Denoising-based Vector AMP

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***Abstract*—The D-AMP methodology, recently proposed by Metzler, Maleki, and Baraniuk, allows one to plug in sophisticated denoisers like BM3D into the AMP algorithm to achieve state-of-the-art compressive image recovery. But AMP diverges with small deviations from the i.i.d.-Gaussian assumption on the measurement matrix. Recently, the VAMP algorithm has been proposed to fix this problem. In this work, we show that the benefits of VAMP extend to D-VAMP.**

Consider the problem of recovering a (vectorized) image $\boldsymbol{x}_0 \in \mathbb{R}^N$ from compressive (i.e., $M \ll N$) noisy linear measurements

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x}_0 + \boldsymbol{w} \in \mathbb{R}^M, \tag{1}$$

known as "compressive imaging." The "sparse" approach to this problem exploits sparsity in the coefficients $\boldsymbol{v}_0 \triangleq \boldsymbol{\Psi}\boldsymbol{x}_0 \in \mathbb{R}^N$ of an orthonormal wavelet transform $\boldsymbol{\Psi}$. The idea is to rewrite (1) as

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{v}_0 + \boldsymbol{w} \quad \text{for} \quad \boldsymbol{A} \triangleq \boldsymbol{\Phi}\boldsymbol{\Psi}^{\mathsf{T}}, \tag{2}$$

recover an estimate $\widehat{\boldsymbol{v}}$ of $\boldsymbol{v}_0$ from $\boldsymbol{y}$, and then construct the image estimate as $\widehat{\boldsymbol{x}} = \boldsymbol{\Psi}^{\mathsf{T}}\widehat{\boldsymbol{v}}$.

Although many algorithms have been proposed for sparse recovery of $\boldsymbol{v}_0$, a notable one is the approximate message passing (AMP) algorithm from [1]. It is computationally efficient (i.e., one multiplication by $\boldsymbol{A}$ and $\boldsymbol{A}^{\mathsf{T}}$ per iteration and relatively few iterations) and its performance, when M and N are large and $\boldsymbol{\Phi}$ is zero-mean i.i.d. Gaussian, is rigorously characterized by a scalar state evolution.

A variant called "denoising-based AMP" (D-AMP) was recently proposed [2] for *direct* recovery of $\boldsymbol{x}_0$ from (1). It exploits the fact that, at iteration t, AMP constructs a pseudo-measurement of the form $\boldsymbol{v}_0 + \mathcal{N}(\boldsymbol{0}, \sigma_t^2\boldsymbol{I})$ with known σ_t^2, which is amenable to any image denoising algorithm. By plugging in a state-of-the-art image denoiser like BM3D [3], D-AMP yields state-of-the-art compressive imaging.

AMP and D-AMP, however, have a serious weakness: they diverge under small deviations from the zero-mean i.i.d. Gaussian assumption on $\boldsymbol{\Phi}$, such as non-zero mean or mild ill-conditioning. A robust alternative called "vector AMP" (VAMP) was recently proposed [4]. VAMP has similar complexity to AMP and a rigorous state evolution that holds under right-rotationally invariant $\boldsymbol{\Phi}$—a much larger class of matrices. Although VAMP needs to know the variance of the measurement noise $\boldsymbol{w}$, an auto-tuning method was proposed in [5].

In this work, we integrate the D-AMP methodology from [2] into auto-tuned VAMP from [5], leading to "D-VAMP." (For a matlab implementation, see http://dsp.rice.edu/software/DAMP-toolbox.)

To test D-VAMP, we recovered the 128×128 *lena*, *barbara*, *boat*, *fingerprint*, *house*, and *peppers* images using 10 realizations of $\boldsymbol{\Phi}$. Table I shows that, for i.i.d. Gaussian $\boldsymbol{\Phi}$, the average PSNR and runtime of D-VAMP is similar to D-AMP at medium sampling ratios. The PSNRs for $\boldsymbol{v}$-based indirect recovery, using Lasso (i.e., "ℓ_1")-based AMP and VAMP, are significantly worse. At small sampling ratios, D-VAMP behaves better than D-AMP, as shown in Fig. 1.

To test robustness to ill-conditioning in $\boldsymbol{\Phi}$, we constructed $\boldsymbol{\Phi} = \boldsymbol{JSPFD}$, with $\boldsymbol{D}$ a diagonal matrix of random ± 1, $\boldsymbol{F}$ a (fast) Hadamard matrix, $\boldsymbol{P}$ a random permutation matrix, and $\boldsymbol{S} \in \mathbb{R}^{M\times N}$ a diagonal matrix of singular values. The sampling rate was fixed at $M/N = 0.1$, the noise variance chosen to achieve SNR=32 dB, and the singular values were geometric, i.e., $s_i/s_{i-1} = \rho \ \forall i > 1$, with ρ chosen to yield a desired condition number. Table II shows that (D-)AMP breaks when the condition number is ≥ 10, whereas (D-)VAMP shows only mild degradation in PSNR (but not runtime).

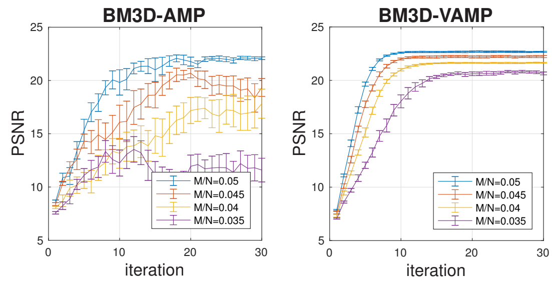

Fig. 1. PSNR versus iteration at several sampling ratios M/N for i.i.d. Gaussian $\boldsymbol{A}$.

TABLE I

AVERAGE PSNR AND RUNTIME FROM MEASUREMENTS WITH I.I.D. GAUSSIAN MATRICES AND ZERO NOISE AFTER 30 ITERATIONS

sampling ratio	10%		20%		30%		40%		50%	
	PSNR	time	PSNR	time	PSNR	time	PSNR	time	PSNR	time
ℓ_1-AMP	**17.7**	**0.5s**	**20.2**	1.0s	**22.4**	1.6s	24.6	2.3s	27.0	3.1s
ℓ_1-VAMP	17.6	**0.5s**	**20.2**	**0.9s**	**22.4**	**1.4s**	**24.8**	**1.8s**	**27.2**	**2.3s**
BM3D-AMP	**25.2**	**10.1s**	**30.0**	8.8s	**32.5**	8.6s	35.1	9.1s	37.4	9.8s
BM3D-VAMP	**25.2**	10.4s	**30.0**	**8.5s**	**32.5**	**8.2s**	**35.2**	**8.5s**	**37.7**	**8.8s**

TABLE II

AVERAGE PSNR AND RUNTIME FROM MEASUREMENTS WITH DHT-BASED MATRICES AND SNR=32 DB AFTER 10 ITERATIONS

condition no.	1		10		10^2		10^3		10^4	
	PSNR	time	PSNR	time	PSNR	time	PSNR	time	PSNR	time
ℓ_1-AMP	17.3	**0.02**	<0	—	<0	—	<0	—	<0	—
ℓ_1-VAMP	**17.4**	0.04	**17.4**	**0.04**	**15.6**	**0.03**	**14.7**	**0.03**	**14.4**	**0.03**
BM3D-AMP	**24.8**	**5.2s**	8.0	—	7.2	—	7.1	—	7.2	—
BM3D-VAMP	**24.8**	5.4s	**24.3**	**5.5s**	**22.6**	**5.3s**	**21.4**	**4.9s**	**20**	**4.5s**

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