Full-Duplex Bidirectional MIMO: Achievable Rates under Limited Dynamic Range

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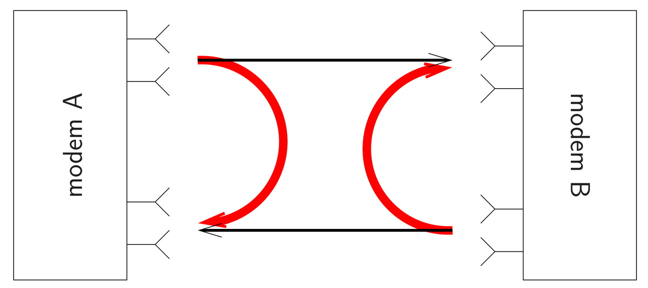
Introduction

We consider the problem of full-duplex communication between two multiple-input, multiple-output (MIMO) wireless modems. By full-duplex, we mean that the two modems perform simultaneous transmission and reception (STAR) at the same carrier frequency. By adapting a full-duplex strategy, there lies potential to nearly double the spectral efficiency over a traditional half-duplex system which either employs time-division-duplexing or frequency-division-duplexing. The fundamental difficulty with STAR is that, due to the close proximity of a given modem's transmit antennas to its receive antennas, the modem's outgoing signal can overwhelm its receiver circuitry, making it impossible to recover the incoming signal.



Typically, this self-interference can be ~100dB. Now, consider a typical ADC with dynamic range ~50dB. Since the self-interference saturates the receiver, we aim to prevent it from happening in the first place (e.g. transmit beamforming).

System Model

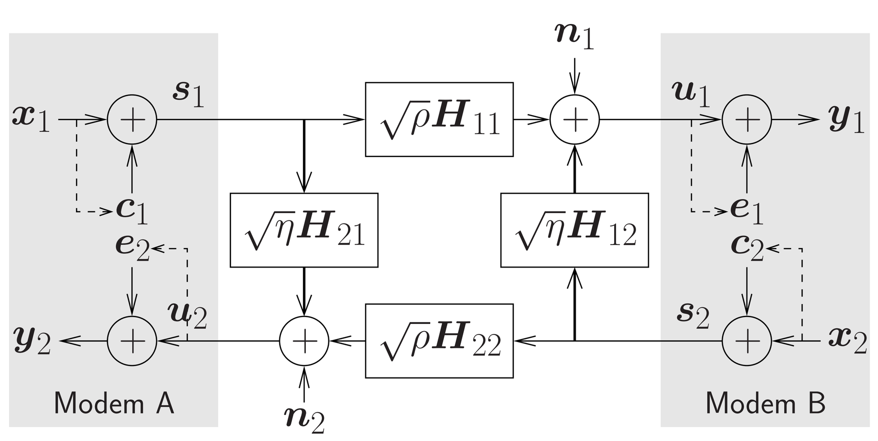


Figure: Our model of bidirectional MIMO communication. The dashed lines denote statistical dependence.

Assumptions:

- n_i : AWGN noise
- Raleigh-fading MIMO channels $H_{ij} \in \mathbb{C}^{N_r \times N_t}$
- Pilot aided LS channel estimates, $\hat{H}_{ij}$
- N_t : number of antennas for *all* transmitters
- N_r : number of antennas for *all* receivers
- ρ : signal-to noise ratio (SNR)
- η : interference-to-noise ratio (INR)

Distortion Model

Transmitter Distortion:

- Modeled as zero-mean Gaussian noise injected per transmit antenna, written as $c_j(t)$.
- Variance is κ times energy of *intended* transmit signal, $Q_j \triangleq \mathrm{Cov}\{x_j(t)\}$.

$$s_j(t) = x_j(t) + c_j(t) \text{ s.t. } \begin{cases} c_j(t) \sim \mathcal{CN}(0, \kappa\,\mathrm{diag}(Q_j)) \\ c_j(t) \perp x_j(t) \\ c_j(t) \perp c_j(t')|_{t' \neq t} \end{cases}$$

- Models additive power-amp noise, non-linearities in DAC and power-amp, and oscillator phase noise.

Receiver Distortion:

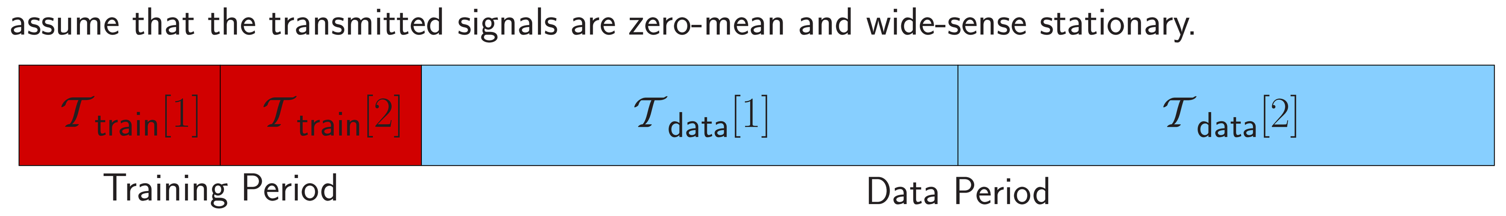
- Modeled as zero-mean Gaussian noise injected per receive antenna, written as $e_i(t)$.
- Variance is β times energy collected at the antenna, $\Phi_i \triangleq \mathrm{Cov}\{u_i(t)\}$.

$$y_i(t) = u_i(t) + e_i(t) \text{ s.t. } \begin{cases} e_i(t) \sim \mathcal{CN}(0, \beta\,\mathrm{diag}(\Phi_i)) \\ e_i(t) \perp u_i(t) \\ e_i(t) \perp e_i(t')|_{t' \neq t} \end{cases}$$

- Models additive gain-control noise, non-linearities in ADC and gain-control, and oscillator phase noise.

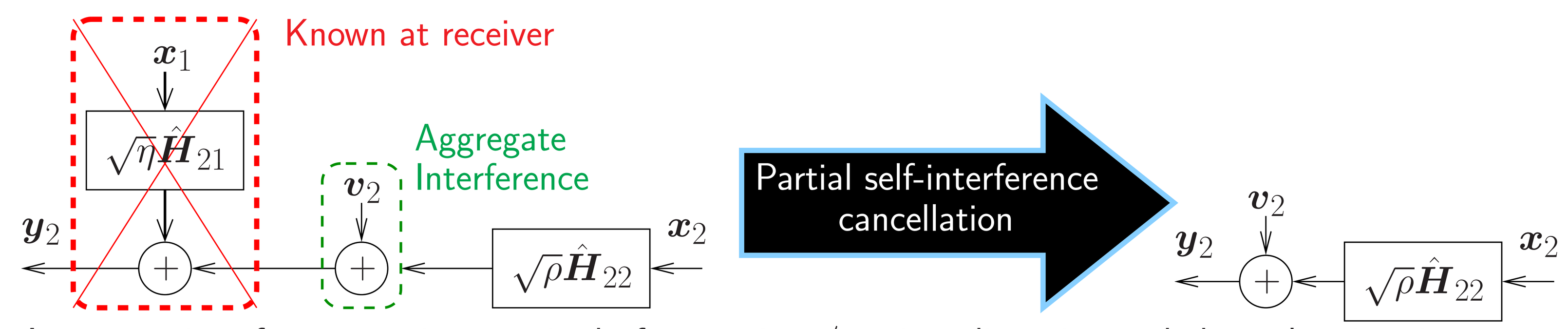
Transmission Protocol

Our signaling epoch $\mathcal{T}$ is partitioned into a training period $\mathcal{T}_{\text{train}}$ and a subsequent data communication period $\mathcal{T}_{\text{data}}$, each of which are partitioned into two sub-periods. Within each of these four sub-periods, we assume that the transmitted signals are zero-mean and wide-sense stationary.



Partial Self-Interference Cancellation

We employ partial self-interference cancellation on the received signal. It is only a partial cancellation, because of channel estimation error, transmitter/receiver distortion. For example, we have



Aggregate interference v_2 is comprised of transmitter/receiver distortion and channel estimation error.

Bounds on Achievable Sum-Rate

- In general, aggregate interference $v_i[l]$ is non-Gaussian (due to channel estimation error)
- We derive tight upper ($\overline{I}(\mathcal{Q})$) and lower-bounds ($\underline{I}(\mathcal{Q})$) on achievable sum-rate for Gaussian case
- With sufficient training $\underline{I}(\mathcal{Q}) \to \overline{I}(\mathcal{Q})$

$$\underbrace{\underline{I}(\mathcal{Q}) = \frac{1}{2}\sum_{i=1}^{2}\sum_{l=1}^{2} \log\det\left(\rho \hat{H}_{ii} Q_i[l] \hat{H}_{ii}^{\mathsf{H}} + \hat{\Sigma}_i[l]\right) - \log\det(\hat{\Sigma}_i[l])}_{\text{Lower Bound}} \le \underbrace{I(\mathcal{Q})}_{\text{Sum-Rate}} \le \underbrace{\overline{I}(\mathcal{Q})}_{\text{Upper Bound}} \quad (1)$$

- $\mathcal{Q} \triangleq (Q_1[1], Q_1[2], Q_2[1], Q_2[2])$, the set of all transmit covariance matrices
- $\hat{\Sigma}_i[l] \triangleq \mathrm{Cov}\{v_i[l] \,|\, \hat{H}_{ii}, \hat{H}_{ij}\}$

Transmit Covariance Optimization

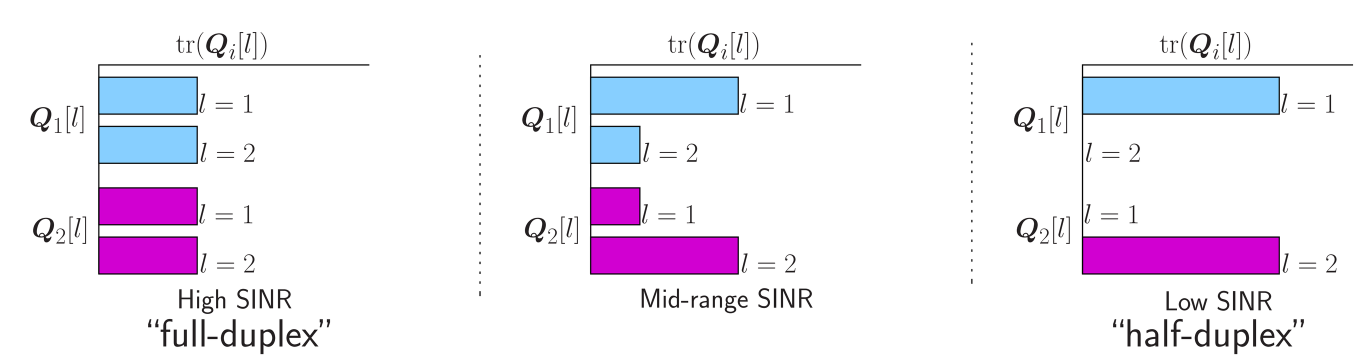
We wish to maximize the sum-rate by finding optimal transmit covariance matrices under a time-averaged power constraint. We have developed a **Gradient Projection** algorithm to solve the following optimization problem. The projection step in essence implements waterfilling.

$$\max_{Q_1[1], Q_1[2], Q_2[1], Q_2[2]} \quad \underline{I}(Q_1[1], Q_1[2], Q_2[1], Q_2[2]) \quad (2a)$$

$$\text{s.t.} \quad \frac{1}{2}\sum_{l=1}^{2} \mathrm{tr}\left(Q_i[l]\right) \le 1, \quad i = 1, 2 \quad \text{Power Constraint} \quad (2b)$$

$$Q_i[l] \succeq 0, \quad \forall i, l \in \{1, 2\}, \quad (2c)$$

Example Transmit Power Allocation



- For mid-range SINR, we expect each transmitter to unevenly share power among time sub-periods

Sum-Rate Approximation

The complicated nature of the optimization problem (2) motivates us to approximate its solution. We approximate using the special case that each H_{ij} is diagonal, with $N_{\min} \triangleq \min\{N_t, N_r\}$ identical entries. This gives us

$$I(\mathcal{Q}) \approx \frac{1}{2}\sum_{i,l} \log\det\left(I + \rho\frac{N_t N_r}{N_{\min}} Q_i[l]\left(I + (\kappa+\beta)\frac{N_t N_r}{N_{\min}}\left[\rho\,\mathrm{diag}(Q_i[l]) + \eta\,\mathrm{diag}(Q_{\bar{j}}[l])\right]\right)^{-1}\right). \quad (3)$$

"Full-Duplex" Analysis:

- When $\eta \ll \rho$, we find the optimal covariances
- $\mathcal{Q}_{\text{FD}} \triangleq (\frac{1}{N_t}I, \frac{1}{N_t}I, \frac{1}{N_t}I, \frac{1}{N_t}I)$.
- $I(\mathcal{Q}_{\text{FD}}) \approx 2N_{\min}\log\left(1 + \frac{\rho}{\frac{N_{\min}}{N_r} + (\kappa+\beta)(\rho+\eta)}\right)$

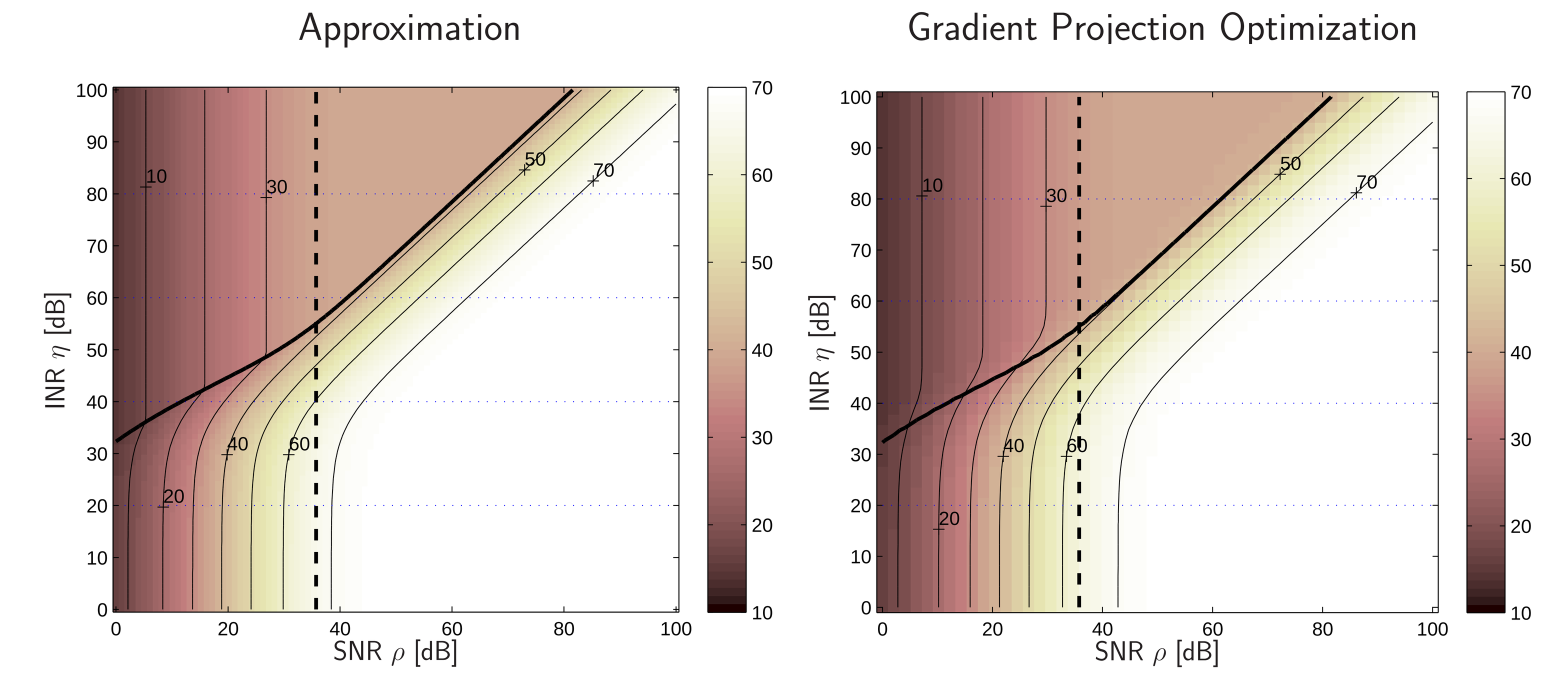
"Half-Duplex" Analysis:

- When $\eta \gg \rho$, we find the optimal covariances
- $\mathcal{Q}_{\text{FD}} \triangleq (\frac{2}{N_t}I, 0, 0, \frac{2}{N_t}I)$.
- $I(\mathcal{Q}_{\text{HD}}) \approx N_{\min}\log\left(1 + \frac{\rho}{\frac{N_{\min}}{2N_r} + (\kappa+\beta)\rho}\right)$

Then, we find the boundary between full and half-duplex as

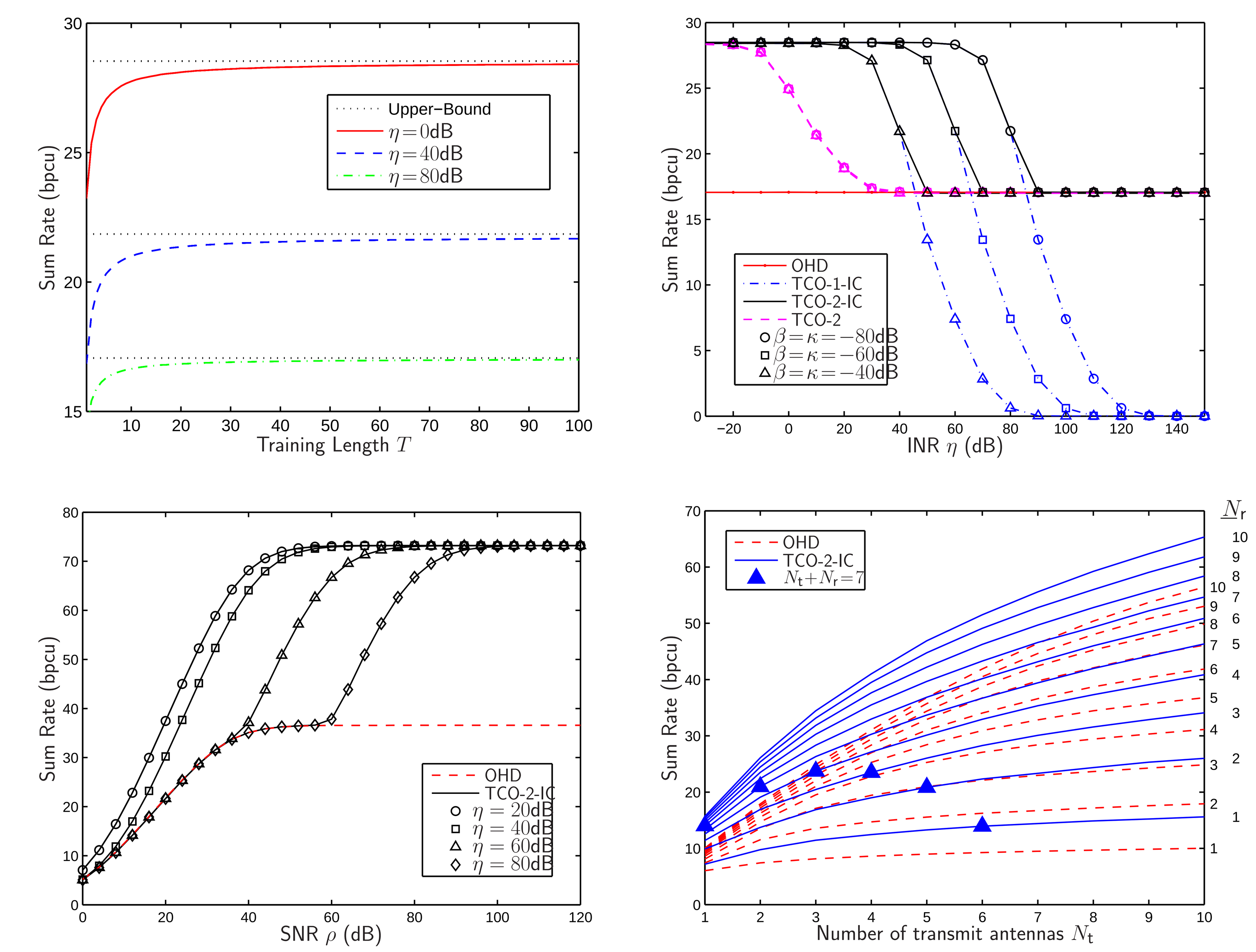
$$\eta = \frac{1}{2}\left(\sqrt{\xi^2 + 2\rho\xi/(\kappa+\beta)} - (\xi - 2\rho)\right) \quad \text{for } \xi \triangleq \frac{N_{\min}}{N_r(\kappa+\beta)} + 2\rho \quad (4)$$

Sum-Rate: Approximation vs. Optimization



The dark curve on both figures is the approximate boundary between full and half-duplex from (4). The dashed line shows the boundary for the SNR-limited and distortion-limited regimes.

Simulation Results



- **OHD:** optized half-duplex
- **TCO-2-IC:** use 2 time intervals, cancel self-interference
- **TCO-1-IC:** use 1 time interval, cancel self-interference
- **TCO-2:** optimize over 2 time intervals, no interference cancellation
- β, κ : Rx/Tx distortion factor
- η : interference-to-noise ratio (INR)
- ρ : signal-to-noise ratio (SNR)

Summary

- Considered the problem of full-duplex bidirectional communication in MIMO modems in which we developed explicit models of limited transmitter/receiver-dynamic range and imperfect CSI.
- Derived upper and lower bounds on the achievable sum-rate that tighten as the number of pilots increases.
- Proposed a transmission scheme based on maximizing the sum-rate lower bound through a non-convex optimization problem
- Derived an analytic approximation of the achievable sum-rate as a function of signal-to-noise ratio, interference-to-noise ratio, transmitter/receiver dynamic range, number of antennas, and number of pilots.

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