On the Achievable Diversity-vs-Multiplexing Tradeoff in Cooperative Channels

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***Abstract*— In this paper, we propose new cooperation protocols for coherent flat-fading channels consisting of two (half-duplex) partners and one cell site. In our work, we differentiate between the multiple access (up-link) and broadcast (down-link) channels. For the cooperative multiple access channel, we upper-bound the achievable diversity-vs-multiplexing tradeoff by that of a point-to-point system with two transmit and one receiver antenna. We then proceed to show that the proposed protocol achieves this optimal diversity-vs-multiplexing tradeoff for all multiplexing gains. For the cooperative broadcast channel, the proposed protocol is shown to achieve the extreme points of the tradeoff curve of the corresponding point-to-point system (i.e., $(d = 2, r = 0)$ and $(d = 0, r = 1)$). For other multiplexing gains (i.e., $0 < r < 1$), the proposed scheme is shown to uniformally dominate all known cooperative protocols. A distinguishing feature of the proposed schemes is that they do not rely on orthogonal subspaces, allowing for a more efficient use of resources. Furthermore, our analysis reveals a fundamental difference between the cooperative multiple access channel and the relay channel model which inspires most known cooperation protocols.**

I. Introduction

Recently, there has been a growing interest in the design and analysis of cooperative transmission schemes for wireless fading channels [1], [2], [3], [4]. In the context of a coherent channel, where the channel state information (CSI) is available only at the receiving end, the basic idea is to leverage the antennas available at the other nodes in the network as a source of *virtual* spatial diversity. Here, we use the same setup as the one considered by Laneman *et al.* in [1], [2]. In these works, the authors proposed several cooperation protocols where the two partners rely on the of use of orthogonal subspaces to repeat each other's signals. Several follow-up works have proposed coding schemes for cooperation which also rely on the use of orthogonal subspaces (e.g., [4]). The use of orthogonal subspaces in these schemes entails a significant price in terms of performance loss for high spectral efficiency scenarios, as observed in [2]. In fact, the authors in [2] pose the following open problem "*a key area of further research is exploring cooperative diversity protocols in the high spectral efficiency regime.*" This problem motivates our work here.

To establish the gain offered by the proposed protocols, we adopt the diversity-vs-multiplexing tradeoff as a performance measure. This powerful tool was introduced by Zheng and Tse for point-to-point multi-input-multi-output (MIMO) channels in [5] and later extended by Tse, Viswanath and Zheng for the (non-cooperative) multiple access channel in [6]. In our work, we differentiate between the multiple access and broadcast scenarios. In the multiple access channel, we propose a novel cooperation protocol where an *artificial* inter-symbol-interferece (ISI) channel is created. We then establish the optimality of this protocol by showing that it achieves the optimal tradeoff curve of a point-to-point MIMO system with two transmit and one receive antennas. We then turn our attention to the cooperative broadcast scenario where the proposed protocol exploits the relay channel formulation. This scheme is shown to achieve the two extreme points (i.e., $(d = 2, r = 0), (d = 0, r = 1)$) of the tradeoff curve of a point-to-point MIMO system with one transmit and two receive antennas. Furthermore, our cooperative broadcast protocol is shown to dominate all known cooperation schemes for $0 < r < 1$. Our results, therefore, establish the strict sub-optimality of cooperation protocols that rely on the use of orthogonal subspaces. Moreover, our results reveal a fundamental difference between the cooperative multiple access scenario and the relay channel formulation which inspired many of the existing cooperation scheme. In fact, as illustrated later, exploiting this difference is one of the enabling keys for achieving the optimal tradeoff in the cooperative multiple access scenario.

II. The Cooperative Multiple Access Channel

A. System Model

We consider the case of two sources and one destination, each equipped with a single antenna. Following in the footsteps of [2], we impose the half duplex constraint on the cooperating partners such that a node can only transmit or receive at any point in time. At this point, we wish to stress that the half duplex constraint is quite restrictive and relaxing it would significantly simplify the design. To simplify presentation, we focus here on the symmetric scenario where the two nodes transmit at the same rate. We adopt a spatially white quasi-static flat Rayleigh fading model where all the channel gains are assumed to be fixed during one code word and change independently from one code word to the next. We further assume a coherent model implying that the inter-source channel gain is known by both sources as well as the destination, whereas the source-destination channel gains are known only by the destination. The additive noise is assumed to be white and Gaussian. We further assume that the variance of the source noise is proportional to that of the destination so that there's always a fixed offset between source and destination signal to noise ratios (SNRs).

B. The Proposed Cooperation Protocol

In the proposed scheme, sources transmit once per frame, where a frame is defined by two consecutive symbols. The two sources, therefore, alternate their transmissions and, when active, each source transmits a linear combination of its current symbol and the (noisy) signal received from its partner during the last time slot (a time slot corresponds to one symbol in this terminology). For source j and frame k, we denote the broadcast and repetition gains by a_j and b_j, respectively, the information symbol by $x_{j,k}$, and the transmitted signal by $t_{j,k}$. At startup the transmitted signals will take the form

$$\begin{aligned}
t_{1,0} &= a_1 x_{1,0} \\
t_{2,0} &= a_2 x_{2,0} + b_2 (h t_{1,0} + w_{2,0}) \\
t_{1,1} &= a_1 x_{1,1} + b_1 (h t_{2,0} + w_{1,0}) \\
t_{2,1} &= a_2 x_{2,1} + b_2 (h t_{1,1} + w_{2,1})
\end{aligned}$$

where h denotes the inter-source unit-variance circularly symmetric complex Gaussian channel gain, $w_{j,k}$ are samples from a circularly symmetric white Gaussian process with variance σ_w^2. The corresponding signals received by the destination are:

$$\begin{aligned}
y_{1,0} &= g_1 t_{1,0} + v_{1,0} \\
y_{2,0} &= g_2 t_{2,0} + v_{2,0} \\
y_{1,1} &= g_1 t_{1,1} + v_{1,1} \\
y_{2,1} &= g_2 t_{2,1} + v_{2,1}
\end{aligned}$$

where g_j is the gain of the channel connecting source j to the destination and $v_{j,k}$ the complex Gaussian destination noise of variance σ_v^2. Note that, in this scheme, sources do not transmit and receive simultaneously. The broadcast and repetition gains $\{a_j, b_j\}$ are chosen to optimize a metric of performance. As a consequence of symmetry, a_1 and a_2, as well as b_1 and b_2, will have the same optimal value. Thus, we assume that broadcast and repetition gains are the same at each source and omit the subscripts, yielding $\{a, b\}$.

In order to characterize the diversity-multiplexing tradeoff achieved by this scheme, we first need to precisely define the signal to noise ratio, SNR, multiplexing gain, r and diversity gain, d. The signal-to-noise ratio is defined as:

$$\text{SNR} \triangleq \frac{E\{|t_{j,k}|^2\}}{\sigma_v^2}$$

Since the diversity-vs-multiplexing tradeoff analysis requires taking the limit SNR $\to \infty$, one can only obtain meaningful results if the ratio σ_w^2/σ_v^2 is held constant. Also, for a destination that receives data at a rate of R bits per channel-use, the multiplexing gain, i.e., r, is defined as:

$$r \triangleq \frac{R}{\log_2 \text{SNR}}$$

To define the diversity gain, d, we recall that the minimal error probability for each source is achieved by the individual ML decoder corresponding to that source. The diversity gain, d, is defined as the smallest exponential rate at which these minimal error probabilities decay at asymptotically large SNRs.

Having these definitions, we next state our result for the symmetric cooperative multiple access channel with two partners

Theorem 1: The optimal diversity-multiplexing tradeoff for the symmetric cooperative multiple access channel with two partners is characterized by:

$$d = 2(1 - r). \tag{1}$$

Furthermore, this optimal tradeoff curve is achieved by the proposed cooperation strategy.

Proof: Due to the space limitation, we only provide a sketch of the proof here. The technical details of the proof are reported in [7]. Realizing that (1) also corresponds to the optimal diversity gain for a point to point communication system with 2 transmit and 1 receive antennas, we only need to show that the proposed cooperation scheme achieves this optimal diversity gain. For this purpose we consider the joint ML decoder for the proposed scheme and characterize its error probability. Note that the error probability of the joint ML decoder upper-bounds the error probabilities of the individual sources' ML decoder. We assume that each of the sources is using a Gaussian random code of codeword length l and data rate R. Note that as the two sources utilize the channel one at a time, the data rate of the destination is also R.

The joint ML decoder's error probability, $\Pr(E)$, can be written as:

$$\Pr(E) = \Pr(E^{\{1\}}) + \Pr(E^{\{2\}}) + \Pr(E^{\{1,2\}})$$

where $E^{\{1\}}$ is the event that the decoder makes an error only in decoding the first source's message, $E^{\{2\}}$

is the similar event in regards to the second source and $E^{\{1,2\}}$ is the event that the decoder makes errors in decoding both of the messages. By using the base rule, $\Pr(E^{\{1\}})$, $\Pr(E^{\{2\}})$ and $\Pr(E^{\{1,2\}})$ can be upper bounded as:

$$\Pr(E^{\{1\}}) \leq \Pr(A) + \Pr(E^{\{1\}}|A_c)$$
$$\Pr(E^{\{2\}}) \leq \Pr(A) + \Pr(E^{\{2\}}|A_c)$$
$$\Pr(E^{\{1,2\}}) \leq \Pr(A) + \Pr(E^{\{1,2\}}|A_c)$$

where A can be any arbitrary event and A_c denotes its complement. In the sequel, we refer to A as the outage event and define it such that by choosing a sufficiently large codeword length, l, the three conditional error probabilities $\Pr(E^{\{1\}}|A_c)$, $\Pr(E^{\{2\}}|A_c)$ and $\Pr(E^{\{1,2\}}|A_c)$ can have an arbitrarily large exponential decay rate. This means that $\Pr(E^{\{1\}})$, $\Pr(E^{\{2\}})$ and $\Pr(E^{\{1,2\}})$ will all be dominated by $\Pr(A)$. Consequently:

$$\Pr(E) \dot{\leq} \Pr(A) \tag{2}$$

where $\dot{\leq}$ denotes exponentially less than or equal as defined in [5].

Description of the outage event, A, becomes much simpler if we define the following variables:

$$v_1 \triangleq -\frac{\log(|g_1|^2)}{\log(\text{SNR})} \qquad v_2 \triangleq -\frac{\log(|g_2|^2)}{\log(\text{SNR})}$$
$$v_3 \triangleq -\frac{\log(|h|^2)}{\log(\text{SNR})} \qquad u \triangleq \frac{\log(|b|^2)}{\log(\text{SNR})}$$

Note that sources' average transmission energy limit imposes the following constraint on the value of u:

$$u \leq v_3 \tag{3}$$

We choose u to be:

$$u \triangleq (v_3)^-$$

where $(x)^-$ means $\min\{x, 0\}$. This choice of u satisfies the constraint given by (3).

Using these definitions, $A^{'}$ (with $A^{'} = A \cap R^{3+}$), can be shown to be the set of all real 3-tuples with nonnegative elements that satisfy one of the following three conditions:

$$(1-v_1)^+ + (1-v_2)^+ < 2r$$
$$\max\{(1-v_1)^+, (1-v_2-v_3)^+\} < r$$
$$\max\{(1-v_1-v_3)^+, (1-v_2)^+\} < r$$

To compute the probability of $A^{'}$, we need the joint pdf of v_1, v_2 and v_3. It is easy to show that for 3-tuples, (v_1, v_2, v_3), with nonnegative elements, the joint pdf takes the form:

$$f(v_1, v_2, v_3) \doteq \text{SNR}^{-(v_1+v_2+v_3)}$$

Thus:

$$\Pr(A^{'}) \doteq \text{SNR}^{-d_{\text{out}}} \tag{4}$$
$$d_{\text{out}} \triangleq \inf_{(v_1,v_2,v_3)\in A^{'}} v_1 + v_2 + v_3$$

Therefore:

$$d_{\text{out}} = 2(1-r) \tag{5}$$

On the other hand, for any $v_i < 0$ the joint pdf of (v_1, v_2, v_3) decays with SNR exponentially. Thus at high SNRs:

$$\Pr(A) \doteq \Pr(A^{'}) \tag{6}$$

From (2), (6) and (4) we conclude:

$$\Pr(E) \dot{\leq} \text{SNR}^{-d_{\text{out}}}$$

This means that the joint ML decoder's error probability decays exponentially at least as fast as d_{out}. The fact that d_{out}, as given by (5), is identical to d, given by (1), completes the proof.

■

III. The Cooperative Broadcast Channel

In this section, we employ the same modelling assumptions used in the cooperative multiple access channel. The key difference here, compared to the cooperative multiple access channel, is the centralized knowledge of the information stream (i.e., **only** one node knows all the information). Noting that the proposed cooperative multiple access scheme relies primarily on exploiting the distributed knowledge of the information between the two sources, one can see the need for a different scheme here. The proposed scheme for this scenario relies on the relay channel formulation.

In the proposed scheme, the source transmits on every time slots in the frame, where a frame is defined by two consecutive time slots. The relay, on the other hand, transmits only once per frame. It simply repeats the (noisy) signal received from the source during the last time slot. It is important to observe that this design is dictated by the half duplex constraint which means that the relay can repeat one symbol in a frame at best. We denote the relay's repetition gain as b. Also for frame k, we denote the information symbols by $x_{j,k}, j = 1, 2$. The signals received by the destination during frame k are:

$$y_{1,k} = g_1 x_{1,k} + v_{1,k}$$
$$y_{2,k} = g_1 x_{2,k} + g_2 b h x_{1,k} + v_{2,k} + g_2 b w_{1,k}$$

where h, g_1 and g_2 denote the gains of the source-relay, source-destination, and relay-destination channels, respectively (These gains are i.i.d circularly symmetric complex Gaussian random variables). $w_{1,k}$ and $v_{j,k}, j = 1, 2$ denote the noises observed by the relay and the destination during the k^{th} frame. It is evident that, in this scheme, the relay is not simultaneously transmitting and receiving at any time slot. Again, the repetition gain b is chosen to optimize a metric of performance as shown later. Now, we define the signal to noise ratio as:

$$\text{SNR} \triangleq \frac{E\{|x_{j,k}|^2\}}{\sigma_v^2}$$

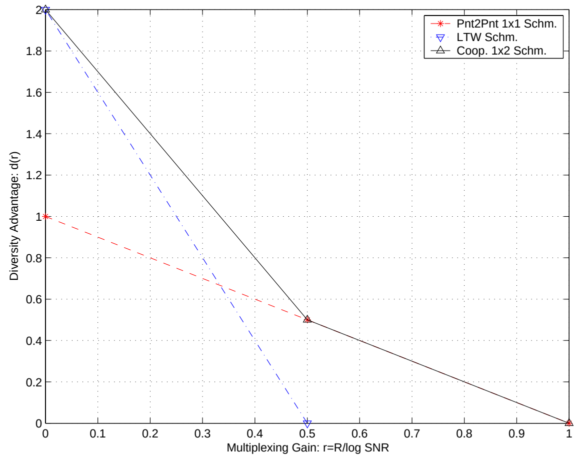

Fig. 1. Diversity-multiplexing tradeoff for the proposed cooperative broadcast scheme.

Similar to the cooperative multiple access scenario, the ratio σ_w^2/σ_v^2 is held constant while the SNR is allowed to grow to its limit. We are now ready to characterize the diversity-multiplexing tradeoff achieved by this scheme.

Theorem 2: The diversity-multiplexing tradeoff achieved by the proposed cooperative broadcast scheme is characterized by:

$$d = \begin{cases} 2(1-\frac{3}{2}r) & \text{if} \quad \frac{1}{2} \geq r \geq 0 \\ 1-r & \text{if} \quad 1 \geq r \geq \frac{1}{2} \end{cases} \tag{7}$$

This curve is shown in Fig. 1.

Proof: The sketch of proof is very similar to that of the cooperative multiple access channel. In particular, we assume that the source is using a Gaussian random code of codeword length, l, and data rate, R, and characterize the error probability of the ML decoder, $\Pr(E)$. We choose R to increase with SNR as given by:

$$R = r\log_2(\text{SNR})$$

The base rule is then used to upper-bound $\Pr(E)$, as:

$$\Pr(E) \leq \Pr(A) + \Pr(E|A_c)$$

where A denotes the outage event. The outage event is chosen such that $\Pr(A)$ always dominates $\Pr(E|A_c)$, thus:

$$\Pr(E) \dot{\leq} \Pr(A)$$

The only thing left is the characterization of A and derivation of its probability, $\Pr(A)$. By defining $v_i, i = 1,2,3$ and u exactly as their counterparts in the cooperative multiple access scenario, one can show that $A^{'}$ (with $A^{'} = A \cap R^{3+}$) is the set of all real 3-tuples with nonnegative elements that satisfy the following condition:

$$(\max\{1-\min\{v_1, v_2+v_3\}, 2(1-v_1)\})^+ < 2r$$

It is then straightforward to see that:

$$\Pr(A^{'}) \doteq \text{SNR}^{-d_{\text{out}}}$$
$$d_{\text{out}} \triangleq \inf_{(v_1,v_2,v_3)\in A^{'}} v_1+v_2+v_3$$

but d_{out} turns out to be identical to d given by (7). Arguments similar to those made for the cooperative multiple access scenario reveals that:

$$\Pr(A) \doteq \Pr(A^{'})$$

thus:

$$\Pr(E) \dot{\leq} \text{SNR}^{-d}$$

which completes the proof. ■

Fig. 1 shows that, while the proposed scheme does not achieve the upper bound of the corresponding point-to-point system, it still dominates the non-cooperative broadcast channel and the scheme proposed by Laneman, Tse, and Wornell (LTW) for all multiplexing gains. In fact, it is straightforward to see that the proposed scheme also dominates other cooperative schemes relaying on the use of orthogonal subspaces (e.g., [4]). The difference between the achievable tradeoff curve by our scheme and that in the corresponding point-to-point system can be traced back to the half-duplex constraint.

IV. Numerical Results

In this section, we report numerical results that quantify the performance gains offered by the proposed schemes in two representative scenarios. These numerical results correspond to outage probabilities and are meant to show that the superiority of our schemes in terms of the diversity-vs-multiplexing tradeoff translate in significant dB gains.

In Fig. 2 and Fig. 3, we compare the proposed schemes with the non-cooperative schemes and LTW amplify and forward scheme. For the LTW scheme, we assume that the link between the source and relay is noiseless. For the proposed cooperative multiple access scheme, the inter-source link is assumed to be 3 dB better than the source-destination channels whereas for the cooperative broadcast scheme the link between the source and the relay is assumed noiseless. In these two figures, the outage probabilities were computed through Mont-Carlo simulations. The fact that our cooperative multiple access scheme achieves a better tradeoff than LTW scheme manifests itself in Fig. 2 as a coding gain that increases with the data rate. On the other hand, Fig. 3 shows the superiority of our cooperative broadcast scheme over both, the non-cooperative scheme (for SNR's greater than 12dB) and LTW scheme, at the particular date rate of 2 bit per channel use.

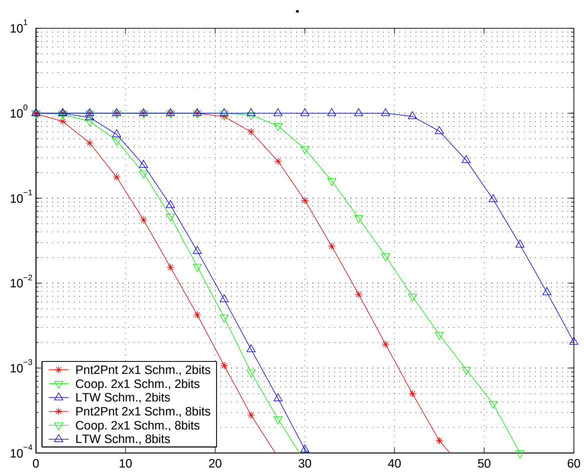

Fig. 2. Comparison of P_{out} vs. SNR for the proposed cooperative multiple access, LTW and point-to-point 2×1 schemes.

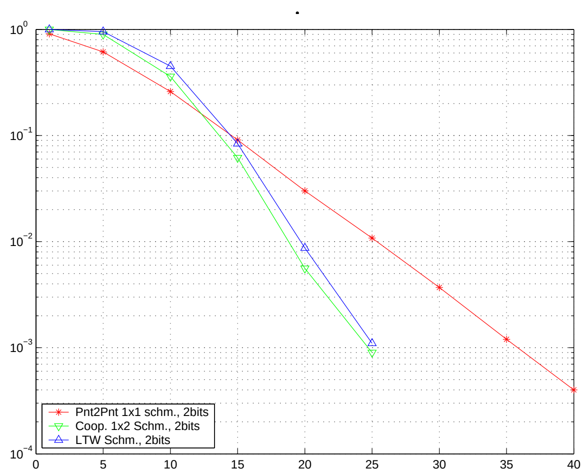

Fig. 3. Comparison of P_{out} vs. SNR for the proposed cooperative broadcast, LTW and point-to-point 1×1 schemes.

V. CONSLUSIONS

In this paper, we considered the design of cooperation protocols for a system consisting of two partners and one cell site. In particular, we differentiated between two scenarios. For the first scenario, i.e., the multiple access channel, we proposed a cooperation protocol that achieves the optimal diversity-vs-multiplexing tradeoff. This scheme exploits the distributed nature of the information stream among the two sources and offers a constructive proof for the achievability of the tradeoff curve of a 2×1 point-to-point system.

In the second scenario, i.e., the broadcast channel, we proposed a scheme that uniformly dominates all cooperation schemes that rely on the use of orthogonal subspaces. The superior diversity-vs-multiplexing curve for the proposed scheme, compared to LTW scheme for example, is a result of allowing the source to transmit continuously. We were still able to exploit the cooperative diversity by creating an *artificial* MIMO channel where the relay repeats every other symbol. On the other hand, in LTW scheme, only one node was allowed to transmit at any point in time, and hence, the maximum multiplexing gain was limited to 0.5.

Finally, it is instructive to contrast the tradeoff curves of the proposed cooperative multiple access and broadcast schemes. From Fig. 1, one can see that for multiplexing gains greater that 0.5, the diversity gain achieved by the proposed cooperative broadcast scheme is identical to that of the non-cooperative protocol. This is due to the fact that the *cooperative* link provided by the relay can not support multiplexing rates greater than 0.5, as a result of the half duplex constraint. Hence, for multiplexing gains larger than 0.5, there is only one link from the source to the destination, and thus, the tradeoff curve is identical to that of a point-to-point system with one transmit and one receive antenna. In the proposed cooperative multiple access protocol, this drawback was avoided by exploiting the availability of two information streams at the two sources. This implies that, with the half duplex constraint, cooperative multiple access schemes inspired by the relay channel formulation ignores a potential source for performance improvement (i.e., the distributed nature of the information across the different nodes).

REFERENCES

[1] J. N. Laneman, G. W. Wornell, “Energy-Efficient Antenna Sharing and Relaying for Wireless Networks,”*Wireless Communications and Networking Conference*, 2000, WCNC. 2000 IEEE , Volume: 1 , 23-28 Sept. 2000 Page(s): 7 -12 vol.1

[2] J. N. Laneman, D. N. C. Tse and G. W. Wornell, “Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior,” *IEEE Transactions on Information Theory*, 2002, submitted.

[3] J. N. Laneman and G. W. Wornell, “ Distributed Space-Time-Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks,” *IEEE Transactions on Information Theory*, Volume: 49 , Issue: 10 , Oct. 2003 Pages:2415 - 2425

[4] A. Stefanov and E. Erkip, “Cooperative Coding for Wireless Networks,” *IEEE Transactions on Communications*, accepted for publication

[5] L. Zheng and D. N. C. Tse. Diversity and Multiplexing: A Fundamental Tradeoff in Multiple Antenna Channels. *IEEE Trans. Info. Theory*, 49:1073 –1096, May 2003.

[6] D. N. C. Tse, P. Viswanath and L. Zheng Diversity-Multiplexing Tradeoff in Multiple Access Channels. *IEEE Trans. on Info. Theory*, 2003, submitted.

[7] K. Azarian, H. El Gamal, and P. Schniter. Towards Optimal Cooperative Diversity Protocols. *preprint*, March. 2004.