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# Asymptotically Optimal Training and Scheduling in the Non-Coherent SIMO Multiple Access Channel

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Channel:

SIMO Multiple Access, block fading channel, with **no CSI – R**

How do we tap the spatial multiplexing and the multiuser diversity gains promised by this SIMO multiuser channel ?

- A *training/scheduling* - based non-coherent communication scheme is designed, identifying the tradeoffs between time and energy spent in gathering CSI and in data transmission



How do these gains and hence the scheme's performance behave with Increasing/vanishing SNR, number of antennas or users ?

- A detailed analysis of the proposed scheme in the *asymptotic regime* is provided
- Proved that the proposed scheme is *scaling-law optimal* by obtaining the degrees of freedom of the non-coherent SIMO multiple access channel
- Proved the order optimality of the proposed scheme in the low SNR regime by deriving the order of decay of the sum capacity of the non-coherent SIMO multiple access channel.



- $n$  independent users, each with one antenna and same average power constraint ( $\rho_{\text{avg}}$ )
- A base station (BS) with  $M$  receive antennas
- Block fading channel model with coherence interval  $T$
- Rayleigh fading, i.e., i.i.d.  $\mathcal{CN}(0, 1)$  coefficients
- Independent AWGN at the BS antennas
- No CSI at the BS (and at the users)
- Training-based non-coherent communication scheme to be deployed (inspired by *Hassibi's* work on single user MIMO)

# Phases of Transmission: Training Phase

- Duration:  $T_\tau$ , number of participating users:  $L$

$$\begin{array}{c} t=1 \\ 2 \\ 3 \\ \vdots \\ T_\tau \end{array} \left[ \begin{array}{c} \text{Received signal} \\ \text{matrix} \end{array} \right]_{T_\tau \times M} = \sqrt{\rho_\tau} \left[ \begin{array}{c} \text{Training} \\ \text{sequence matrix} \\ (S_\tau) \end{array} \right]_{T_\tau \times L} \left[ \begin{array}{c} \text{Channel matrix} \\ \text{of the } L \\ \text{training users} \end{array} \right]_{L \times M} + \left[ \begin{array}{c} \text{AWGN} \end{array} \right]_{T_\tau \times M}$$

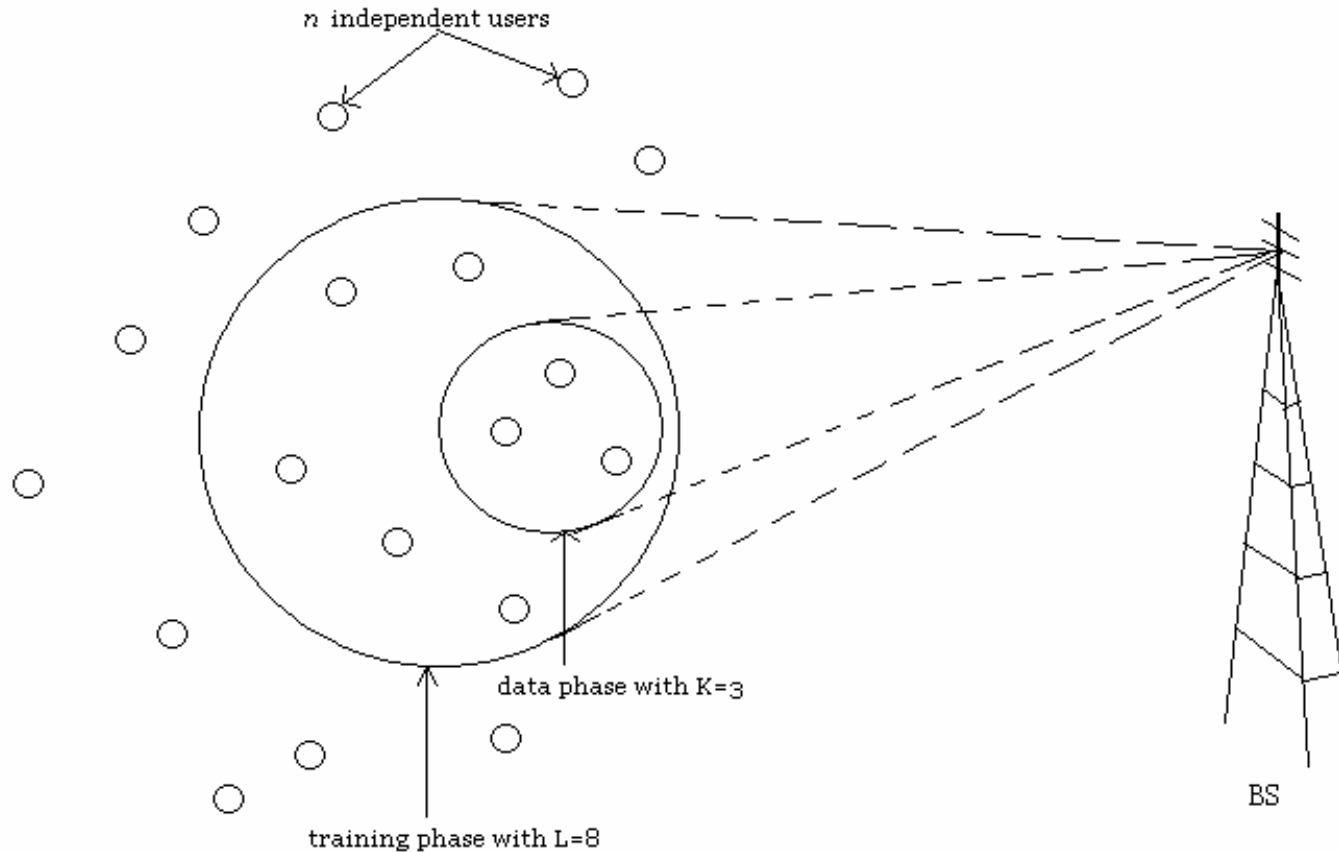
- Training power level of each user:  $\rho_\tau$
- MMSE estimate of channel coefficients obtained at the end of the training phase



- Duration:  $T_d = T - T_\tau$
- A subset of  $K$  users out of the  $L$  trained are scheduled to transmit data (choice of users is based on a metric to be introduced soon...)

$$\begin{array}{c}
 t = T_\tau + 1 \\
 \vdots \\
 T_\tau + 2 \\
 \vdots \\
 T
 \end{array}
 \left[ \begin{array}{c}
 \text{Received signal} \\
 \text{matrix} \\
 T_d \times M
 \end{array} \right]
 = \sqrt{\rho_d}
 \left[ \begin{array}{c}
 \text{Data symbol} \\
 \text{matrix} \\
 T_d \times K
 \end{array} \right]
 \left[ \begin{array}{c}
 \text{Channel} \\
 \text{estimate matrix} \\
 \text{of the } K \\
 \text{scheduled users} \\
 K \times M
 \end{array} \right]
 + \underbrace{\left[ \begin{array}{c}
 \text{Additive} \\
 \text{noise} \\
 T_d \times M
 \end{array} \right]}_{\text{noise due to estimation error} + \text{AWGN}}$$

- Data power level of each user:  $\rho_d$





- Our metric ( $C_{LB}$ ) is a lower bound to the sum capacity of the SIMO multiple access channel
- Precisely,  $C_{LB}$  is the sum rate achieved by the proposed scheme when the additive noise is at its worst distribution with the best matched signal distribution.
- Gaussian distribution corresponds to the best case signal and worst case noise, as proved by *Medard, Hassibi* et al





Training sequence,  $S_\tau$ :

- Identified an effective SNR term influencing the metric
- The necessary and sufficient condition for  $S_\tau$  that maximizes this effective SNR is obtained as,

$$S_\tau^* S_\tau = T_\tau I_L$$

- This implies  $T_\tau \geq L$



Power share factor,  $\alpha$  :

- In any coherence interval, total energy = energy used in training + energy spent on data
- Designed  $\alpha$  such that energy spent on data =  $\alpha \times$  total energy
- $\alpha$  that maximizes effective SNR is obtained as,

$$\alpha_{\text{opt}} = \begin{cases} \frac{1}{2} & T_d = L \\ \gamma - \sqrt{\gamma(\gamma - 1)} & T_d > L \\ \gamma + \sqrt{\gamma(\gamma - 1)} & T_d < L \end{cases} \quad \text{where} \quad \gamma = \frac{L + \rho_{\text{avg}} n T}{\rho_{\text{avg}} n T \left[ 1 - \frac{L}{T_d} \right]}$$



Training period,  $T_\tau$ :

- For  $0 < T_d \leq T - L$ , the objective function monotonically increases with  $T_d$
- Combined with the condition  $T_\tau \geq L$ , optimum value of  $T_\tau = L$
- With  $\alpha$  and  $T_\tau$  results,

$$\textit{total data power} > \textit{average power} > \textit{total training power}$$

when  $T_d < T_\tau$  and vice versa



- Signal design: Gaussian symbols, i.i.d. across space and time with variance  $\rho_d$
- Training period:  $T_\tau = L$
- Training sequence: designed such that  $S_\tau^* S_\tau = T_\tau I_L$ . Standard  $L$  - dimensional basis vectors will serve the purpose
- Power share: optimum value of  $\alpha$  obtained earlier



## User Selection Protocol:

- $L$  users selected randomly/round-robin to train their channels
- Subset of  $K$  users giving the maximum mutual information is scheduled to transmit data
- With potentially short  $T$ , interleaving is necessary. Each user maintains a codebook of rate  $\frac{T}{n(T-L)} C_{LB}$  and interleaves its codewords across the coherence intervals it transmits in



As SNR ( $\rho_{\text{avg}}$ )  $\rightarrow \infty$  :

- Trivial scheduling ( $K=L$ ) is optimal. All trained users should transmit data
- $\min(n, M, \lfloor \frac{T}{2} \rfloor)$  users should be allowed to train *and* transmit data
- Power gain obtained by exploiting the statistical diversity shows *inside the log function* of the sum rate expression
- This gain could not compensate for the loss in the *prelog factor* when a strict subset of trained users is scheduled to transmit data



As SNR ( $\rho_{\text{avg}}$ )  $\rightarrow \infty$ :

- Sum rate of the proposed scheme has a scaling law = degrees of freedom of the non-coherent, single user,  $n \times M$  channel derived by *Zheng* and *Tse*
- As a corollary, non-coherent, SIMO multiple access channel has the *same degrees of freedom* as its single user counterpart given by
 
$$\frac{T - n^*}{T} n^* \quad \text{with} \quad n^* = \min(n, M, \lfloor \frac{T}{2} \rfloor)$$
- Our scheme is *scaling-law optimal*



As the number of users,  $n \rightarrow \infty$  :

- Every time  $n$  doubles, the sum rate, in bits per channel use (*bpcu*), increases by the channel's degrees of freedom
- Explanation: as  $n$  increases with the number of *active* users bounded, the power per active user increases as a consequence of our power constraint
- What happens with the Broadcast channel?





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- What happens with the Broadcast channel?
- But, is there a *catch*?...



As SNR ( $\rho_{\text{avg}}$ )  $\rightarrow \infty$ :

- Per-user rate,

$$\left(\frac{T - \min(n, M, \lfloor \frac{T}{2} \rfloor)}{T}\right) \left(\frac{\min(n, M, \lfloor \frac{T}{2} \rfloor)}{n}\right) \log(\rho_{\text{avg}}) + O(1) \quad \text{as } \rho_{\text{avg}} \rightarrow \infty$$

drops *monotonically* with the number of users ( $n$ ) as long as  $n \geq 1$

- Thus, the per-user *capacity* of the non-coherent, SIMO multiple access channel also drops monotonically with  $n$
- Compare this with the coherent per-user capacity

$$\frac{\min(n, M)}{n} \log(\rho_{\text{avg}}) + O(1) \quad \text{as } \rho_{\text{avg}} \rightarrow \infty$$



As the number of BS antennas,  $M \rightarrow \infty$  :

- *Channel Hardening* occurs leading to vanishing scheduling gain in the system
- $\min(n, \lfloor \frac{T}{2} \rfloor)$  users should be allowed to train. *All of them* should transmit.
- Compare with the  $\text{SNR} \rightarrow \infty$  case
- Doubling  $M$  gives the same effect as a *3 dB rise in SNR* on the sum rate



As SNR (  $\rho_{\text{avg}}$  )  $\rightarrow 0$  :

- Sum rate lower bound (  $C_{\text{LB}}$  ) of the proposed scheme decays *quadratically* with SNR. Thus the proposed scheme is potentially *order of decay* sub-optimal
- Only one user should be allowed to train and transmit data
- Explanation: more users trained  $\Rightarrow$  reduced training power per user  $\Rightarrow$  low quality channel estimates  $\Rightarrow$  poor effective SNR which leads to sub-optimality in this power constrained regime
- No provision to tap the scheduling gain here



Solution: Incorporate “Flash” signaling:

- Motivated by the following observation made in previous works (e.g., by *Zheng, Rao* et al): *flashy/peaky signaling can be used to achieve linear rate of decay with SNR in single user non-coherent channels*
- Assuming  $\text{SNR} < 1$ , our previous scheme is implemented for SNR-fraction of coherence intervals where the transmitters are otherwise silent
- Result: per-user power conditioned on active coherence intervals becomes unity, independent of SNR
- Sum rate of this modified scheme achieves a linear decay with SNR



Regarding our flash-modified scheme:

- Corollary: Sum capacity of the non-coherent SIMO multiple access channel also decays linearly with vanishing SNR
- The modified scheme is, thus, order optimal
- Since the effective per-user power is 1 (independent of the vanishing SNR), it can be argued that non-trivial scheduling becomes advantageous at low SNR
- Compare this with the high SNR scenario



- A training/scheduling - based communication scheme for the non-coherent SIMO multiple access channel is designed
- The non-coherent SIMO multiple access channel has the same degrees of freedom as the non-coherent, single user MIMO channel
- The proposed scheme is scaling-law optimal



- Studied the performance of the proposed scheme as the number of users or the base station antennas grow unbounded
- Sum capacity of the non-coherent SIMO multiple access channel decays linearly with vanishing SNR
- With a “flash” based modification, demonstrated the order optimality of the proposed scheme as  $\text{SNR} \rightarrow 0$
- Proved that non-trivial scheduling is sub-optimal at high SNR and optimal at low SNR



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