1 and true channel order L_1 are unknown. This yields[1]

$$\mathcal{E}_{\nu|\text{moe}} = \lambda_0 \left| \mathbf{v}_0^H \mathbf{C}_{\delta_{\text{moe}},L}^{(1)H} \mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1} \mathbf{C}_{\delta_1,L_1}^{(1)} \mathbf{g}^{(1)} \right|^{-2} - \sigma_s^2, \quad (11)$$

where $\{\lambda_0, \mathbf{v}_0\}$ is the minimum eigenvalue/vector pair of the matrix $\mathbf{C}_{\delta_{\text{moe}},L}^{(1)H} \mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1} \mathbf{C}_{\delta_{\text{moe}},L}^{(1)}$. Recall $\mathbf{C}_{\delta,L}^{(k)}$, $\mathbf{g}^{(k)}$ were defined in (4).

As in [4], $\mathcal{E}_{\nu|\text{moe}}$ can be compared to $\mathcal{E}_{\nu|\text{mse}}$ by rewriting (9):

$$\begin{aligned} \mathcal{E}_{\nu|\text{mse}} &= \left(\mathbf{g}^{(1)H} \mathbf{C}_{\delta_1,L_1}^{(1)H} \mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1} \mathbf{C}_{\delta_1,L_1}^{(1)} \mathbf{g}^{(1)}\right)^{-1} - \sigma_s^2 \\ &= \left(\textstyle\sum_{i=0}^{L_1-1} \lambda_i \left|\mathbf{v}_i^H \mathbf{g}^{(1)}\right|^2\right)^{-1} - \sigma_s^2. \end{aligned} \quad (12)$$

Under perfect knowledge of channel-order and delay, (11) becomes $\mathcal{E}_{\nu|\text{moe}} = \left(\lambda_0 \left|\mathbf{v}_0^H \mathbf{g}^{(1)}\right|^2\right)^{-1} - \sigma_s^2$. Thus, (12) implies that $\mathcal{E}_{\nu|\text{moe}} \geq \mathcal{E}_{\nu|\text{mse}}$ with equality iff $\lambda_i = 0$ for $i > 0$. But, since λ_0 is the smallest of the eigenvalues $\{\lambda_i\}$, this is only possible for $L_1 = 1$.

To summarize, the MOE technique in (10) will exhibit nonzero extra UMSE for all nontrivial channels, and its MSE performance will be dependent on the assumed channel length L.

4.2. A Subspace Technique

Recently, Gesbert has devised a subspace-based (SS) method for blind acquisition/equalization [5]. The SS receiver is specified as

$$\mathbf{f}_{\text{ss}} := \arg \min_{\mathbf{f}:\|\mathbf{f}\|_2 = 1} \left\| \left(\boldsymbol{\mathcal{V}}_{M_k} \;\; \mathbf{R}_{\mathbf{r},\mathbf{r}} \boldsymbol{\mathcal{U}}_\ell^{(1)}\right)^H \mathbf{f} \right\|_2 \quad (13)$$

where $\boldsymbol{\mathcal{U}}_\ell^{(1)}$ is an basis for orthogonal complement of the "desired-signal space" and $\boldsymbol{\mathcal{V}}_{M_k}$ is an orthonormal basis for the "noise

[1] Derivation omitted due to lack of space; contact authors for details.

space.". Under particular conditions, it is possible to define $\boldsymbol{\mathcal{U}}_\ell^{(1)}$ and $\boldsymbol{\mathcal{V}}_{M_k}$ so that a Wiener receiver $\mathbf{f}_{\nu|\text{mse}}$ (for some ν) is the unique receiver orthogonal to both $\mathbf{R}_{\mathbf{r},\mathbf{r}}\boldsymbol{\mathcal{U}}_\ell^{(1)}$ and $\boldsymbol{\mathcal{V}}_{M_k}$, thus ensuring that $\mathbf{f}_{\text{ss}} = \mathbf{f}_{\nu|\text{mse}}$.

In [5], it is suggested to choose $\boldsymbol{\mathcal{V}}_{M_k}$ as the $PN_r - \sum_k M_k$ least dominant eigenvectors of $\mathbf{R}_{\mathbf{r},\mathbf{r}}$. Denoting by ℓ the maximum number of symbols possibly contributed by any user to a received segment of duration T, it is then suggested that $\boldsymbol{\mathcal{U}}_\ell^{(1)}$ be chosen as[2] the orthogonal complement of $\mathbf{C}^{(1)}_{0,\ell N-N+1}$. For these choices of $\boldsymbol{\mathcal{V}}_{M_k}$ and $\boldsymbol{\mathcal{U}}_\ell^{(1)}$, [6] proves that when

(a) N_r is an integer multiple of N,

(b) $M_k = N_r/N + \ell - 1 \;\; \forall k$,

(c) $K(N_r/N + \ell - 1) \le P(N_r - 1 + N(1-\ell))$, and

(d) $\big(\boldsymbol{\mathcal{H}}, \mathbf{C}^{(1)}_{0,\ell N-N+1}\big)$ has nullspace of dimension 1,

the minimizing solution $\mathbf{f}_{\text{ss}}$ is unique and equals $\mathbf{f}_{\nu|\text{mse}}$ for some ν.

Taking a closer look, conditions (a) and (b) are simplifications rather than requirements, and can be removed under proper reformulation of (c). With minor manipulation, (c) can be rewritten

$$N_r \;\ge\; N(\ell-1)\frac{1+K/NP}{1-K/NP} + \frac{1}{1-K/NP} \qquad (14)$$

and interpreted as a cell-load-dependent length requirement. (See Figure 1.) Condition (d) has the following interpretation: if we consider $\mathbf{C}^{(1)}_{0,\ell N-N+1}$ to be a basis for the desired signal $\boldsymbol{h}_\nu^{(1)}$ (i.e., for some delay ν there exists $\mathbf{g}$ such that $\boldsymbol{h}_\nu^{(1)} = \mathbf{C}^{(1)}_{0,\ell N-N+1}\mathbf{g}$), then (d) requires that the span of $\mathbf{C}^{(1)}_{0,\ell N-N+1}$ be small enough to exclude undesired user signals and inter-symbol interference signals (i.e., $\{\boldsymbol{h}_m^{(k)} : k \ne 1,\; \boldsymbol{h}_m^{(1)} : m \ne \nu\} \notin \text{span}(\mathbf{C}^{(1)}_{0,\ell N-N+1})$).

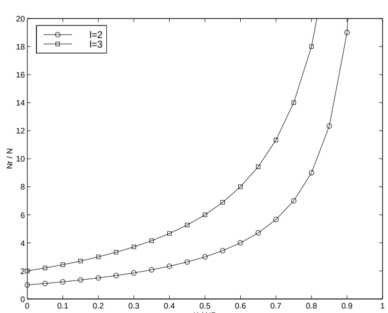

Figure 1: Normalized receiver length N_r/N required to guarantee $\mathbf{f}_{\nu|\text{mse}} = \mathbf{f}_{\text{ss}}$ versus cell-load K/PN, for $\ell = 2, 3$.

In an asynchronous environment, it not hard to find situations where (d) fails. First, realize that if $0 \le \delta_k < N$ is unknown and if $L_k > 2$, then up to $\ell = 3$ of the k^{th} user's symbols may affect a length-T observation. For $\ell = 3$, [5] suggests choosing $\boldsymbol{\mathcal{U}}_\ell^{(1)}$ as the orthogonal complement of $\mathbf{C}^{(1)}_{0,2N+1}$. But if, for example, $\delta_1 \le N - L_1 + 1$, then $\{\boldsymbol{h}_\nu^{(1)}, \boldsymbol{h}_{\nu+1}^{(1)}\} \in \text{span}\big(\mathbf{C}^{(1)}_{0,2N+1}\big)$. This is evident from the definition of $\mathbf{C}^{(1)}_{0,2N+1}$ in (4) and from the construction

$$\begin{array}{rcl} \boldsymbol{h}_\nu^{(1)} &=& (\,0\cdots0 \;\; \boxed{\text{nonzero}} \;\;\;\;\;\;\;\;\;\; 0\cdots0 \;\; 0\cdots0\,)^t, \\ \boldsymbol{h}_{\nu+1}^{(1)} &=& (\,0\cdots0 \;\; 0\cdots0 \;\;\;\;\;\; \boxed{\text{nonzero}} \;\; 0\cdots0\,)^t. \\ && \longleftarrow \;\;\; P(2N+\delta_1+L_1-1) \;\;\; \longrightarrow \end{array}$$

[2]Note that this $\boldsymbol{\mathcal{U}}_\ell^{(1)}$ requires an observation length $N_r \ge \ell N$.

For satisfaction of (d), the best choice of $\boldsymbol{\mathcal{U}}_\ell^{(1)}$ would be the orthogonal complement of $\mathbf{C}^{(1)}_{\delta_1,L_1}$, but δ_1 and L_1 are unknown.

Problems are also expected in the estimation of $\boldsymbol{\mathcal{V}}_{M_k}$ since we do not expect knowledge of $\sum_{k=1}^K M_k$. (Recall that M_k specifies the number of symbols that the k^{th} user contributes to the N_r-chip observation interval and is dependent on L_k and δ_k. In general, K specifies the total number of in-cell *plus out-of-cell* users.)

To summarize, the SS methods are capable of determining a Wiener receiver under restrictive set of circumstances: a potentially large number of receiver parameters (see Figure 1), a good assumption on the total number of interfering symbols $\sum_{k=1}^K M_k$, and satisfaction of a rank condition which, in general, requires a particular relationship between the desired user's synchronization delay and channel length. Finally, we note that even though a particular Wiener receiver may be obtained, it may not estimate the desired user sequence at the MSE-optimal system delay.

4.3. A Minimum Entropy Technique

Recently, the authors have applied minimum entropy (ME) concepts to blind acquisition and equalization of CDMA signals [7]. This application of ME can be motivated by the tendency for the desired symbol stream to have a very non-Gaussian distribution (e.g., BPSK) but for the noise-plus-interference to have a distribution that is much closer to Gaussian. Various measures of "distance from Gaussianity" may be applied, e.g., Shannon entropy, kurtosis, or constant-modulus (CM) cost [8].

A specific ME technique was proposed in [7] which suggested (locally) minimizing the pre-whitened CM cost $J_{\text{cm}}(\mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1/2}\mathbf{f})$.

$$J_{\text{cm}}(\mathbf{f}) := \mathrm{E}\{\big||\mathbf{f}^H\mathbf{r}(n)|^2 - 1\big|^2\}. \qquad (15)$$

The CM criterion has found extensive use in single-user applications and has been noted for its excellent MSE behavior under a wide range of operating conditions (see [8] and the references therein). For example, the CM criterion is known for good MSE performance even when $\boldsymbol{\mathcal{H}}$ is *not* full column rank. For i.i.d. BPSK sources and i.i.d. Gaussian noise, the expressions in [8] can be used to show that the pre-whitening yields

$$J_{\text{cm}}(\mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1/2}\mathbf{f}) = -2\|\mathbf{f}^H\mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1/2}\boldsymbol{\mathcal{H}}\|_4^4 + 3\|\mathbf{f}\|_2^4 + \|\mathbf{f}\|_2^2 + 1. \quad (16)$$

The difficulty in using the CM criterion for blind acquisition/ equalization of multiuser signals has to do with the fact that J_{cm} is multimodal. Roughly speaking, the different minima of J_{cm} correspond to different combinations of {user,delay}. For acquisition of user $k = 1$, we are interesting in $\mathbf{f}_{\text{cm}}$ locally minimizing J_{cm} at a particular subset of minima. Because estimation performance may vary over this subset of CM minimizers, we are really interested in finding the $\mathbf{f}_{\text{cm}}$ minimizing J_{cm} at the minimum corresponding to the MSE-optimal system delay for user 1.

Determining $\mathbf{f}_{\text{cm}}$ usually involves gradient descent of J_{cm} from a particular initialization. Thus, successful application of J_{cm} to blind acquisition/equalization requires a good initialization procedure. In [7], the authors suggested hypothesizing a number of initializations and choosing the one which gives the minimum kurtosis output stream, where (normalized, pre-whitened) kurtosis is defined as follows:

$$\kappa_y := \mathrm{E}\{|\mathbf{f}^H\mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1/2}\mathbf{r}(n)|^4\}/(\mathrm{E}\{|\mathbf{f}^H\mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1/2}\mathbf{r}(n)|^2\})^2.$$

Pre-whitening is used to increase the effectiveness of kurtosis-based initialization and to decrease the initialization sensitivity of the CM criterion [7]. In the P-sensor case, when $\mathbf{R}_{\mathbf{r},\mathbf{r}}^{-1/2}$ is Hermitian, and when the codes have good cross-correlation properties (e.g., Gold codes), it might be convenient to select a set of PN initialization hypotheses equal to the columns of $\mathbf{C}_{0,N}^{(1)}$.

Since closed form expressions for the UMSE of $\mathbf{f}_{\text{cm}}$ are difficult to derive, we rely on MATLAB's gradient search routine "fminu" (initialized as above) to compute $\mathbf{f}_{\text{cm}}$ locally minimizing (16).

5. NUMERICAL COMPARISONS

In all simulations, we use Gold codes with $N=31$ and user delays $\{\tau_k\}$ uniformly distributed over $[0,T)$. The propagation channels are based on root-raised-cosine pulse shaping (with excess BW 0.2) and 5 ray multipath channels such that the last 4 rays are normally distributed in amplitude with std. dev. 0.3, and uniformly distributed in delay over $[0, 5T_c)$, relative to the first ray. We allow 12 T_c of channel duration, though the bulk of the impulse response energy may fall within as little as 8 T_c. Due to a lack of space, we restrict ourselves to the case of equal user powers. The background noise is zero-mean AWGN with SNR = 20 dB (referenced to the average in-cell user's power). We chose equalizer length $N_r = 3N$ to allow for $\ell = 3$ in the SS method, and kept $P=1$.

Figure 2 shows the UMSE performance of the MOE criterion as a function of the channel length estimate L for 10 users and 25 users (averaged over 100 Monte-Carlo runs). With 10 users, setting L equal to the true channel order $L_1 = 12$ seems to be a good idea. But, as the other-user interference grows, the constraint $\mathbf{C}_{\delta,L}^{(1)H}\mathbf{f} = \mathbf{g}$ robs the equalizer of valuable degrees of freedom. Thus, MOE channel-length specification is complicated by a dependence on interference level.

Figure 3 compares the performance of the various criteria versus K, the number of users. As a reference, we plot Wiener performance for the *optimal* system delay ν. The MOE constraint order L was chosen as the true order L_1, and as expected, extra UMSE performance degrades as K increases. Two versions of the SS technique were examined. The first, labeled "SS," operates blindly using $\boldsymbol{\mathcal{U}}_\ell^{(1)}$ and $\boldsymbol{\mathcal{V}}_{M_k}$ recommended for $\ell = 3$ in [5]. The second, labeled "SS(δ_1,L_1)," uses exact knowledge of delay δ_1 and length L_1 in choosing $\boldsymbol{\mathcal{U}}_\ell^{(1)}$ as the orthogonal complement of $\mathbf{C}_{\delta_1,L_1}^{(1)}$. As discussed in Section 4.2, this modification was proposed as a means of keeping condition 4.2.(d) satisfied, and its relative success demonstrates both the importance of 4.2.(d) and its typical lack of satisfaction in asynchronous environments. Though not visible in Figure 3, the SS(δ_1,L_1) technique *did* find Wiener equalizers for K satisfying (14), though these did not always correspond to the MSE-optimal system delay. Finally, note that the ME-based criterion achieved *nearly zero* extra UMSE for all K!

6. CONCLUSION

In this paper, we have examined the UMSE performance of MOE, SS, and ME criteria. While the ME method inherits the desirable robustness properties of the CM criterion on which it is based, the MOE and SS criterion do not seem to perform well in environments with significant asynchronism, multipath, and user load.

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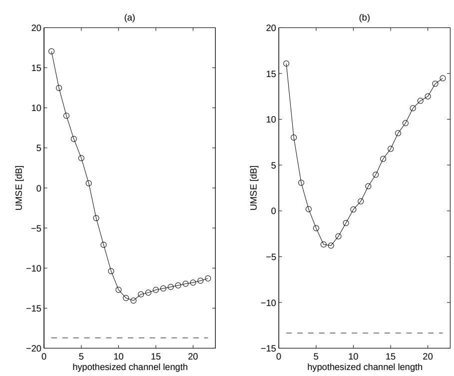

Figure 2: Averaged UMSE of MOE receiver versus channel length estimate L for (a) 10 users, and (b) 25 users.

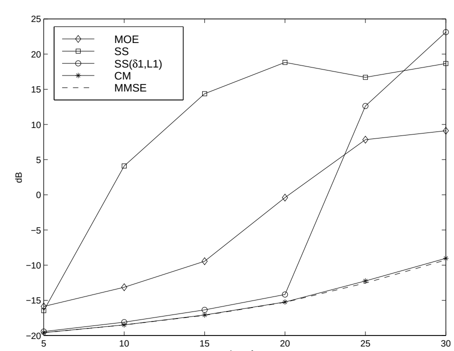

Figure 3: Averaged UMSE of blind criteria versus # users K.