Full-Duplex MIMO Relaying: Achievable Rates Under Limited Dynamic Range

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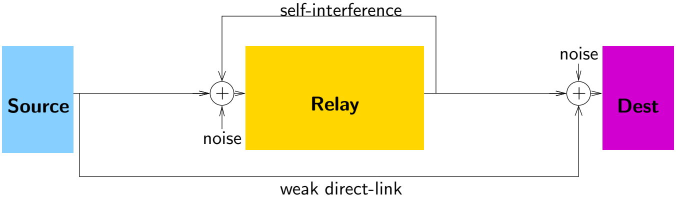
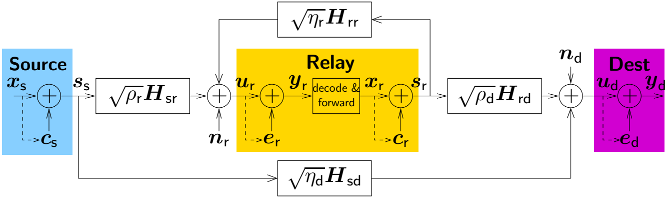
Introduction



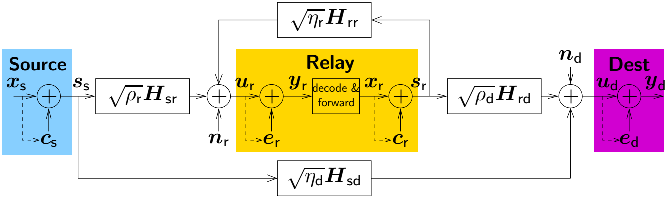
- Source communicates to Destination through decode and forward Relay
- MIMO at all terminals
- Relay operates in full-duplex mode
- Fundamental challenges:
  - high self-interference (as high as 100dB!)
  - limited dynamic range due to non-ideal transmitter and receiver hardware (power amp noise, non-linearities in ADC/DAC, oscillator phase noise, AGC noise)
- Fundamental question:
  - What is the maximum achievable rate of such systems?

System Model



- $\boldsymbol{H}_{ij}$ are MIMO Rayleigh fading propagation channels, assumed to be unknown and static.
- $\boldsymbol{n}_i$ is AWGN thermal noise of unit variance.
- ρ_i represents SNR and η_i represents INR.
- Dynamic range limitation modeled by signal-power dependent additive interference $\boldsymbol{c}_j$ at transmitters and $\boldsymbol{e}_i$ at receivers.
  - Facilitates tractable achievable-rate analysis.
  - Recent work (e.g. Rice, Lincoln) confirms the fidelity of this model.

Dynamic Range Limitation Model



- Each receive chain is corrupted by additive Gaussian interference with power proportional to the intended receive power; similar for each transmit chain

$$\boldsymbol{e}_i(t) \sim \mathcal{CN}(\mathbf{0}, \beta \, \mathrm{diag}(\boldsymbol{\Phi_i})), \quad \boldsymbol{e}_i(t) \perp\!\!\!\perp \boldsymbol{u}_i(t), \quad \boldsymbol{e}_i(t) \perp\!\!\!\perp \boldsymbol{e}_i(t')\big|_{t' \neq t}$$

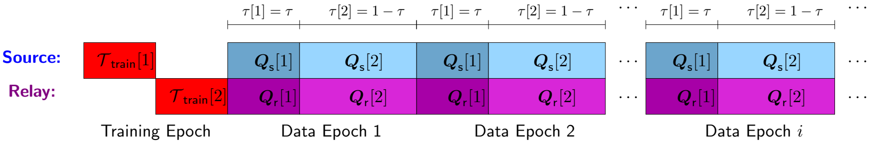
$$\boldsymbol{c}_j(t) \sim \mathcal{CN}(\mathbf{0}, \kappa \, \mathrm{diag}(\boldsymbol{Q_j})), \quad \boldsymbol{c}_j(t) \perp\!\!\!\perp \boldsymbol{x}_j(t), \quad \boldsymbol{c}_j(t) \perp\!\!\!\perp \boldsymbol{c}_j(t')\big|_{t' \neq t}$$

where $\boldsymbol{\Phi}_i = \mathrm{Cov}(\boldsymbol{u}_i)$ and $\boldsymbol{Q}_j = \mathrm{Cov}(\boldsymbol{x}_j)$.

Transmission Protocol

- During Epoch i, the source communicates the i^{th} packet to the relay, while the relay simultaneously communicates the $(i-1)^{st}$ packet to the destination. $\rightsquigarrow$ Enables full-duplex communication.
- Before the first data epoch, we have a training epoch where we perform least-squares channel estimation.
- Data communication parameters (e.g. transmit covariance matrices) are designed to maximize the achievable rate.

Two Periods Per Epoch



- We allow two distinct transmit covariance matrices per data epoch.
- The two periods per data epoch can differ in duration.

Partial Interference Cancellation

- We show that the relay's received signal can be modeled as

$$\boldsymbol{y}_{\mathsf{r}}(t) \quad = \quad \sqrt{\rho_{\mathsf{r}}}\hat{\boldsymbol{H}}_{\mathsf{sr}}\boldsymbol{x}_{\mathsf{s}}(t) + \boldsymbol{v}_{\mathsf{r}}(t)$$

  where $\boldsymbol{v}_{\mathsf{r}}$ is the aggregate interference including transmitter/receiver dynamic-range induced self-noise, channel-estimation error, and thermal noise. Similarly, we can write $\boldsymbol{y}_{\mathsf{d}}$ with interference $\boldsymbol{v}_{\mathsf{d}}$.

- We write the relay's aggregate interference as

$$\begin{aligned}\boldsymbol{v}_{\mathsf{r}}(t) \quad \triangleq \quad & \sqrt{\eta_{\mathsf{r}}}\hat{\boldsymbol{H}}_{\mathsf{rr}}\boldsymbol{x}_{\mathsf{r}}(t) + \sqrt{\rho_{\mathsf{r}}}\hat{\boldsymbol{H}}_{\mathsf{sr}}\boldsymbol{c}_{\mathsf{s}}(t) - \boldsymbol{D}_{\mathsf{sr}}^{\frac{1}{2}}\tilde{\boldsymbol{H}}_{\mathsf{sr}}(\boldsymbol{x}_{\mathsf{s}}(t) + \boldsymbol{c}_{\mathsf{s}}(t)) + \boldsymbol{n}_{\mathsf{r}}(t) \\ & + \sqrt{\eta_{\mathsf{r}}}\hat{\boldsymbol{H}}_{\mathsf{rr}}\boldsymbol{c}_{\mathsf{r}}(t) - \boldsymbol{D}_{\mathsf{rr}}^{\frac{1}{2}}\tilde{\boldsymbol{H}}_{\mathsf{rr}}(\boldsymbol{x}_{\mathsf{r}}(t) + \boldsymbol{c}_{\mathsf{r}}(t)) + \boldsymbol{e}_{\mathsf{r}}(t)\end{aligned}$$

  where $\sqrt{\eta_{\mathsf{r}}}\hat{\boldsymbol{H}}_{\mathsf{rr}}\boldsymbol{x}_{\mathsf{r}}(t)$ is known by the relay and can be eliminated using interference cancellation.

Lower-Bounding the Achievable Rate

- Mutual information characterization is complicated by the fact that the aggregate interference $\boldsymbol{v}_i$ is non-Gaussian when channel-estimation error is non-zero.

- We therefore lower-bound the mutual information by replacing $\boldsymbol{v}_i$ with a Gaussian noise of identical covariance, i.e.,

$$\underline{I}_{\mathsf{sr}}(\boldsymbol{\mathcal{Q}}[l]) \quad = \quad \log\det\left(\boldsymbol{I} + \rho_{\mathsf{r}}\hat{\boldsymbol{H}}_{\mathsf{sr}}\boldsymbol{Q}_{\mathsf{s}}[l]\hat{\boldsymbol{H}}_{\mathsf{sr}}^{\mathsf{H}}\hat{\boldsymbol{\Sigma}}_{\mathsf{r}}^{-1}[l]\right)$$

  where $\hat{\boldsymbol{\Sigma}}_{\mathsf{r}} = \mathrm{Cov}(\boldsymbol{v}_{\mathsf{r}} \,|\, \hat{\boldsymbol{H}}_{\mathsf{sr}}, \hat{\boldsymbol{H}}_{\mathsf{rr}})$ and $\boldsymbol{\mathcal{Q}}[l] \triangleq \{\boldsymbol{Q}_{\mathsf{s}}[l], \boldsymbol{Q}_{\mathsf{r}}[l]\}$. A similar expression is found for $\underline{I}_{\mathsf{rd}}(\boldsymbol{\mathcal{Q}}[l])$.

- We can also upper-bound the mutual information by ignoring the channel estimation error component.

Maximizing the Achievable-Rate Lower-Bound

- For full-duplex operation, the Source $\rightarrow$ Destination rate is bottlenecked by the smallest of $\{I_{\mathsf{sr}}, I_{\mathsf{rd}}\}$.
- Therefore, our metric is

$$\underline{I}_{\tau}(\boldsymbol{\mathcal{Q}}) = \min\left\{ \underbrace{\sum_{l=1}^{2} \tau[l] \underline{I}_{\mathsf{sr}}(\boldsymbol{\mathcal{Q}}[l])}_{\triangleq \underline{I}_{\mathsf{sr},\tau}(\boldsymbol{\mathcal{Q}})}, \underbrace{\sum_{l=1}^{2} \tau[l] \underline{I}_{\mathsf{rd}}(\boldsymbol{\mathcal{Q}}[l])}_{\triangleq \underline{I}_{\mathsf{rd},\tau}(\boldsymbol{\mathcal{Q}})} \right\}$$

where $\boldsymbol{\mathcal{Q}} \triangleq \{\boldsymbol{Q}_{\mathsf{s}}[1], \boldsymbol{Q}_{\mathsf{s}}[2], \boldsymbol{Q}_{\mathsf{r}}[1], \boldsymbol{Q}_{\mathsf{r}}[2]\}$.

- Our optimization problem becomes $\max_{\boldsymbol{\mathcal{Q}}} \underline{I}_{\tau}(\boldsymbol{\mathcal{Q}})$ with power and positivity constraints

$$\boldsymbol{\mathcal{Q}} \in \mathbb{Q}_{\tau} \triangleq \left\{ \begin{array}{lcl} \sum_{l=1}^{2} \tau[l] \operatorname{tr}\left(\boldsymbol{Q}_{\mathsf{s}}[l]\right) \leq 1 & , & \boldsymbol{Q}_{\mathsf{s}}[l] \geq \mathbf{0} \quad \forall l \in \{1,2\} \\ \sum_{l=1}^{2} \tau[l] \operatorname{tr}\left(\boldsymbol{Q}_{\mathsf{r}}[l]\right) \leq 1 & , & \boldsymbol{Q}_{\mathsf{r}}]l] \geq \mathbf{0} \quad \forall l \in \{1,2\} \end{array} \right. .$$

Transmit Covariance Optimization

- We convert the maximin problem to a weighted sum-rate optimization problem

$$\max_{\zeta\in[0,1]} \max_{\boldsymbol{\mathcal{Q}}\in\mathbb{Q}_\tau} \left(\zeta \underline{I}_{\mathsf{sr},\tau}(\boldsymbol{\mathcal{Q}}) + (1-\zeta)\underline{I}_{\mathsf{rd},\tau}(\boldsymbol{\mathcal{Q}})\right)$$

  where we find ζ via bisection search.

- To maximize τ-weighted sum-rates $\underline{I}_{\mathsf{sr},\tau}(\boldsymbol{\mathcal{Q}})$ and $\underline{I}_{\mathsf{sr},\tau}(\boldsymbol{\mathcal{Q}})$, we have developed a Gradient Projection algorithm.

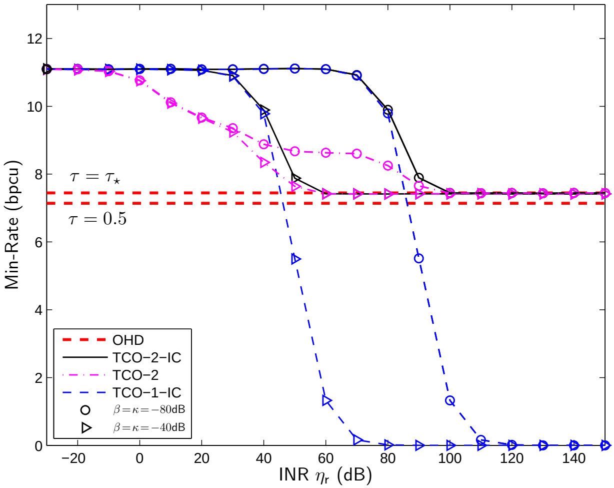
- The projection step is performing waterfilling over both spatial and temporal degrees of freedom.

- Finally we maximize with respect to the time-share τ using a grid search.

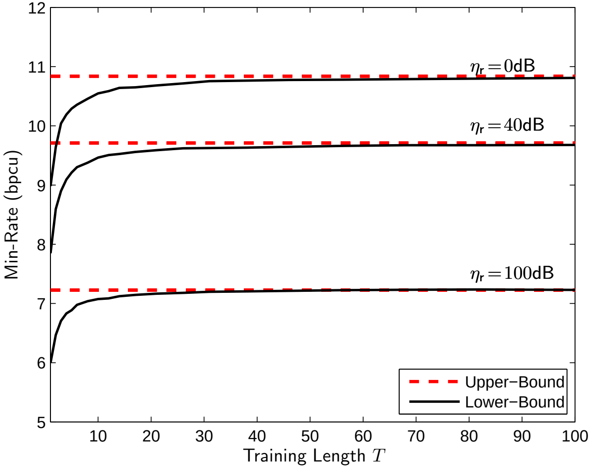
Numerical Results

- We will now show the achievable-rate bounds in the following plots:
  - versus INR η_{r}
  - versus training length
- In the plots, we show our proposed scheme as well as the following schemes:
  - Half-duplex with optimized covariance matrices and time-sharing parameter τ
  - Our proposed scheme without performing interference cancellation
  - Our proposed scheme using only one period per data epoch

Achievable-Rate Lower-Bound vs. INR η_r



Achievable-Rate Lower-Bound vs Training Length T



Conclusion

- We characterized the achievable-rate of MIMO decode-and-forward full-duplex relaying.
- We considered dynamic range limitations at the transmitter and receiver, as well as channel-estimation error from the training-based least-squares.
- Our solution required solving a non-convex optimization problem, for which we applied the projected gradient method.
- An analytic approximation that writes mutual information as an explicit function of the SNRs, INRs, numbers of antennas, and dynamic-range parameters κ and β was also derived (see paper).

Thanks!

Backup Slides

Gradient Projection Algorithm

We find the achievable-rate lower-bound via Gradient Projection:

for $k = 1, 2, 3, \ldots$

$$\boldsymbol{P}_{\mathsf{r}}^{(k)}[1] = \boldsymbol{Q}_{\mathsf{r}}^{(k)}[1] + \boldsymbol{G}_{\mathsf{r}}^{(k)}[1]$$

$$\boldsymbol{P}_{\mathsf{r}}^{(k)}[2] = \boldsymbol{Q}_{\mathsf{r}}^{(k)}[2] + \boldsymbol{G}_{\mathsf{r}}^{(k)}[2]$$

$$\left(\tilde{\boldsymbol{Q}}_{\mathsf{r}}^{(k)}[1], \tilde{\boldsymbol{Q}}_{\mathsf{r}}^{(k)}[2]\right) = \mathcal{P}_{\mathbb{Q}_\tau}\left(\boldsymbol{P}_{\mathsf{r}}^{(k)}[1], \boldsymbol{P}_{\mathsf{r}}^{(k)}[2]\right)$$

$$\boldsymbol{Q}_{\mathsf{r}}^{(k+1)}[1] = \boldsymbol{Q}_{\mathsf{r}}^{(k)}[1] + \gamma^{(k)}\left(\tilde{\boldsymbol{Q}}_{\mathsf{r}}^{(k)}[1] - \boldsymbol{Q}_{\mathsf{r}}^{(k)}[1]\right)$$

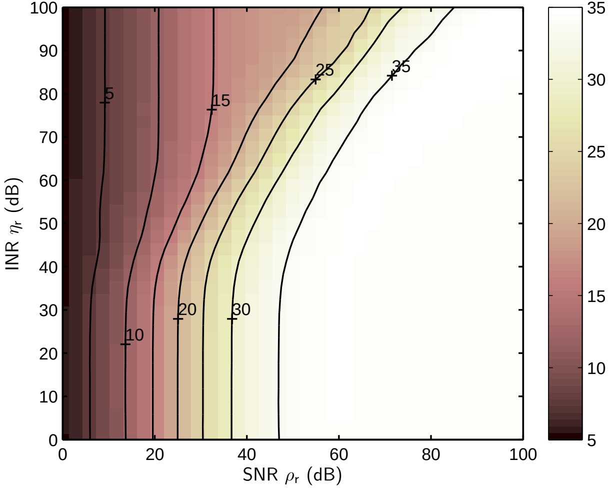
$$\boldsymbol{Q}_{\mathsf{r}}^{(k+1)}[2] = \boldsymbol{Q}_{\mathsf{r}}^{(k)}[2] + \gamma^{(k)}\left(\tilde{\boldsymbol{Q}}_{\mathsf{r}}^{(k)}[2] - \boldsymbol{Q}_{\mathsf{r}}^{(k)}[2]\right)$$

$\langle$Similar repeated for $\boldsymbol{Q}_{\mathsf{s}}[1]$ and $\boldsymbol{Q}_{\mathsf{s}}[2]\rangle$

end

where $\boldsymbol{G}_{\mathsf{r}}^{(k)}[l]$ is the gradient, and $\mathcal{P}_{\mathbb{Q}_\tau}(\cdot)$ projects the period 1 and period 2 covariances onto the constraint set. $\gamma^{(k)}$ is chosen via the Armijo stepsize rule.

Achievable-Rate Lower-Bound Contour over SNR and INR



Analytic Approximation of Achievable Rate

- The complicated nature of the optimization problem motivates us to approximate its solution
- Making simplifying assumptions, we are able to find straightforward optimal transmit covariance matrices for both full-duplex and half-duplex operation.
- Our analytic approximate solution is simply the maximum of the full-duplex and half-duplex approximate solutions.

Analytic Approximation Contour over SNR and INR