A Factor-Graph Approach to Joint OFDM Channel Estimation and Decoding in Impulsive Noise Channels

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Outline

- Uncoordinated interference in communication systems
- Effect of interference on OFDM systems
- Prior work on OFDM receivers in uncoordinated interference
- Message-passing OFDM receiver design
- Simulation results

Uncoordinated Interference

- Typical Scenarios:
  - Wireless Networks:
    Ad-hoc Networks, Platform Noise, non-communication sources
  - Powerline Communication Networks:
    Non-interoperable standards, electromagnetic emissions

- Statistical Model:

Gaussian Mixture (GM)

$$p(n) = \sum_{k=0}^{K-1} \pi_k \mathcal{N}(n; 0, \gamma_k)$$

$K \in \mathbb{N}$: # of comp.

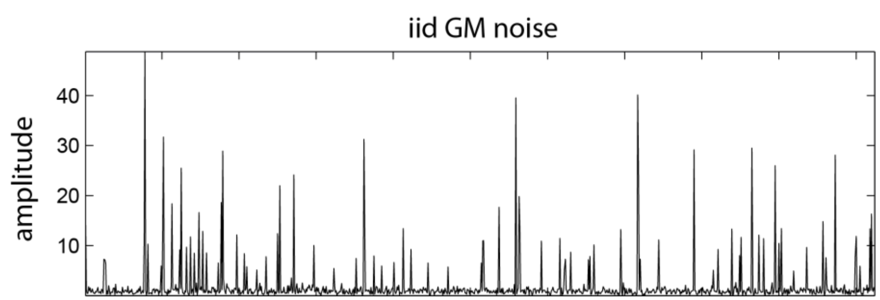
π_i: comp. probability

γ_i: comp. variance

Interference Model

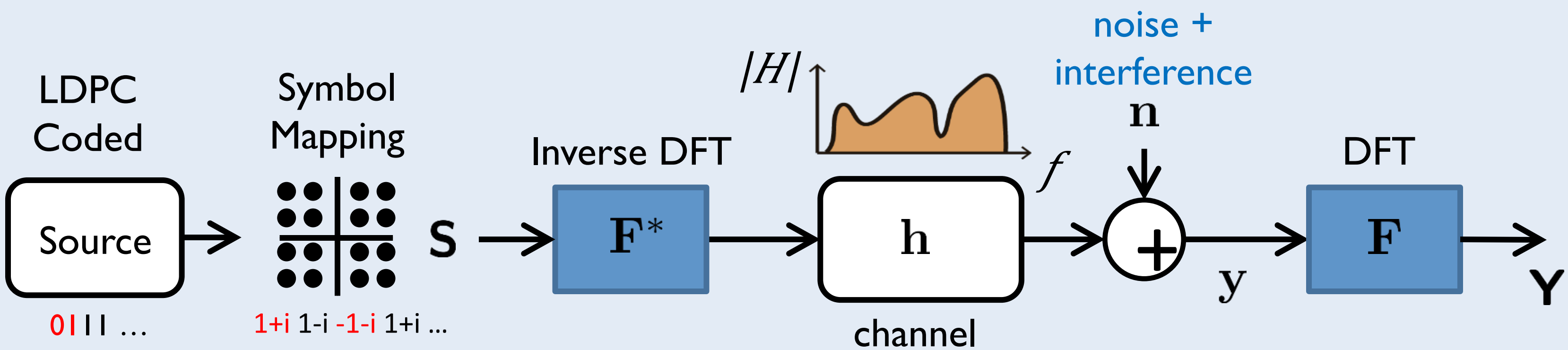
Two impulsive components:

- 7% of time/20dB above background
- 3% of time/30dB above background



OFDM Basics

System Diagram



Noise Model

$$n_j = g_j + i_j$$

where
- n_j : total noise
- g_j : background noise
- i_j : interference

and
- $g_j \sim \mathcal{N}(0, N_0)$
- i_j : GM or GHMM

Receiver Model

- After discarding the cyclic prefix:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{H}\mathbf{F}^*\mathbf{S} + \mathbf{g} + \mathbf{i}$$

- After applying DFT:

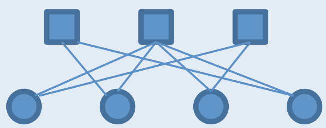
$$\mathbf{Y} = \mathbf{F}\mathbf{y} = \mathbf{F}\mathbf{H}\mathbf{F}^*\mathbf{S} + \mathbf{F}\mathbf{n} = \mathbf{H} \circ \mathbf{S} + \mathbf{G} + \mathbf{I}$$

- Subchannels: $\mathsf{Y}_k = \mathsf{H}_k \mathsf{S}_k + \mathsf{G}_k + \mathsf{I}_k$

OFDM Symbol Structure

Coding

- Added redundancy protects against errors



Data Tones

$$Y_k = H_k S_k + G_k + I_k$$

- Symbols carry information
- Finite symbol constellation
- Adapt to channel conditions

Pilot Tones

$$Y_k = H_k p + G_k + I_k$$

- Known symbol (p)
- Used to estimate channel

Null Tones

$$Y_k = G_k + I_k$$

- Edge tones (spectral masking)
- Guard and low SNR tones
- Ignored in decoding

pilots → linear channel estimation → symbol detection → decoding

But, there is unexploited information and dependencies

Prior OFDM Designs

Category	References	Method	Limitations
Time-Domain preprocessing (PP)	[Haring2001]	Time-domain signal estimation	• ignore OFDM signal structure • performance degrades with increasing SNR and modulation order
	[Zhidkov2008, Tseng2012]	Time-domain signal thresholding	
Sparse Signal Reconstruction	[Caire2008, Lampe2011]	Compressed sensing	• utilize only known tones • don't use interference models • complexity
	[Lin2011]	Sparse Bayesian Learning (SBL)	
Iterative Receivers	[Mengi2010, Yih2012]	Iterative preprocessing & decoding	• suffer from preprocessing limitations • ad-hoc design
	[Haring2004]	Turbo-like receiver	

All don't consider the non-linear channel estimation, and don't use code structure

Joint MAP-Decoding

- The MAP decoding rule of LDPC coded OFDM is:

$$\arg\max_{b_m \in \{0,1\}} P(b_m|\mathbf{Y};\Theta) \quad \forall m$$

- Can be computed as follows:

$$P(b_m|\mathbf{Y};\Theta) = \sum_{\mathbf{b}_{\setminus \mathbf{m}}} P(\mathbf{b}|\mathbf{Y};\Theta) \propto \sum_{\mathbf{b}_{\setminus \mathbf{m}}} p(\mathbf{Y}|\mathbf{b};\Theta)P(\mathbf{b})$$

$$\propto \sum_{\mathbf{S},\mathbf{c},\mathbf{b}_{\setminus \mathbf{m}}} \prod_{k=0}^{N-1} \int_{\mathbf{i},\mathbf{h}} p(\mathsf{Y}_k|\mathsf{S}_k,\mathbf{h},\mathbf{i};\Theta)p(\mathbf{h};\theta_{\mathbf{h}})p(\mathbf{i};\theta_{\mathbf{i}})P(\mathbf{S}|\mathbf{c})P(\mathbf{c}|\mathbf{b})$$

depends on linearly-mixed N noise samples and L channel taps

non iid & non-Gaussian

LDPC code

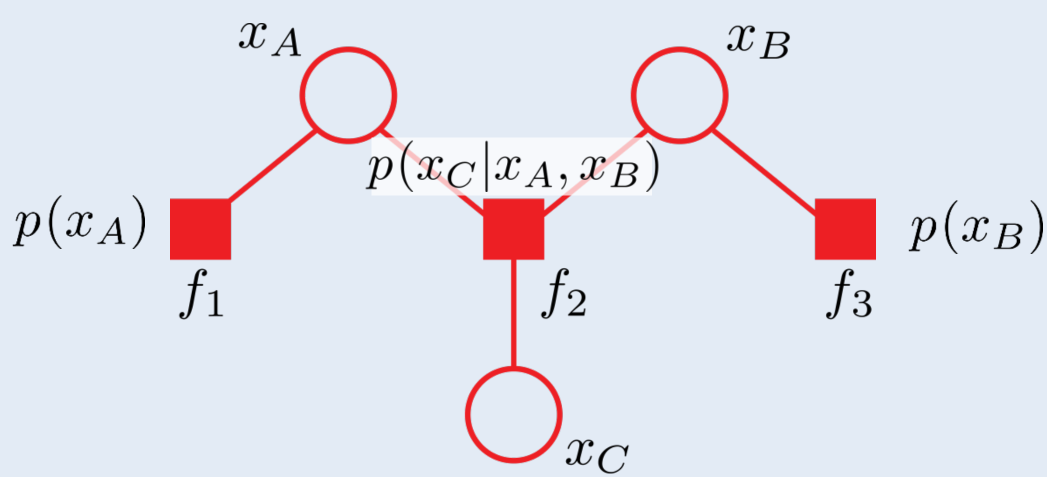
Very high dimensional integrals and summations !!

Belief Propagation on Factor Graphs

- Graphical representation of pdf-factorization
- Two types of nodes:
  - variable nodes denoted by circles
  - factor nodes (squares): represent variable "dependence"
- Consider the following pdf:

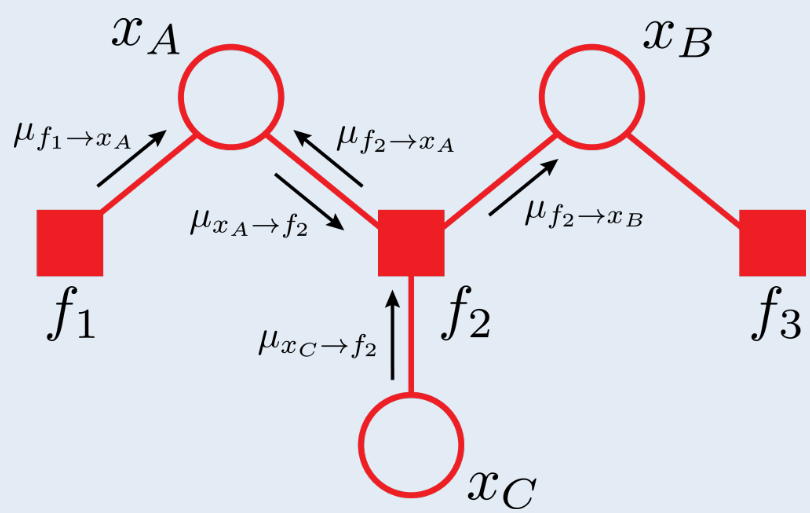
$$p(x_A, x_B, x_C) = p(x_A)p(x_B)p(x_C|x_A, x_B)$$

- Corresponding factor graph:



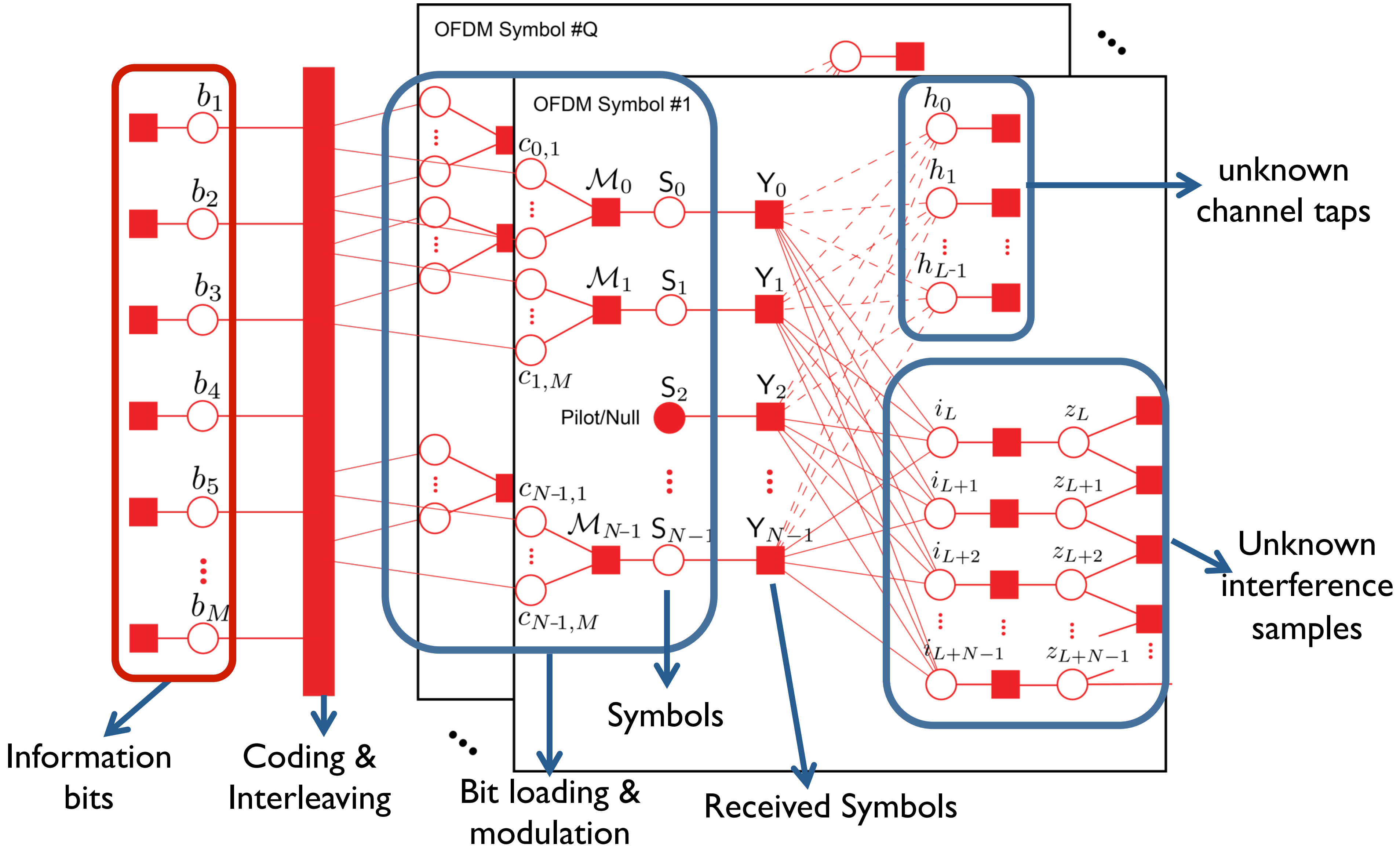
Belief Propagation on Factor Graphs

- Approximates MAP inference by exchanging messages on graph

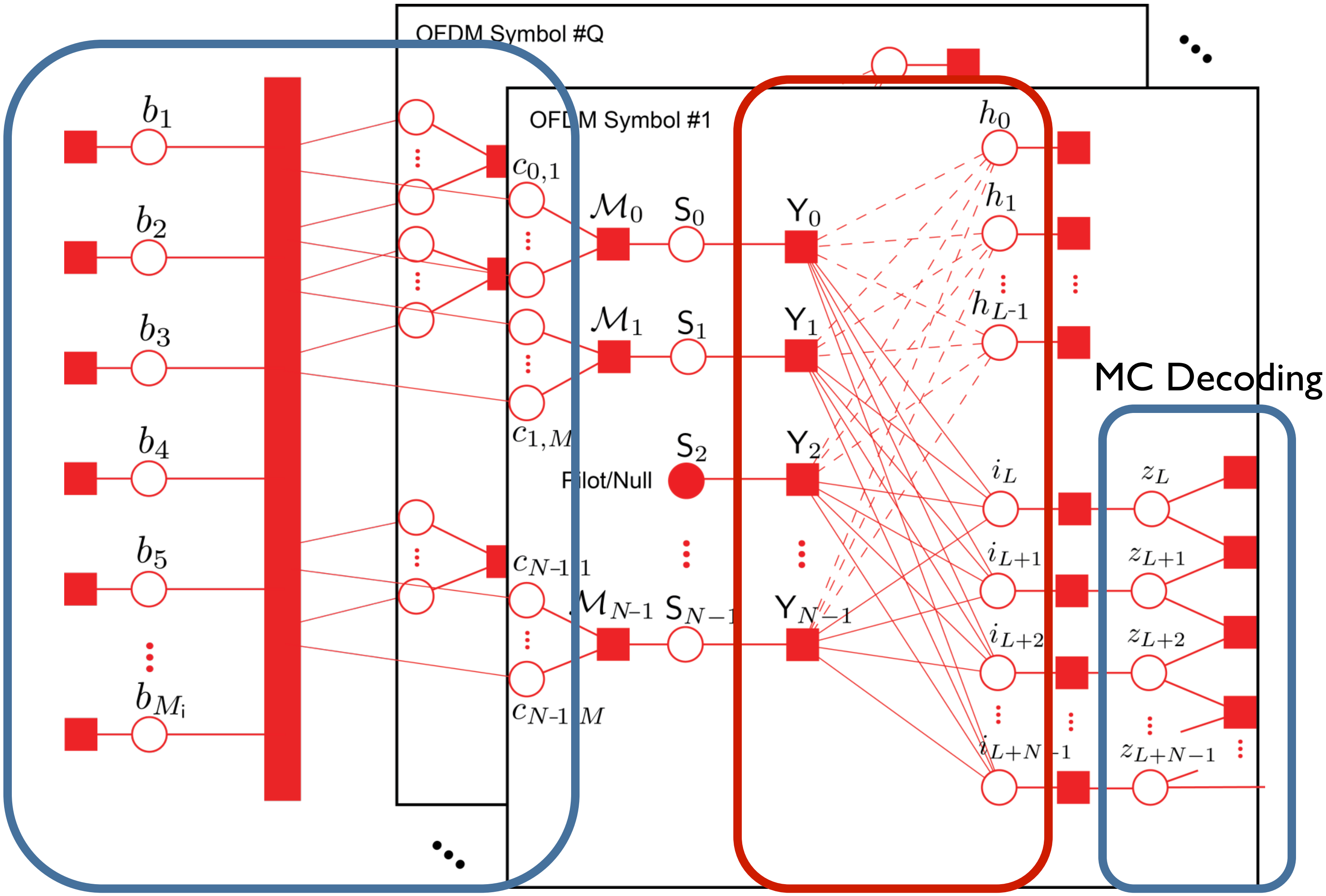


- Factor message = factor's belief about a variable's p.d.f.
- Variable message = variable's belief about its own p.d.f.
- Variable operation = multiply messages to update p.d.f.
- Factor operation = merges beliefs about variable and forwards
- Complexity = number of messages = node degrees

Coded OFDM Factor Graph



BP over OFDM Factor Graph



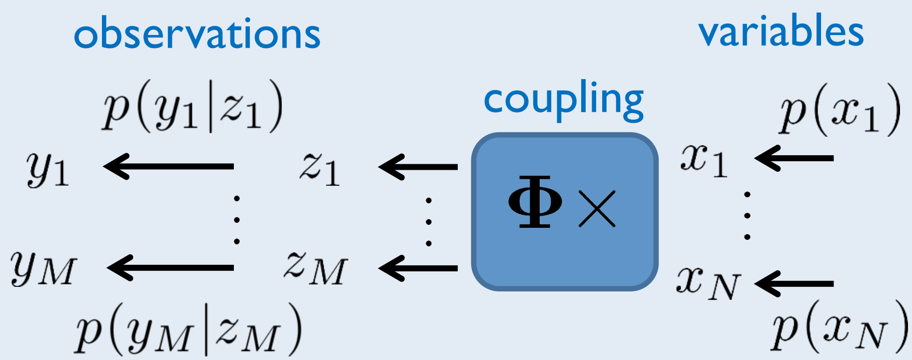
LDPC Decoding via BP *[MacKay2003]*

Node degree=N+L!!!

Generalized Approximate Message Passing

[Donoho2009,Rangan2010]

Estimation with Linear Mixing



- Generally a hard problem due to coupling
- Regression, compressed sensing, …
- OFDM systems:

Interference subgraph

given $\mathbf{H}$

$\mathbf{z} = \mathbf{I}$ and $\mathbf{x} = \mathbf{i}$

$\mathbf{\Phi} = \mathbf{F}$

channel subgraph

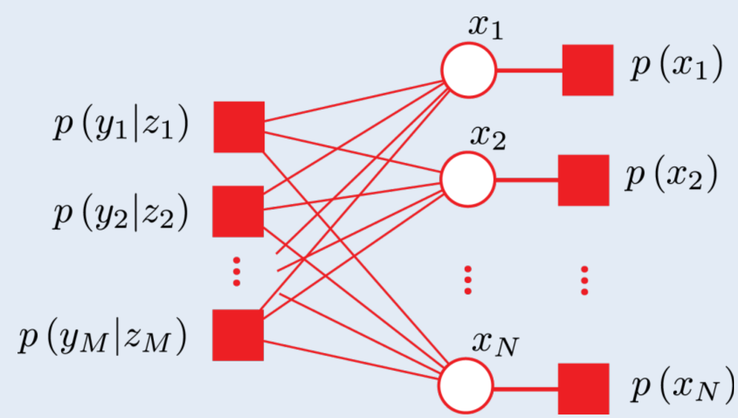
given $\mathbf{I}$

$\mathbf{z} = \mathbf{H}$ and $\mathbf{x} = \mathbf{h}$

$\mathbf{\Phi} = \sqrt{N}\mathbf{F}_{1:L}$

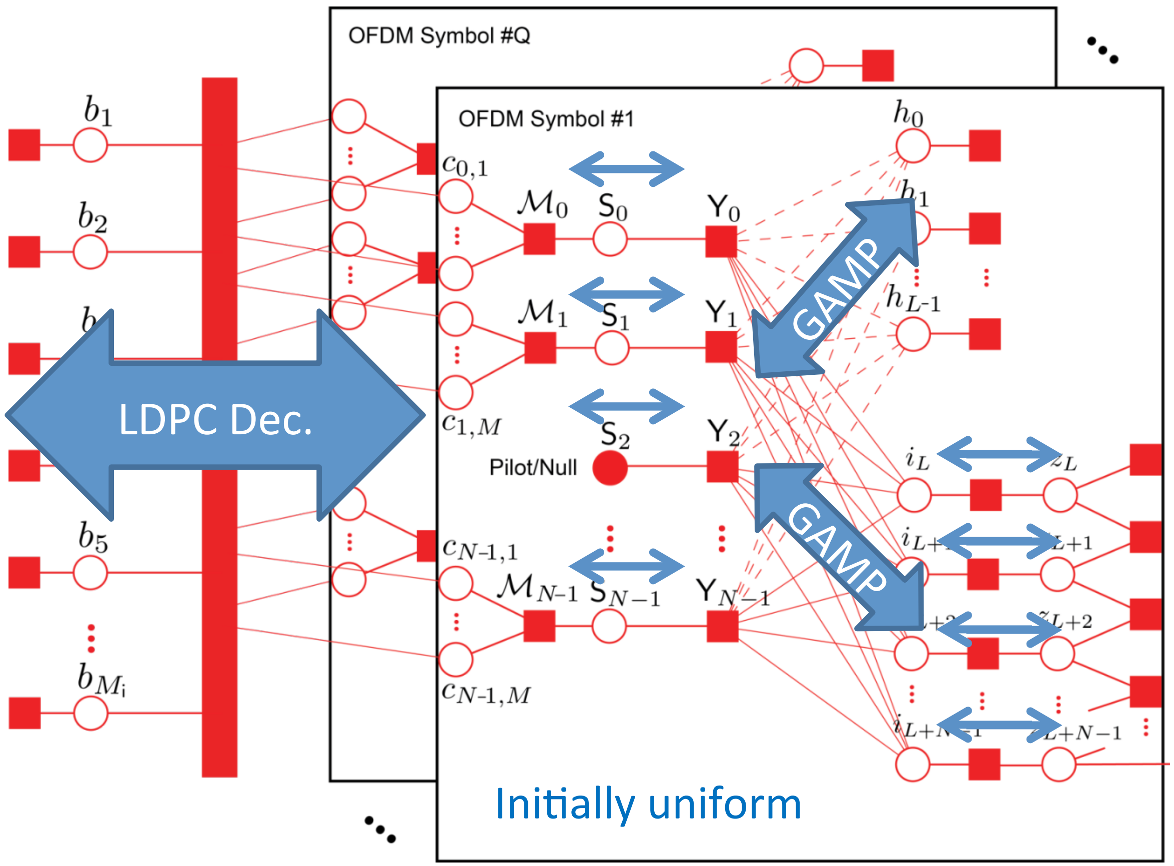
3 types of output channels for each

Decoupling via Graphs



- If graph is sparse use standard BP
- If dense and "large" $\rightarrow$ Central Limit Theorem
- At factors nodes treat z as Normal
- Depend only on means and variances of incoming messages
- Non-Gaussian output $\rightarrow$ quad approx.
- Similarly for variable nodes
- Series of scalar MMSE estimation problems: $O(N+M)$ messages

Message-Passing Receiver



Schedule

Turbo Iteration:

1. coded bits to symbols
2. symbols to **Y**
3. Run channel GAMP
4. Run noise "equalizer"
5. **Y** to symbols
6. Symbols to coded bits
7. Run LDPC decoding

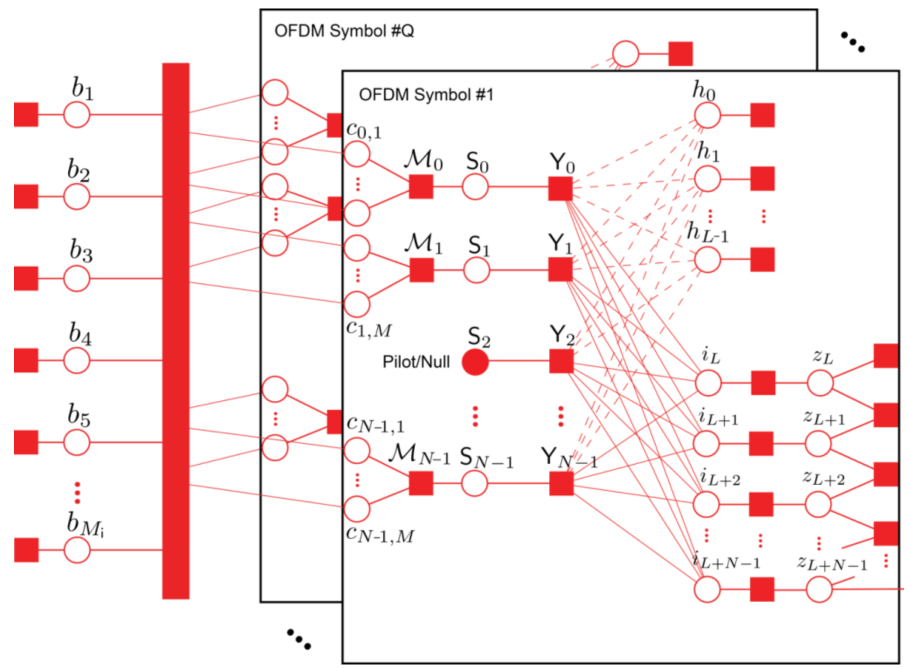
Equalizer Iteration:

1. Run noise GAMP
2. MC Decoding
3. Repeat

Receiver Design & Complexity

Design Freedom

- Not all samples required for sparse interference estimation
- Receiver can pick the subchannels:
  - Information provided
  - Complexity of MMSE estimation
- Selectively run subgraphs
  - Monitor convergence (GAMP variances)
  - Complexity and resources
- GAMP can be parallelized effectively



Operation	Complexity per iteration
MC Decoding	$\mathcal{O}(N)$
LDPC Decoding	$\mathcal{O}(M_c + C)$
GAMP	$\mathcal{O}(\min[N \log N, \lvert U\rvert^2])$

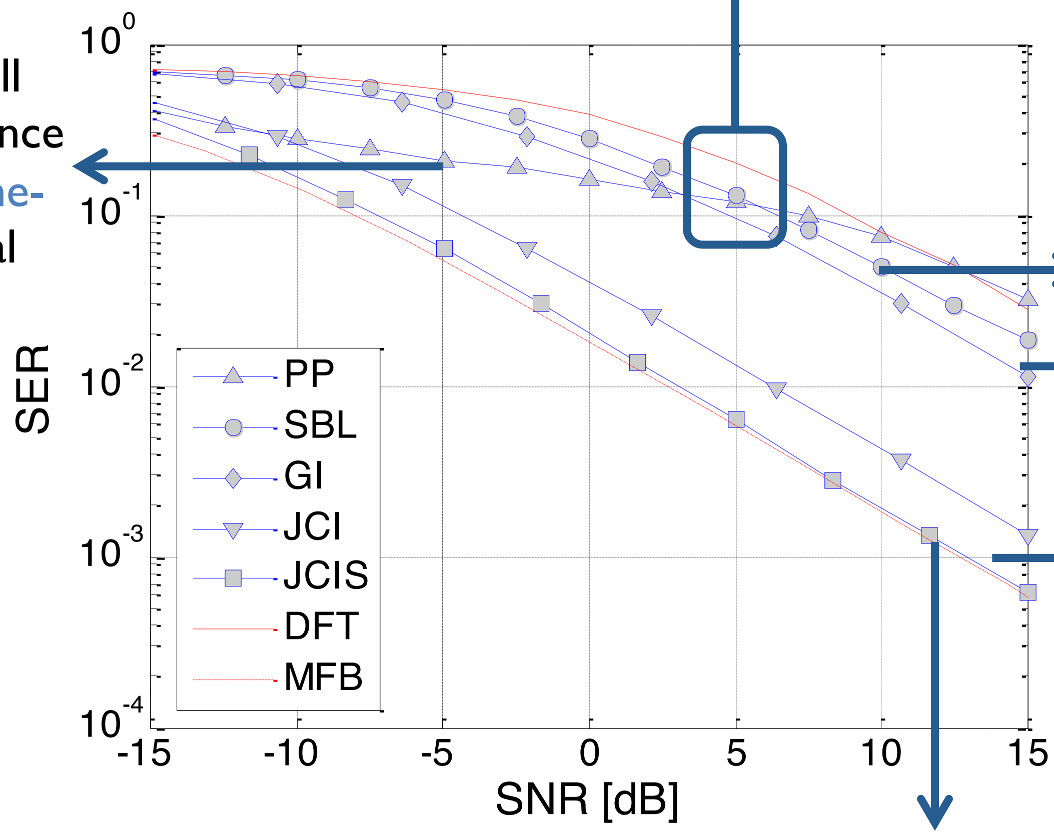
Notation

N : # tones
M_c : # coded bits
C : # check nodes
U : set of used tones

Simulation - Uncoded Performance

use LMMSE channel estimate

performs well when interference dominates time-domain signal



use only known tones, requires matrix inverse

2.5dB better than SBL

within 1dB of MF Bound
15db better than DFT

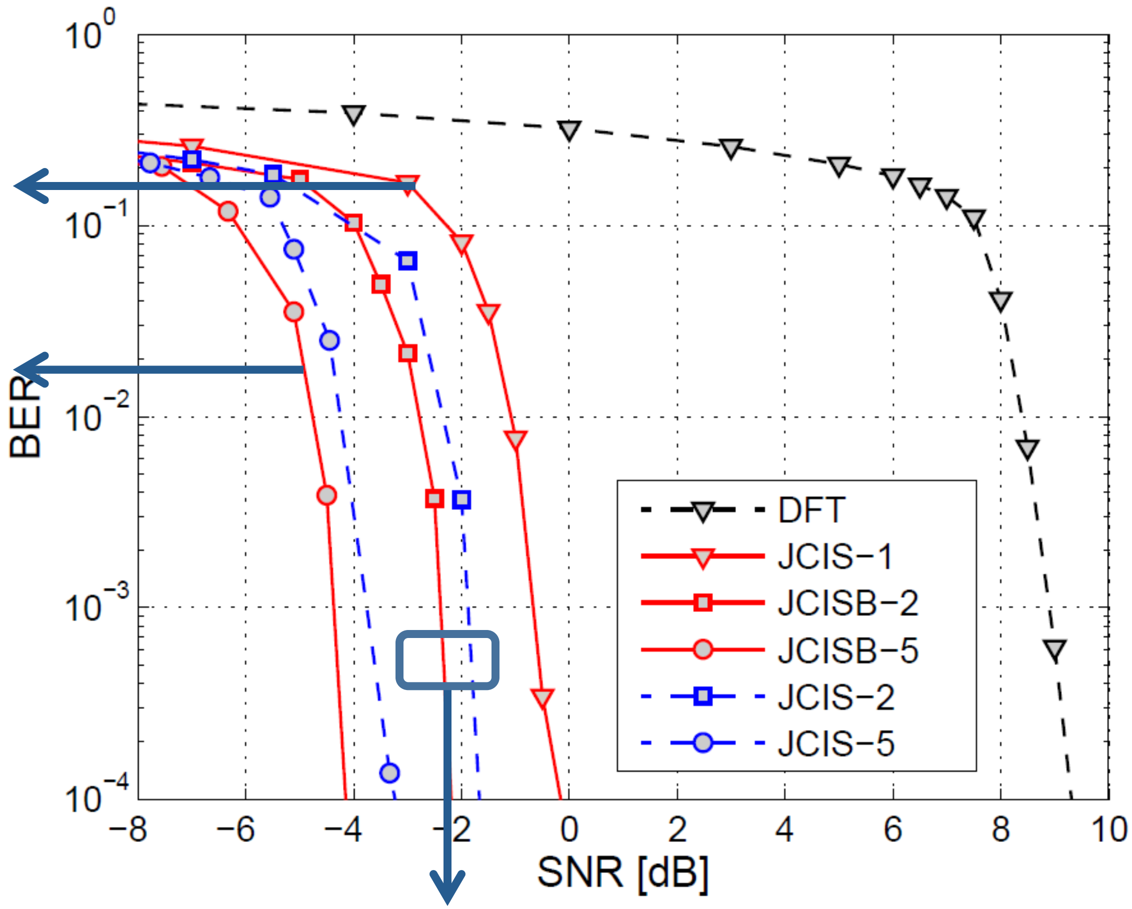
Settings
5 Taps
GM noise
4-QAM
256 tones
15 pilots
80 nulls

Matched Filter Bound: Send only one symbol at tone k

Simulation - Coded Performance

one turbo iteration gives 9db over DFT

5 turbo iterations gives 13dB over DFT



Settings

10 Taps
GM noise
16-QAM
N=1024
150 pilots
Rate ½
L=60k

Integrating LDPC-BP into JCNED by passing back bit LLRs gives 1 dB improvement

Summary

- Huge performance gains if receiver account for uncoordinated interference
- The proposed solution combines all available information to perform approximate-MAP inference
- Asymptotic complexity similar to conventional OFDM receiver
- Can be parallelized
- Highly flexible framework: performance vs. complexity tradeoff