A Low-Complexity Receiver for CP-OFDM in Doubly-Selective Channels

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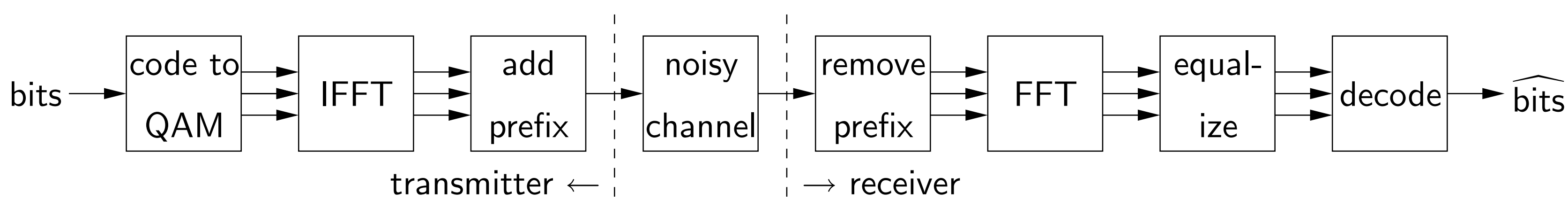
December 3, 2003

Cyclic Prefix OFDM (CP-OFDM):

- Uses FFT for efficient modulation/demodulation.
- $\mathcal{O}(\log N)$ operations/symbol for FFT length N.
- Perfect interference suppression when
  - cyclic prefix length $>$ channel delay spread,
  - channel is time-invariant over the FFT-block duration.

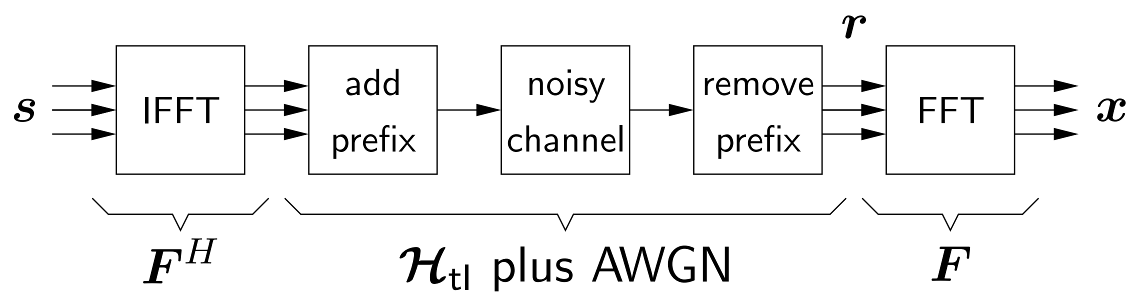


Doubly-Selective Channel:

- Long OFDM symbol motivated by reduction of cyclic-prefix redundancy.
- Channel time-variation present in applications with high mobility and/or high carrier frequency.

$\rightsquigarrow$ *Need a way to handle significant channel time-variation within an OFDM symbol interval.*

System Model:



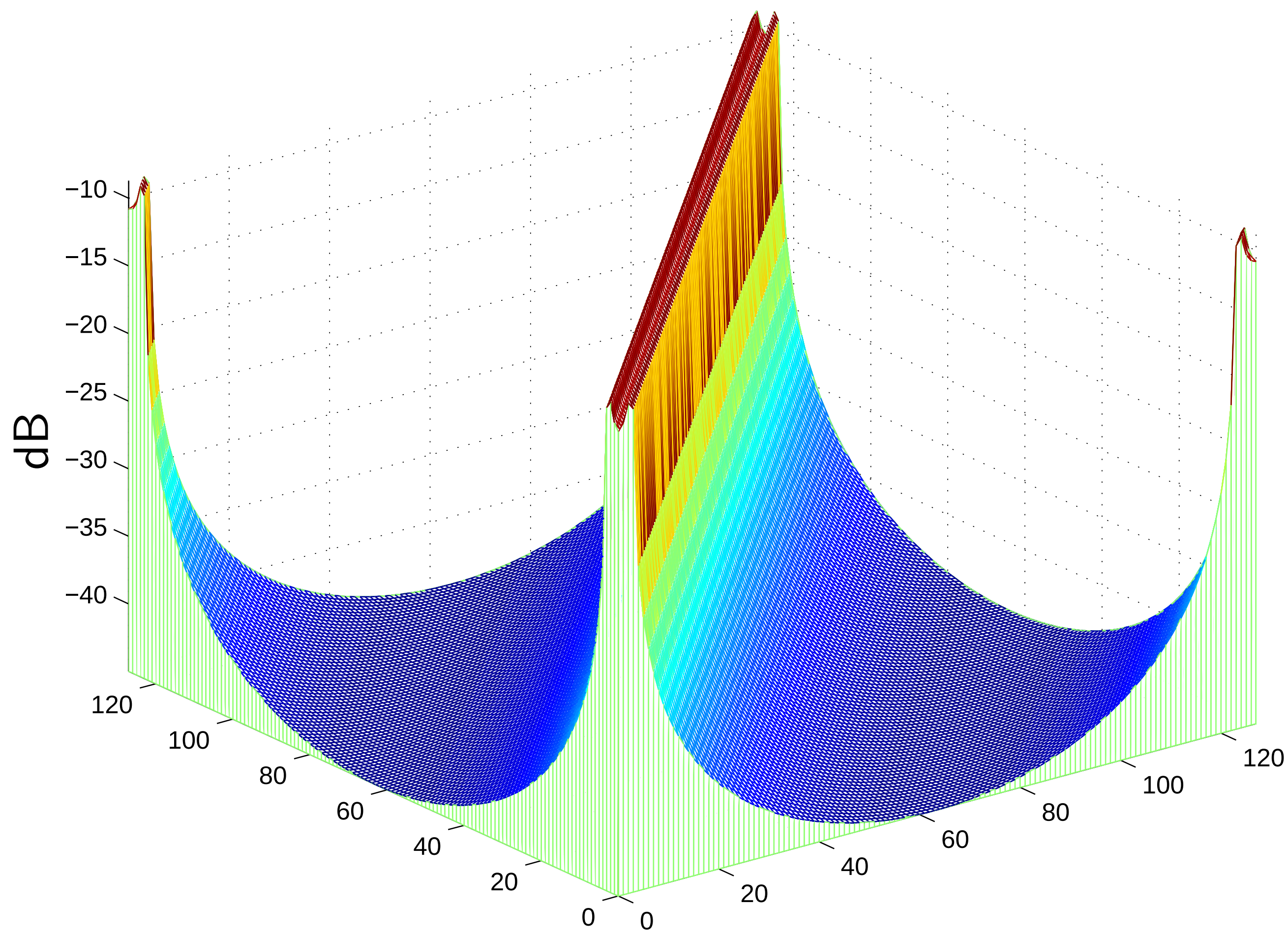
$$
\begin{aligned}
\boldsymbol{r} &= \mathcal{H}_{\text{tl}} \boldsymbol{F}^H \boldsymbol{s} + \boldsymbol{\nu} \\
\boldsymbol{x} &= \underbrace{\boldsymbol{F} \mathcal{H}_{\text{tl}} \boldsymbol{F}^H}_{\mathcal{H}_{\text{df}}} \boldsymbol{s} + \underbrace{\boldsymbol{F} \boldsymbol{\nu}}_{\boldsymbol{w}}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{H}_{\text{tl}} &= \text{circular-convolution matrix} \\
\mathcal{H}_{\text{df}} &= \text{sub-carrier coupling matrix}
\end{aligned}
$$

$\mathcal{H}_{\text{df}}$ diagonal iff channel is LTI (assuming adequate prefix length)

Rayleigh $\mathrm{E}\{|[\mathcal{H}_{\mathsf{df}}]_{m,n}|^2\}$ for $N = 128$ and $f_{\mathsf{d}} = 0.03$:



Subcarrier Coupling Matrix $\mathcal{H}_{\mathsf{df}}$:

$$\mathcal{H}_{\mathsf{df}} = \begin{pmatrix} h_{\mathsf{df}}(0,0) & h_{\mathsf{df}}(-1,1) & \dots & h_{\mathsf{df}}(1-N,N-1) \\ h_{\mathsf{df}}(1,0) & h_{\mathsf{df}}(0,1) & \dots & h_{\mathsf{df}}(2-N,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ h_{\mathsf{df}}(N-1,0) & h_{\mathsf{df}}(N-2,1) & \dots & h_{\mathsf{df}}(0,N-1) \end{pmatrix}$$

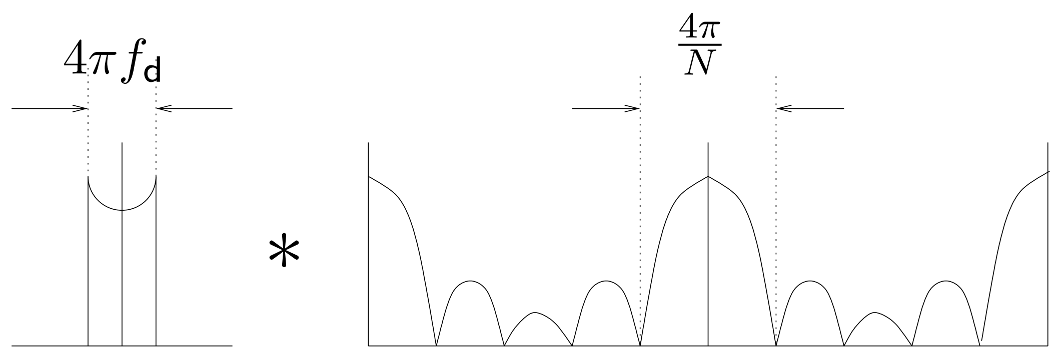
$$\begin{aligned} h_{\mathsf{df}}(\nu,k) &:= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{N-1} h_{\mathsf{tl}}(n,l) e^{-j\frac{2\pi}{N}n\nu} e^{-j\frac{2\pi}{N}lk} \\ &= \text{response at carrier } k+\nu \text{ to an impulse applied at carrier } k \end{aligned}$$

$$h_{\mathsf{tl}}(n,l) := \text{response at time } n \text{ to an impulse applied at time } n-l$$

ICI: Doppler Spread meets Finite Block Length:

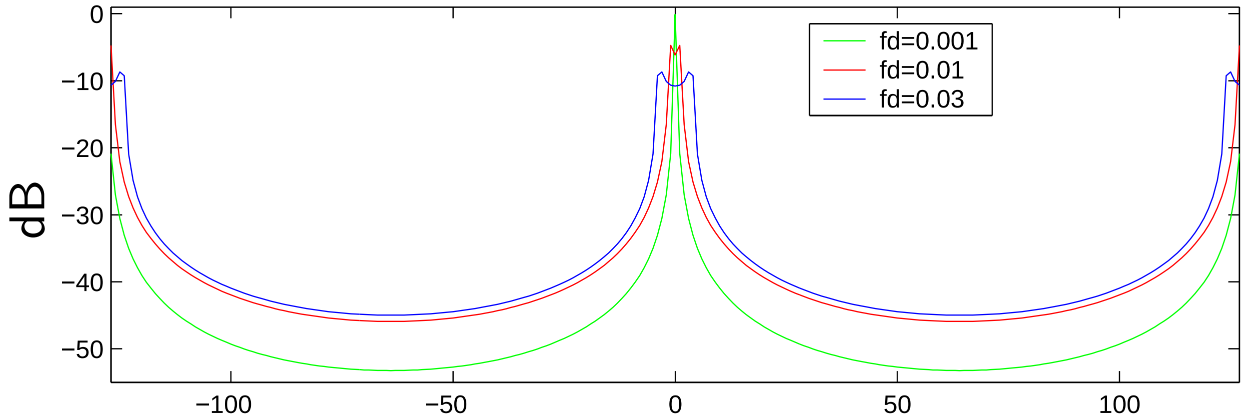
$$\mathrm{E}\left\{|h_{\mathsf{df}}(\nu,k)|^2\right\} = \overbrace{\left(\frac{I_{[0,2\pi f_{\mathsf{d}})}(|\phi|)\ \sum_l \sigma_l^2}{\sqrt{(2\pi f_{\mathsf{d}})^2-\phi^2}}\right)}^{\text{assuming WSSUS Rayleigh}} * \overbrace{\left(\frac{\sin(\phi N/2)}{N\sin(\phi/2)}\right)^2}^{\text{block length } N}\Bigg|_{\phi=\frac{2\pi}{N}\nu}$$

= Samples of

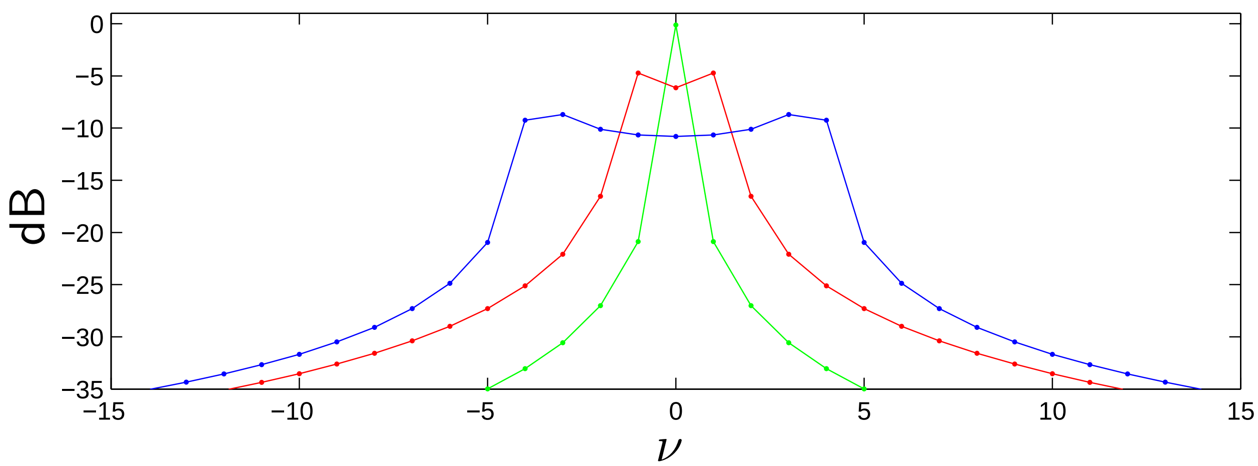


Note: *Zero Doppler spread* $\Rightarrow$ *Sample at sinc nulls* $\Rightarrow$ *Zero ICI.*

Rayleigh $\mathrm{E}\{|h_{\mathsf{df}}(\nu,\cdot)|^2\}$ for $N = 128$ and various f_{d}:



(zoomed)



CP-OFDM Equalization/Detection:

Objective: Recover finite-alphabet vector $\boldsymbol{s}$ from $\boxed{\boldsymbol{x} = \boldsymbol{\mathcal{H}}_{\text{df}}\,\boldsymbol{s} + \boldsymbol{w}}$.

Classical Strategies:

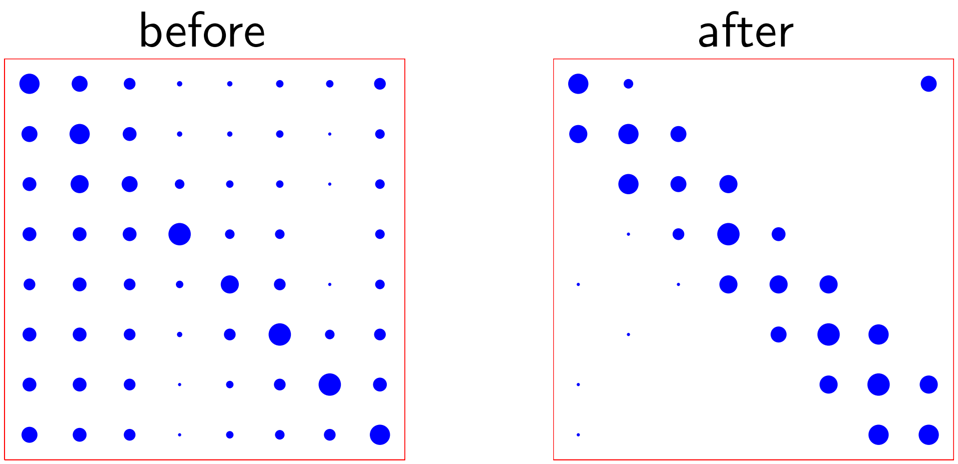
- ZF, LS: $\hat{\boldsymbol{s}}_{\text{zf}} = \text{slice}\Big[\boldsymbol{\mathcal{H}}_{\text{df}}^{-1}\boldsymbol{x}\Big]$
- MMSE: $\hat{\boldsymbol{s}}_{\text{mmse}} = \text{slice}\Big[\boldsymbol{\mathcal{H}}_{\text{df}}^{H}\big(\boldsymbol{\mathcal{H}}_{\text{df}}\boldsymbol{\mathcal{H}}_{\text{df}}^{H} + \sigma_w^2\boldsymbol{I}\big)^{-1}\boldsymbol{x}\Big]$
- MLSD: $\hat{\boldsymbol{s}}_{\text{mlsd}} = \arg\max_{\boldsymbol{s}} \|\boldsymbol{x} - \boldsymbol{\mathcal{H}}_{\text{df}}\boldsymbol{s}\|^2$

LTV channel: $\rightsquigarrow$ Equalization requires $\geq O(N^3)$ operations

$\rightsquigarrow$ Low-complexity advantage of CP-OFDM is lost!

Linear Pre-Processing to Simplify Detection:

- Use linear pre-processing to simplify detection.
  - Want to make $\mathcal{H}_{\text{df}}$ <u>sparse</u>
  - ICI-response "shortening"
  - Reminiscent of ISI-shortening for single-carrier MLSD
- Time-domain windowing = Doppler-domain convolution!



Low-Complexity Pre-Processing = Windowing:

- Apply time-domain window $\boldsymbol{b}$ before receiver's FFT:

$$\breve{\boldsymbol{x}} = \boldsymbol{F}\,\mathcal{D}(\boldsymbol{b})\boldsymbol{r}$$

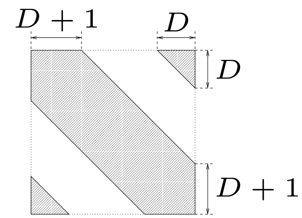
- Equivalent to (circularly) filtering the columns of $\boldsymbol{\mathcal{H}}_{\mathsf{df}}$...

$$\begin{aligned}
\breve{\boldsymbol{x}} &= \underbrace{\boldsymbol{F}\,\mathcal{D}(\boldsymbol{b})\boldsymbol{F}^H}_{\mathcal{C}(\boldsymbol{\beta})}\underbrace{\boldsymbol{F}\boldsymbol{r}}_{\boldsymbol{x}} \\
&= \mathcal{C}(\boldsymbol{\beta})\Big(\boldsymbol{\mathcal{H}}_{\mathsf{df}}\,\boldsymbol{s} + \boldsymbol{w}\Big) \quad \text{where } \boldsymbol{\beta} = \tfrac{1}{\sqrt{N}}\boldsymbol{F}\boldsymbol{b} \\
&= \underbrace{\mathcal{C}(\boldsymbol{\beta})\boldsymbol{\mathcal{H}}_{\mathsf{df}}}_{\text{effective channel}}\,\boldsymbol{s} + \underbrace{\mathcal{C}(\boldsymbol{\beta})\boldsymbol{w}}_{\text{colored noise}}
\end{aligned}$$

- This $\mathcal{O}(N)$ processing cannot *perfectly* suppress ICI, but it can come close...

Max-SINR Window Coefficients:

- Say we allow $2D$ diagonals of controlled ICI.



- Max-SINR window coefficients $\boldsymbol{b}_\star$ are

$$\boldsymbol{b}_\star = \text{gen-evec}_{\text{max}}\Big(\boldsymbol{A}\odot\boldsymbol{R}^*,\ \operatorname{diag}(\boldsymbol{R}+\sigma^2\boldsymbol{I})-\boldsymbol{A}\odot\boldsymbol{R}^*\Big)$$

where, for WSSUS Rayleigh fading,

$$[\boldsymbol{A}]_{m,n} = \frac{\sin\big(\frac{\pi}{N}(2D+1)(n-m)\big)}{N\sin\big(\frac{\pi}{N}(n-m)\big)}$$

$$[\boldsymbol{R}]_{n,m} = J_0\big(2\pi f_{\mathsf{d}}(n-m)\big)\sum_{l=0}^{N_h-1}\sigma_l^2$$

- Note that $\boldsymbol{b}_\star$ is a function of $\Big\{D, N, f_{\mathsf{d}}, \text{SNR} = \frac{\sum\sigma_l^2}{\sigma^2}\Big\}$

Symbol Estimation:

- After windowing to preserve $2D+1$ diagonals, have

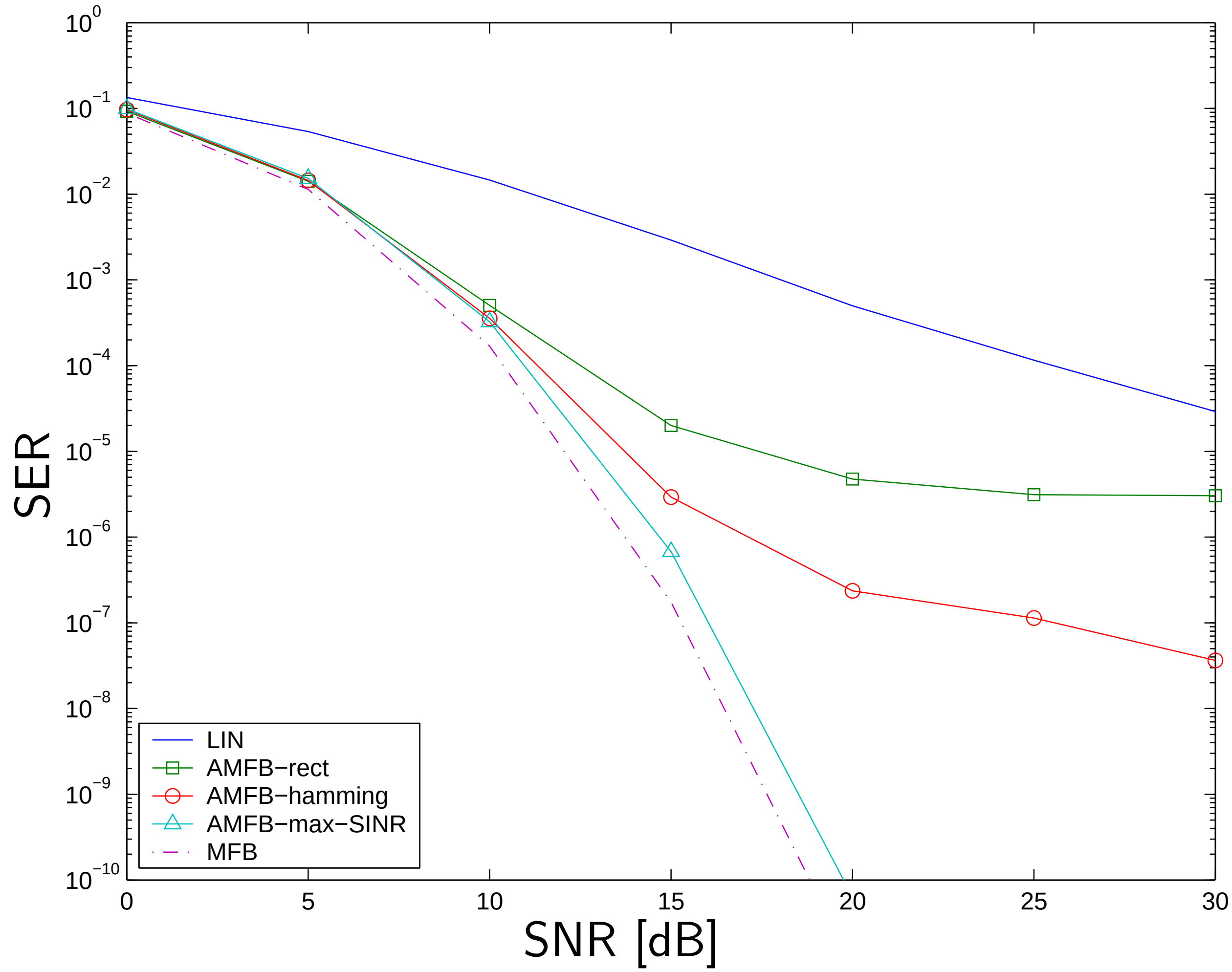
$$\breve{\boldsymbol{x}} = \underbrace{\mathcal{M}_{\rm D}\big(\mathcal{C}(\boldsymbol{\beta})\boldsymbol{\mathcal{H}}_{\sf df}\big)}_{\breve{\boldsymbol{\mathcal{H}}}_{\sf df}}\boldsymbol{s} + \underbrace{\overline{\mathcal{M}}_{\rm D}\big(\mathcal{C}(\boldsymbol{\beta})\boldsymbol{\mathcal{H}}_{\sf df}\big)}_{\approx \boldsymbol{0}}\boldsymbol{s} + \mathcal{C}(\boldsymbol{\beta})\boldsymbol{w}$$

  where $\mathcal{M}_{\rm D}(\cdot)$ is a mask operator and $\overline{\mathcal{M}}_{\rm D}(\cdot)$ its complement.

- Goal: Estimate $\{s_0, \ldots, s_{N-1}\}$ given $\breve{\boldsymbol{\mathcal{H}}}_{\sf df}$, $\mathcal{C}(\boldsymbol{\beta})$, and $\breve{\boldsymbol{x}}$.

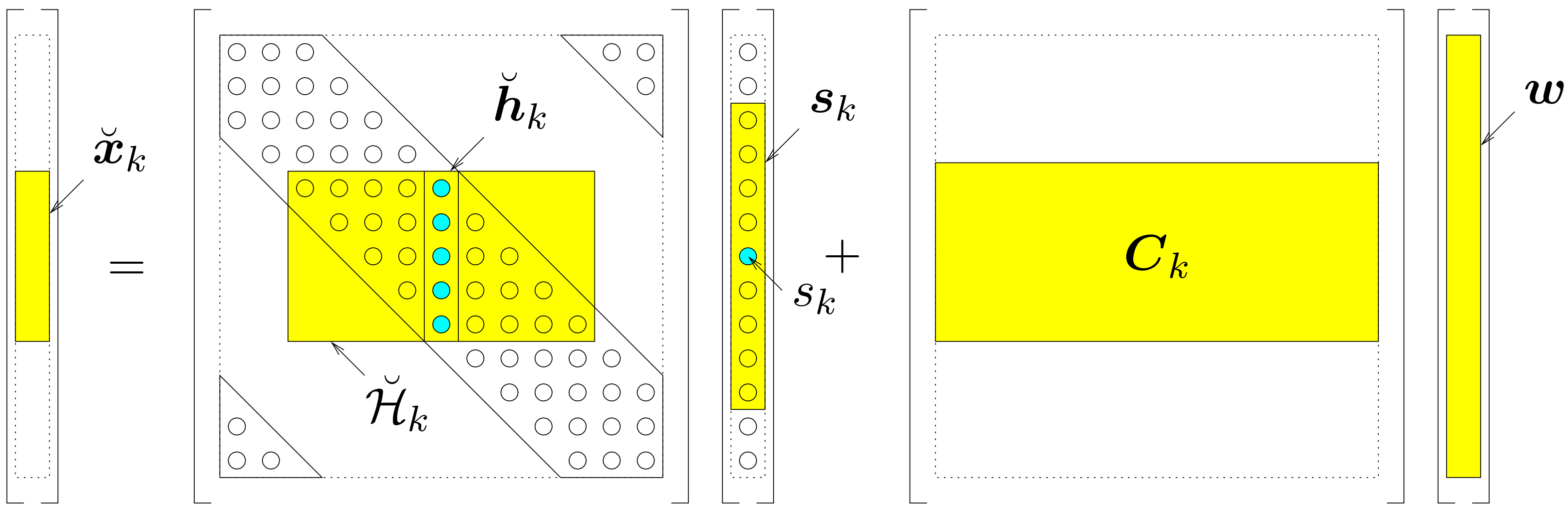
- Benchmarks:
  - Linear MMSE.
  - MFB: uses true $\mathcal{C}(\boldsymbol{\beta})\boldsymbol{\mathcal{H}}_{\sf df}$ with perfect interference cancellation.
  - AMFB: uses sparse $\breve{\boldsymbol{\mathcal{H}}}_{\sf df}$ with perfect interference cancellation.

Effect of Windowing on AMFB

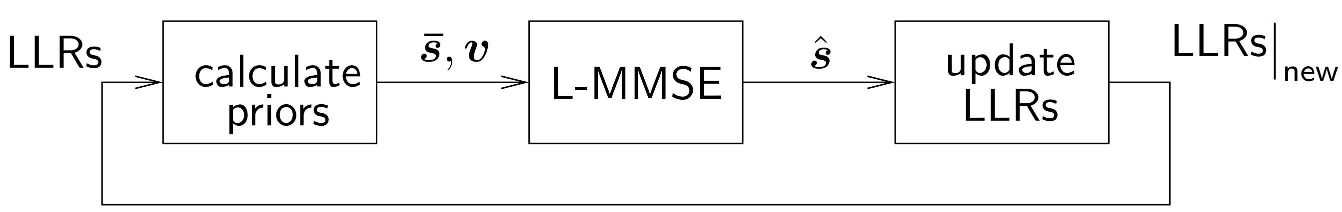


Iterative MMSE Estimation:

L-MMSE estimate of s_k:



Block Iteration for $\hat{\boldsymbol{s}}$:



Variations on Iterative Joint Estimation:

1. *Block Iterative Estimation* (BIE):
   Estimate block of symbols $\{\hat{s}_0, \ldots, \hat{s}_{N-1}\}$ then update block of priors $\{\bar{s}_0, \ldots, \bar{s}_{N-1}\}$ and $\{v_0, \ldots, v_{N-1}\}$. Repeat...

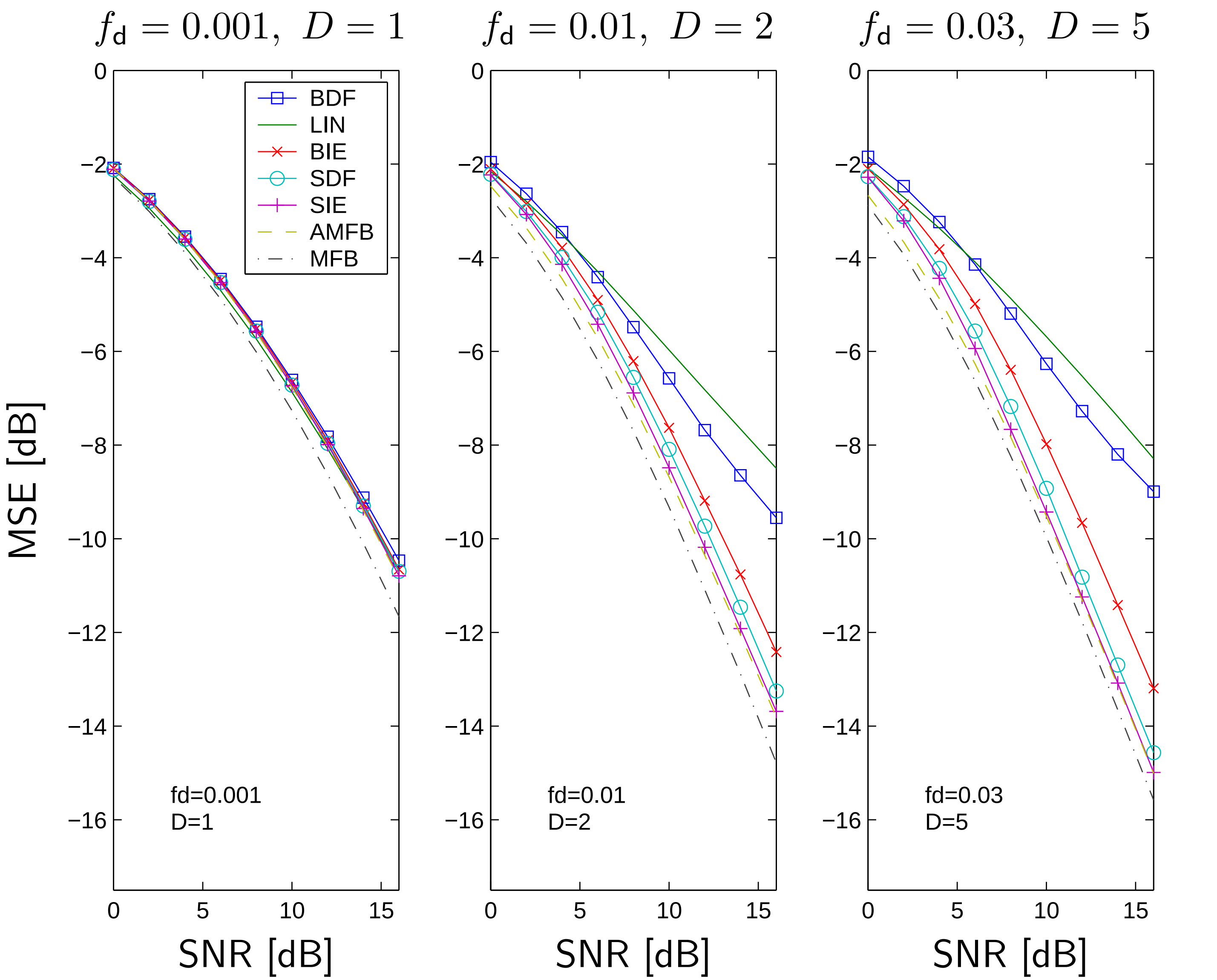
2. *Sequential Iterative Estimation* (SIE):
   For each k, compute $\hat{s}_k$ then immediately update priors $\bar{s}_k$ and v_k. Repeat with $k \to \langle k+1 \rangle_N$...

3. *Block Decision Feedback* (BDF):
   Like BIE, but $\bar{s}_k^{(i+1)} = \mathrm{sgn}(\hat{s}_k^{(i)})$ and $v_k^{(i+1)} = 0$.

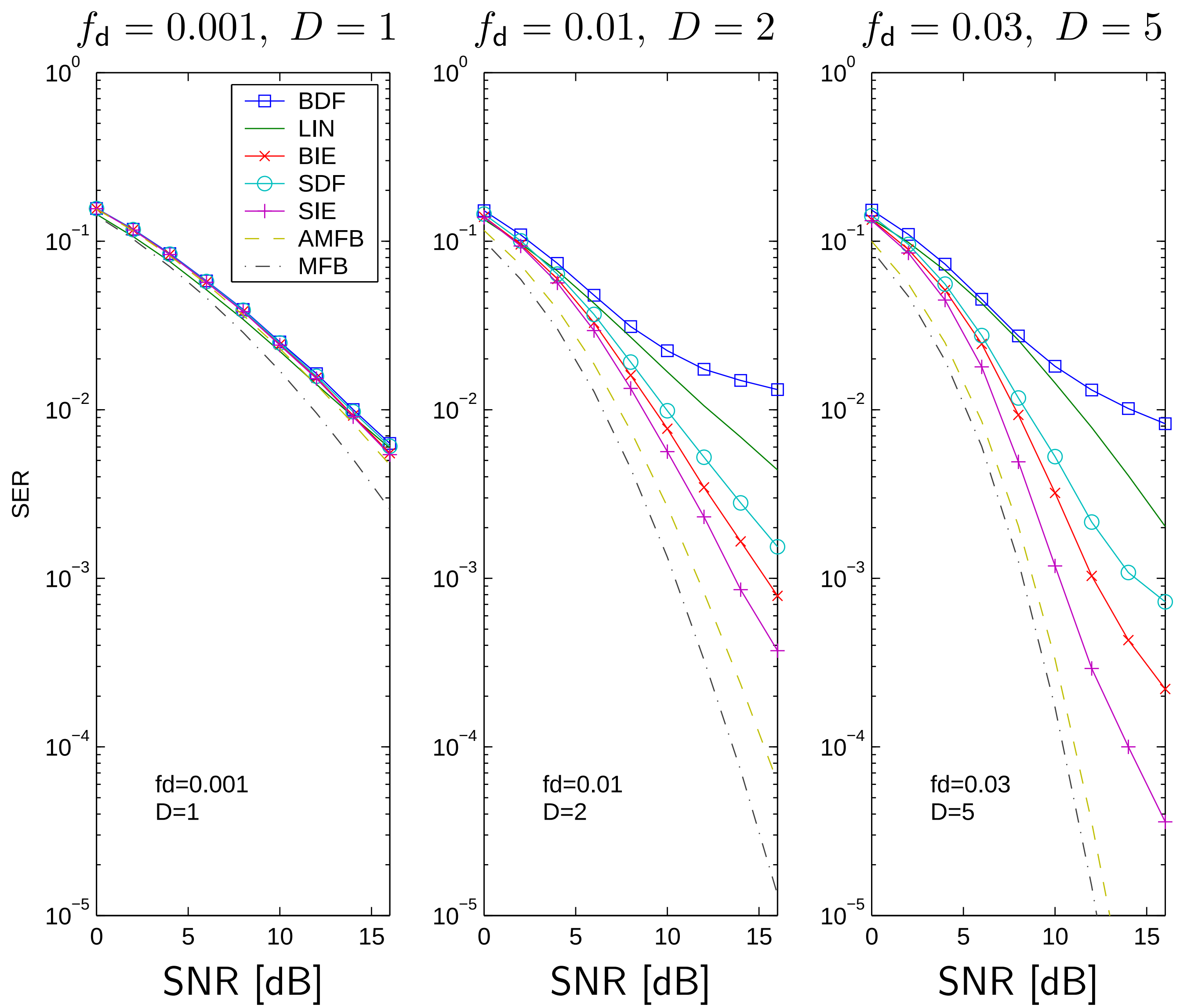
4. *Sequential Decision Feedback* (SDF):
   Like SIE, but $\bar{s}_k^{(i+1)} = \mathrm{sgn}(\hat{s}_k^{(i)})$ and $v_k^{(i+1)} = 0$.

Exploit banded $\breve{\mathcal{H}}_{\mathsf{df}}$; Need only $\mathcal{O}(D^2 N)$ operations per iteration.

MSE versus SNR after 2 iterations $(N = 128,\ \boldsymbol{b} = \hat{\boldsymbol{b}}_\star)$:



SER versus SNR after 2 iterations $(N = 128,\ \boldsymbol{b} = \hat{\boldsymbol{b}}_\star)$:



Summary:

- CP-OFDM reception complicated by time-selectivity.
- Proposed a two-stage CP-OFDM receiver for doubly-selective channels:
  1. SINR-optimal windowing,
  2. Iterative MMSE estimation.
- Like classical CP-OFDM receivers, requires $\mathcal{O}(\log N)$ ops/symbol.
- Performance:
  - MSE is $\sim$ 1–2dB from MFB.
  - Uncoded error rate is ~ 3dB from MFB.
  - Soft decoding can be easily incorporated in increase performance.