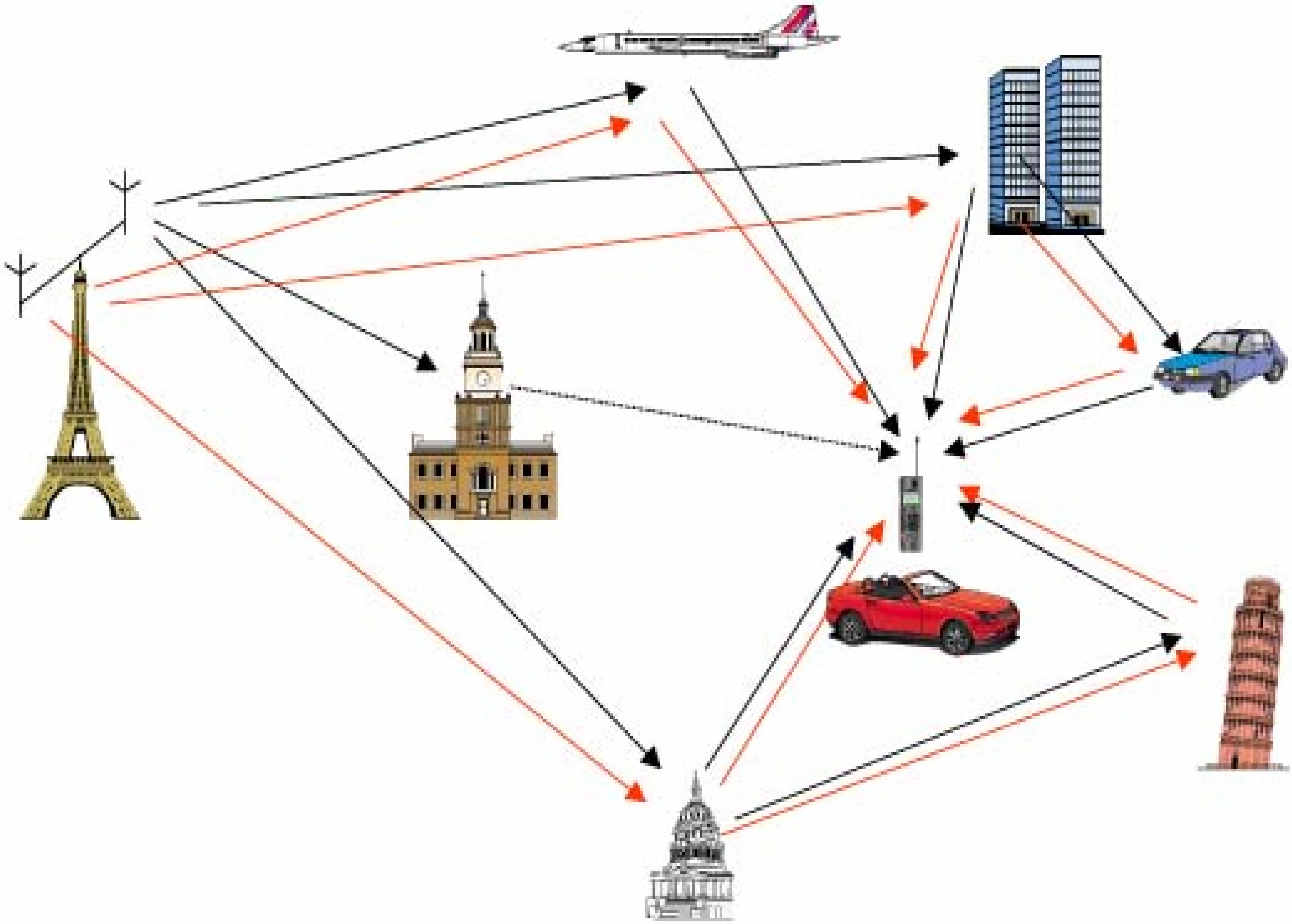
Design of Multi-carrier Modulation for Doubly Selective Channels based on a Complexity Constrained Achievable Rate Metric

Sibasish Das

Phil Schniter

The Propagation Environment



Courtesy: Dr. Youjian Liu, Univ. of Colorado, Boulder

The Doubly Selective Channel

- **Channel modeled as linear time-varying system:**

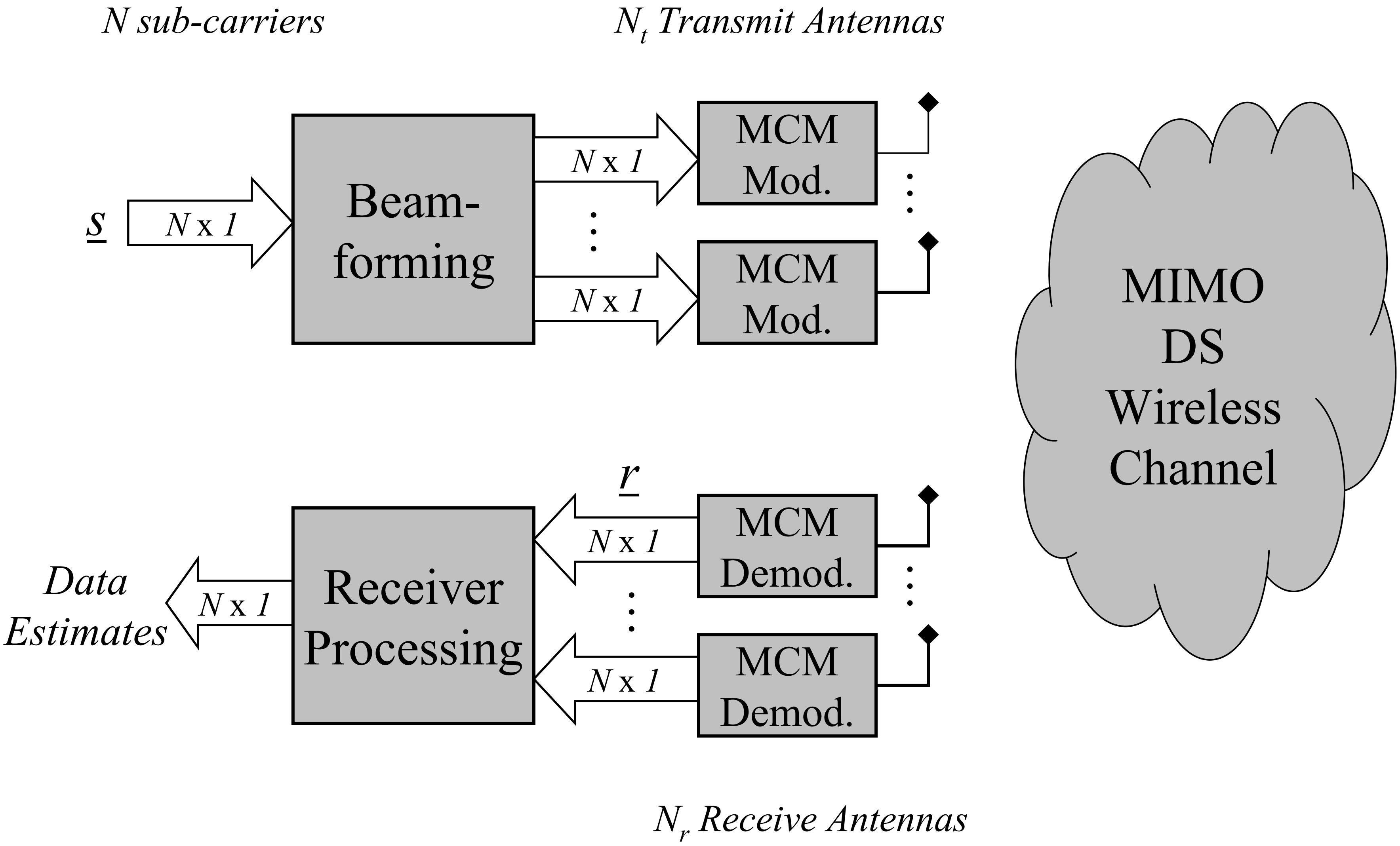
$$r_n = \sum_{l=0}^{N_h-1} h_{n,l} t_{n-l} + v_n$$

  - Large delay spread $\Rightarrow \quad N_h >> 1$
  - Rapid time variations modeled by WSSUS Rayleigh fading

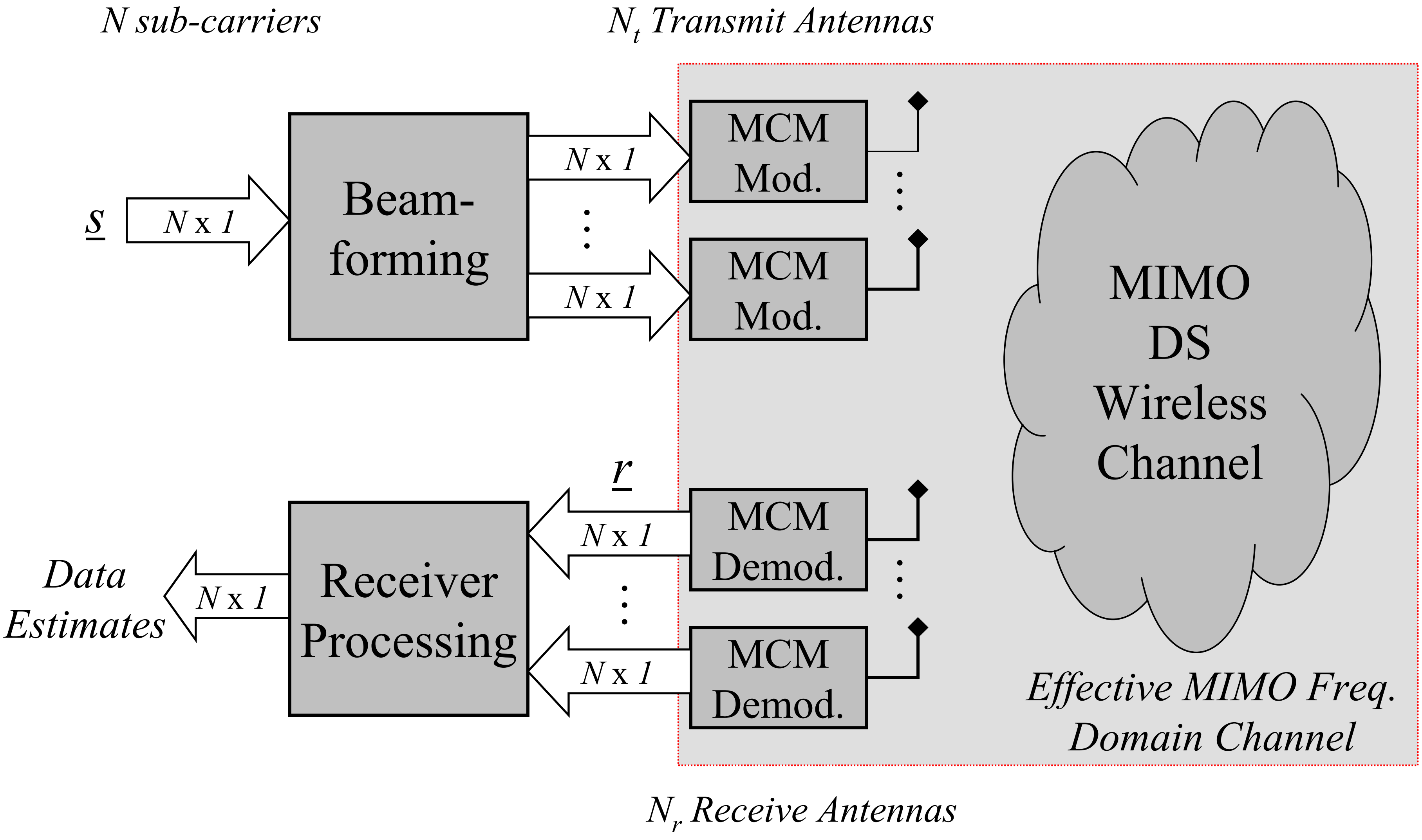
$$E[h_{n_1,l_1} h^*_{n_2,l_2}] = \sigma^2_{l_1} J_0\left(2\pi f_d (n_1 - n_2)\right)\delta_{l_1-l_2}$$

- **Popular to use multi-carrier modulation (MCM) on such channels**

MIMO - MCM


N sub-carriers
N_t Transmit Antennas
$\underline{s}$
N x 1
Beam-forming
N x 1
N x 1
MCM Mod.
MCM Mod.
MIMO DS Wireless Channel
$\underline{r}$
N x 1
N x 1
MCM Demod.
MCM Demod.
Receiver Processing
N x 1
Data Estimates
N_r Receive Antennas

MIMO - MCM


N sub-carriers
N_t Transmit Antennas
$\underline{s}$
N x 1
Beam-forming
N x 1
N x 1
MCM Mod.
MCM Mod.
MIMO DS Wireless Channel
$\underline{r}$
N x 1
N x 1
MCM Demod.
MCM Demod.
Receiver Processing
N x 1
Data Estimates
Effective MIMO Freq. Domain Channel
N_r Receive Antennas

The Challenge

- Delay spread ⇒ inter-symbol interference (ISI)

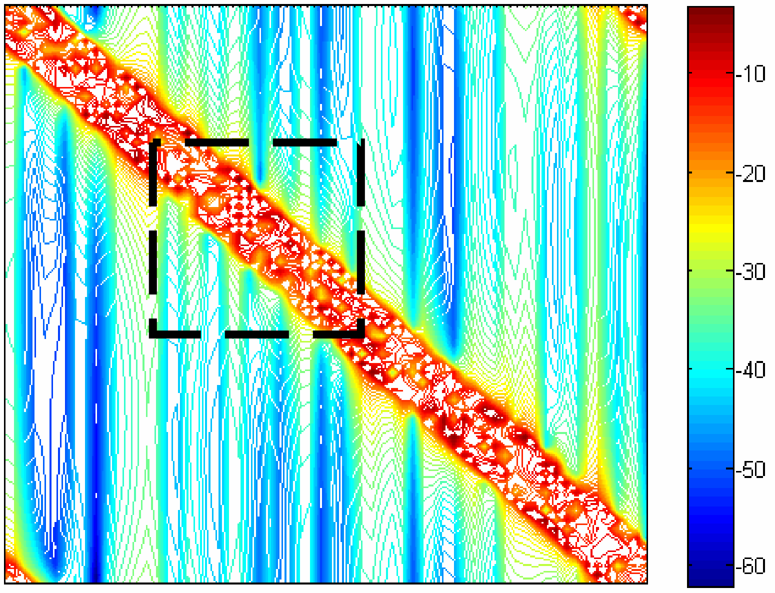
- Doppler spread ⇒ inter-carrier interference (ICI)

- Optimal receiver has very high complexity
  - When $N=512$, $N_t = N_r = 2$
  - More than $\underline{1\text{x}10^6}$ complex ICI coefficients !!

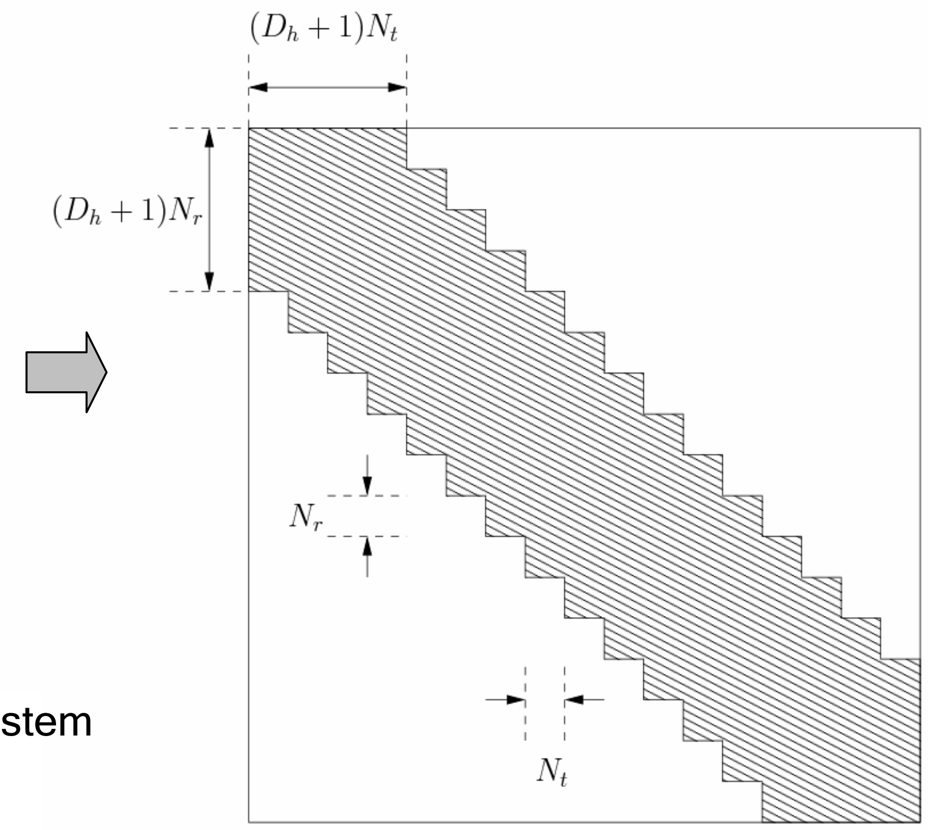
Can the structure of the MIMO Frequency Domain Channel be exploited?

MIMO - Sub-carrier Coupling Matrix

- Use guards / pulse shaping to suppress ISI
- ICI Profile
- D_h = Significant ICI radius

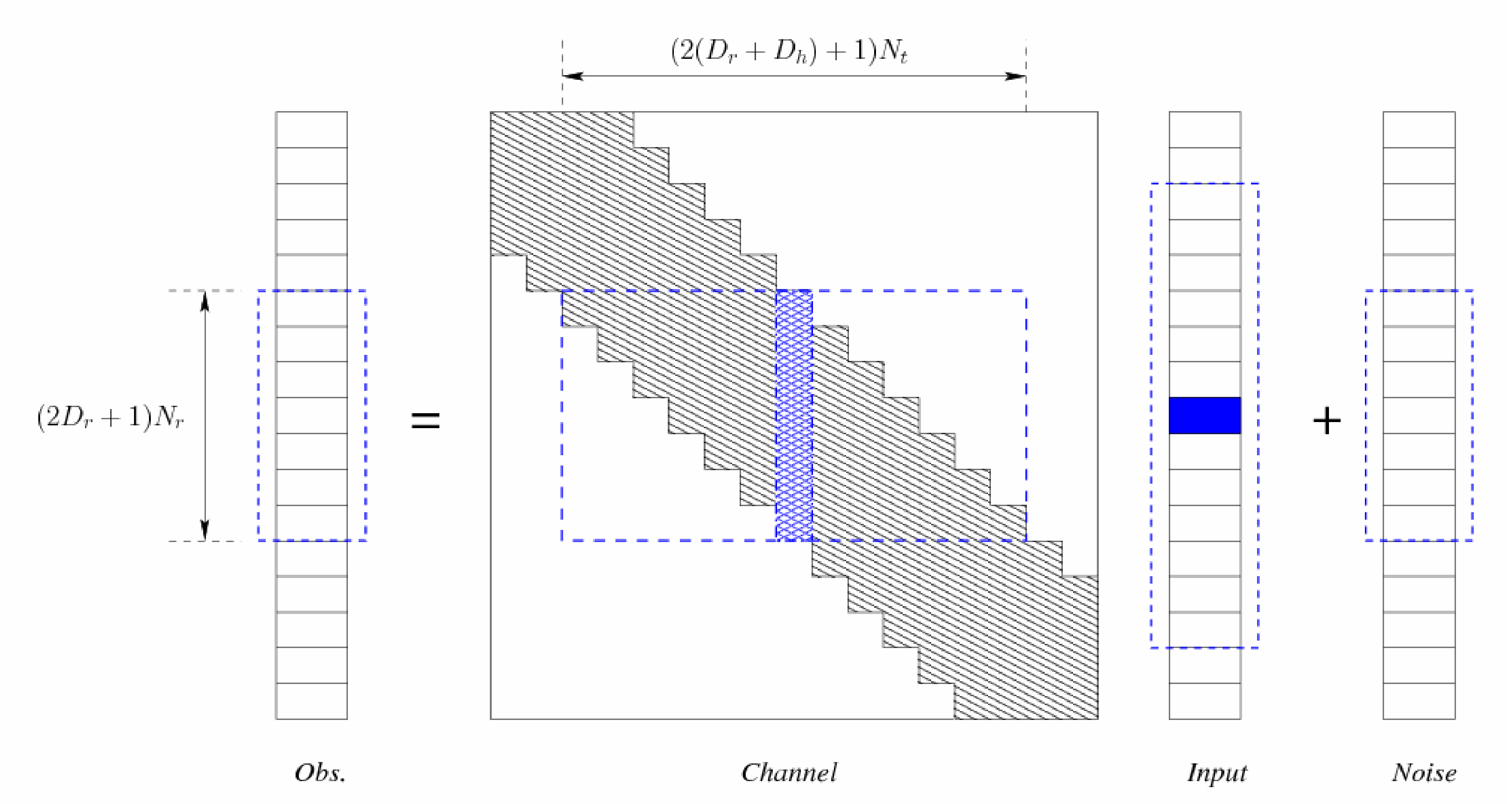


ICI Power Profile (dB) of 2x2 MIMO-OFDM system
with N = 256 sub-carriers



Local-ICI Processing

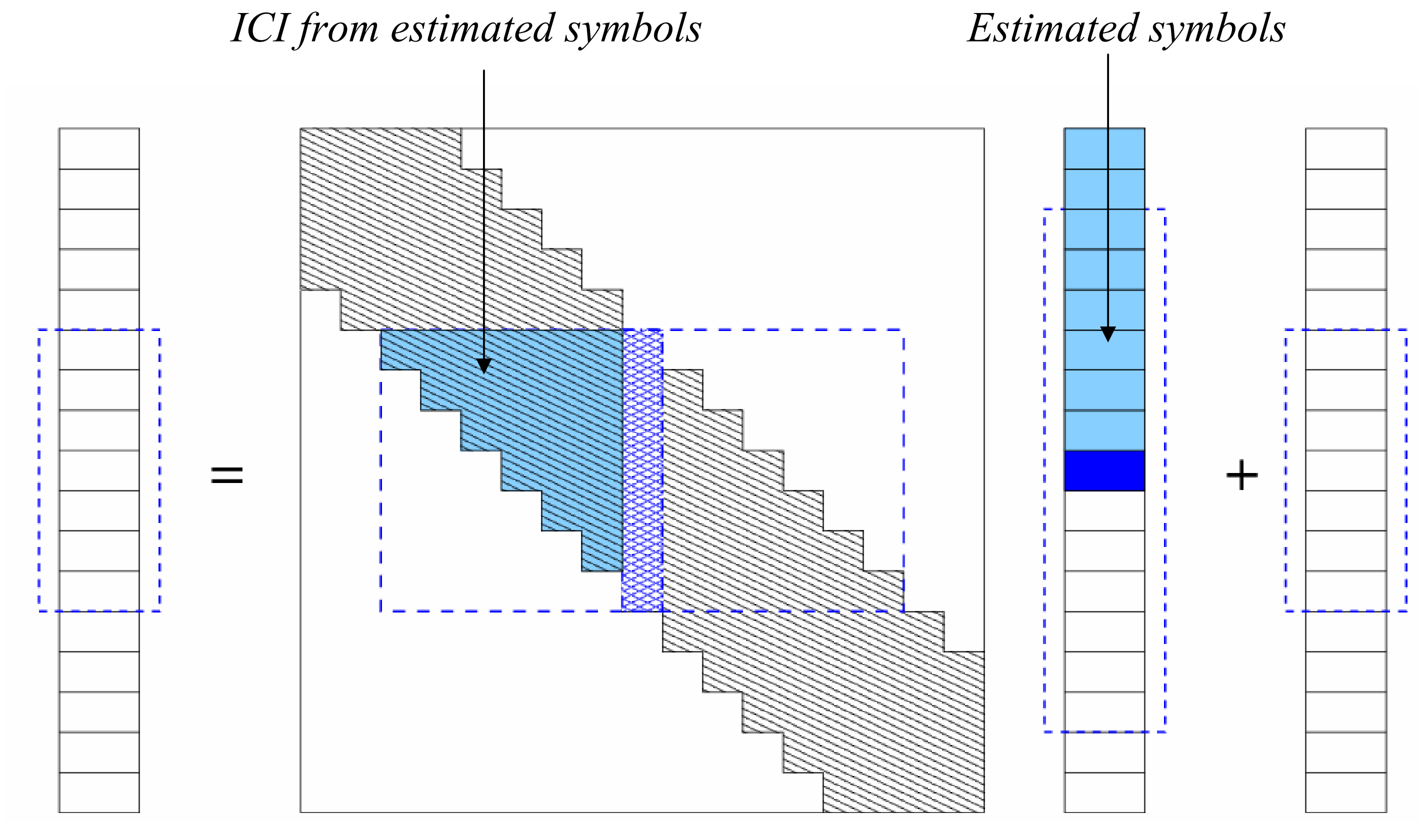
– Consider a local-ICI processing receiver which:



- Local-ICI processing performance/complexity controlled by “D_r”

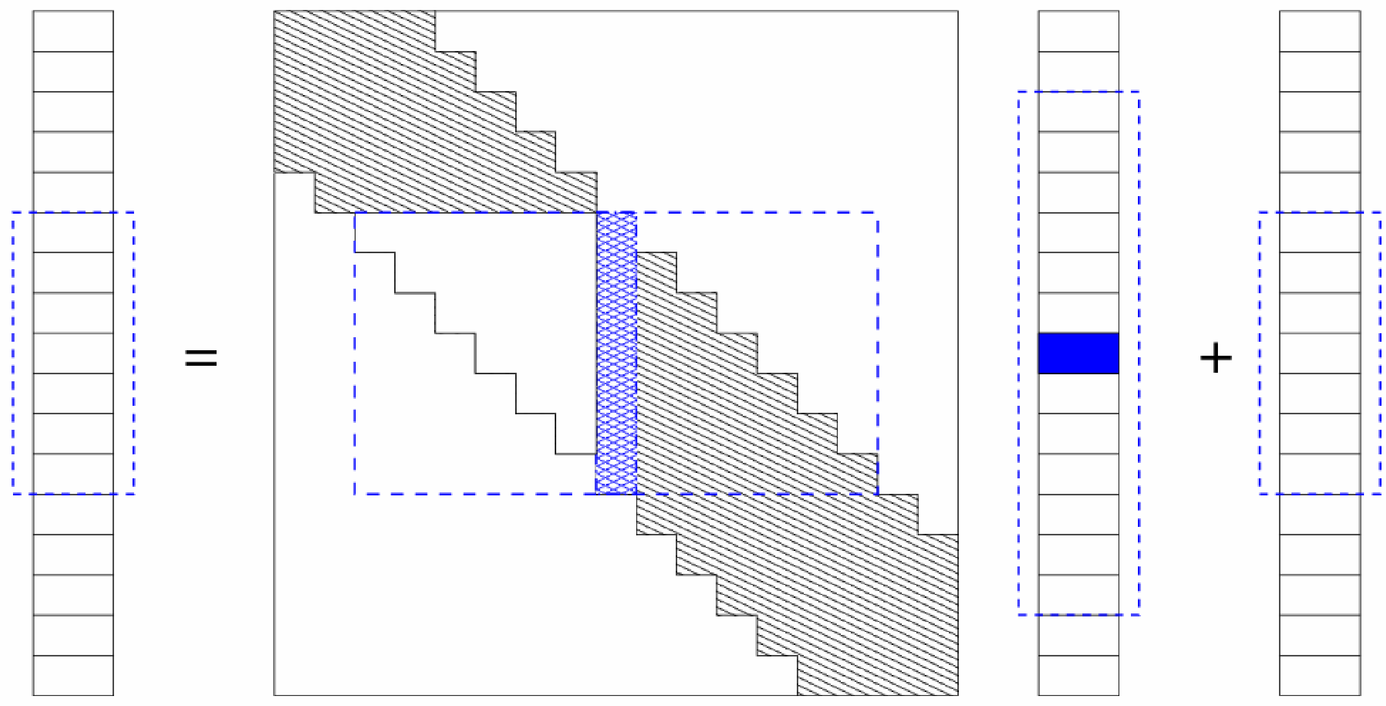
Local-ICI Processing

- Consider a local-ICI processing receiver which:
  - Cancels ICI using previous symbol estimates ⇒ Partial-SIC (P-SIC)



Local-ICI Processing

- Consider a local-ICI processing receiver which:
  - Cancels ICI using previous symbol estimates ⇒ Partial-SIC (P-SIC)
  - Performs local linear combining after P-SIC
  - Maximizes rate on each sub-carrier



Measuring Performance

– Local-ICI processing applicable to many MCM schemes…

– Choice of MCM scheme affects:
  - Decoding Complexity
  - Spectral Efficiency
  - Diversity Exploitation

– “Practical” performance metric should incorporate effects of all the above

– We measure the ***achievable ergodic rate*** when local-ICI processing is used on a generic MIMO-MCM system

Achievable Rate Metric (ARM)

- Assumptions:
  - Coding & Decoding over large blocks of MCM symbols
  - Each sub-carrier coded independently
  - Use of i.i.d. (complex) Gaussian codebooks

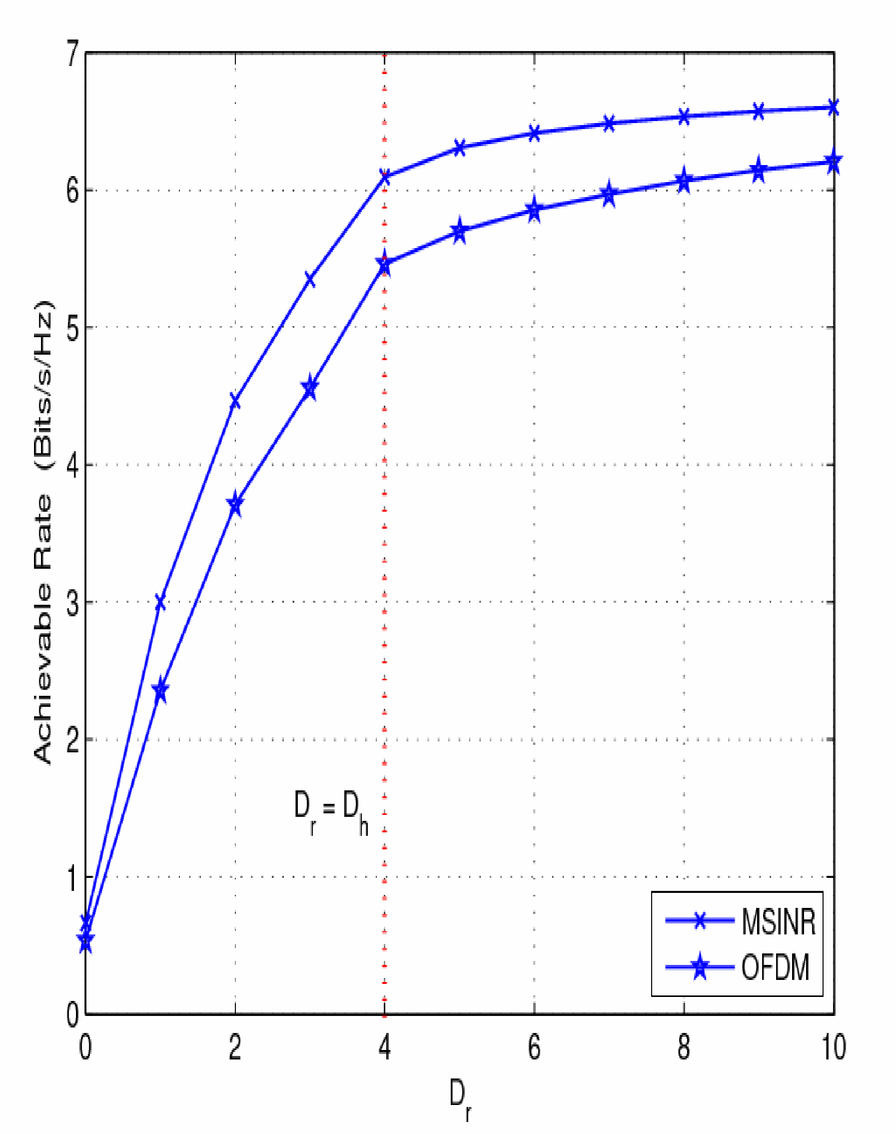
- ARM Definition:

$$R = E_{\mathbf{H}}\left[\sum_{k=0}^{N-1}\log(1+\gamma_k)\right]$$

  - γ_k: SINR for k^{th} sub-carrier using partial SIC and local linear combining.
  - SINR is function of beamforing vectors, local linear combiner and ICI coefficients.

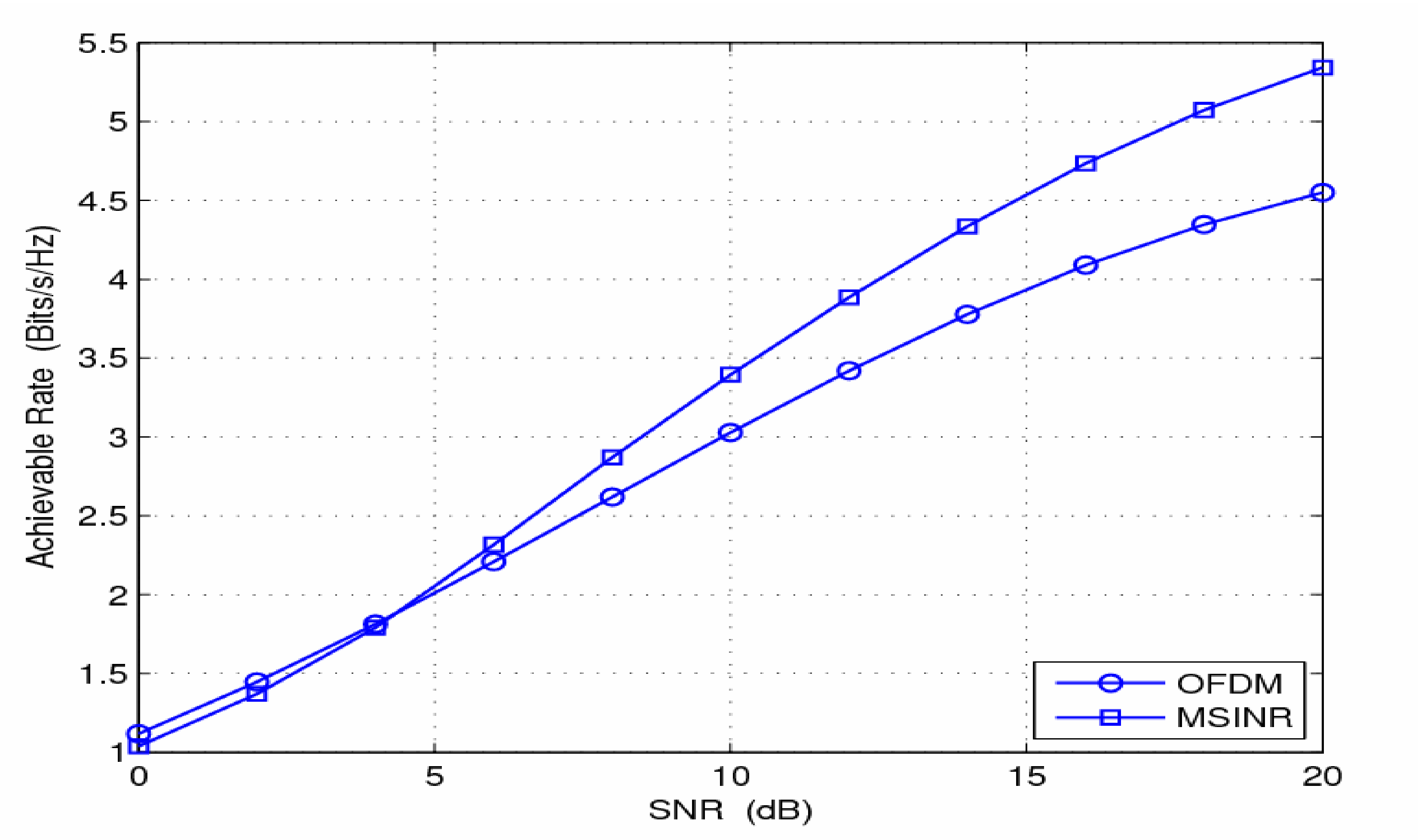
- ARM Advantages:
  - Characterizes trade-off between complexity (D_r) and performance (R)
  - Compares performance of local-ICI processing on various MCM schemes at equal complexity

Rate Complexity Trade-off



- $N = 256, N_t = 1, N_r = 2,$
  $f_d T_c = 0.016, SNR = 20\ dB$
- Achievable rate saturates when $D_r > D_h$
- “Sweet spot” at $D_r = D_h$
- When $D_r = D_h$,
  - Achievable rate is more than ***90%*** of maximum achievable rate
  - Complexity is less than ***0.5%*** of optimal processing

Performance at $D_r = D_h = 1$



$$N = 128\,,\ N_t = 1\,, N_r = 2\,, f_d T_c = 0.008$$

Beamforming

- Wish to design beamforming vectors for sub-carriers
  - Assume CSI at transmitter
  - Desire good performance in the presence of ICI
  - Assume local-ICI processing receiver

- Consider a MIMO-OFDM system
  - MIMO-OFDM ⇒ White Noise
  - MIMO-OFDM ⇒ No ISI
  - Equal power across sub-carriers

Beamforming Approaches

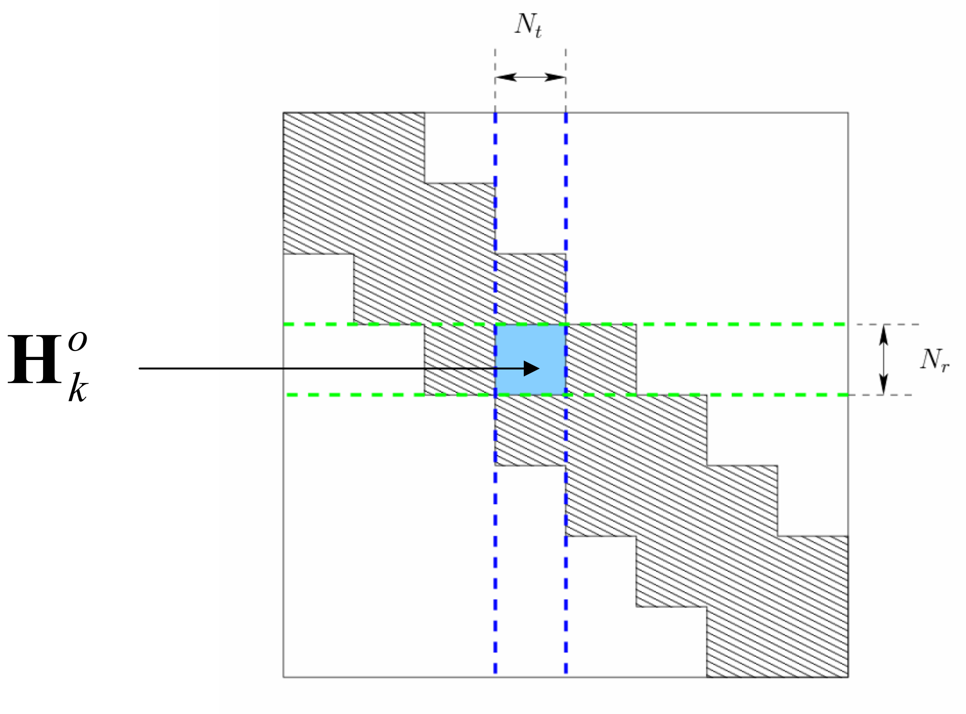
- Traditional Max-SNR…
  - e.g., [*Bolcskei et al, T.Comm., Feb'02*]
  - Not designed for ICI

- "Doubly selective" Max-SNR…
  - Maximize total received energy per sub-carrier

- Approximate Max-ARM

Traditional Max-SNR Beamforming

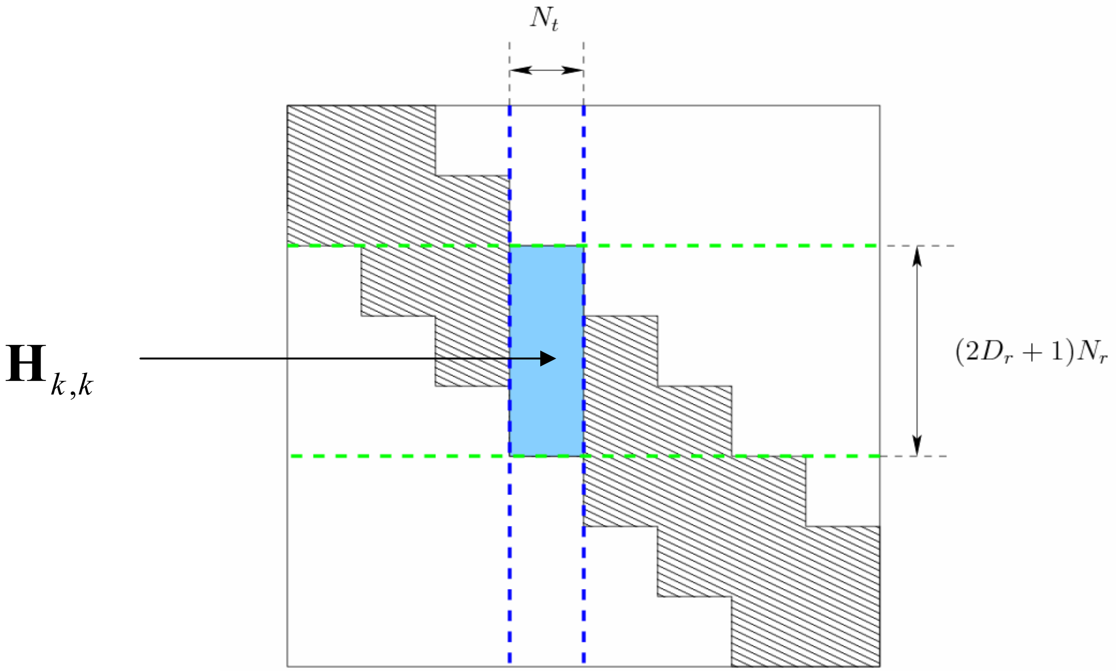
- Maximize SNR for each sub-carrier ***independently***
- "signal" defined as $\mathbf{z}_k^H \mathbf{H}_k^o \mathbf{x}_k s_k$
- Receiver performs antenna combining, but not sub-carrier combining (D_r=0)



  - Beamforming vector for k^{th} sub-carrier: $\mathbf{x}_k = v_*(\mathbf{H}_k^{oH} \mathbf{H}_k^o)$

"Doubly-Selective" Max-SNR Beamforming

- Maximize SNR for each sub-carrier ***independently***
- Receiver considers both antenna and sub-carrier combining (D_r=D_h)
- "signal" defined as $\mathbf{z}_k^H \mathbf{H}_{k,k} \mathbf{x}_k s_k$



  - Beamforming vector for k^{th} sub-carrier: $\mathbf{x}_k = \nu_*(\mathbf{H}_{k,k}^H \mathbf{H}_{k,k})$

Max-ARM Beamforming

- Motivation
  - SNR-maximization does not penalize ICI effects on other sub-carriers!
  - Can an ICI penalty be incorporated into beamformer design?

- Ideal solution: Maximize ARM
  - Leads to a difficult optimization problem!

$$R = E_{\mathbf{H}}\left[\sum_{k=0}^{N-1} \log(1 + \gamma_k)\right]$$

$$\gamma_k = \frac{\mathcal{E}_{sig}(\mathbf{x}_k, \mathbf{z}_k, \mathbf{H})}{\mathcal{E}_{ici}(\{\mathbf{x}_{k'}\}_{k' \in \mathcal{K}_+(k)}, \mathbf{z}_k, \mathbf{H}) + \mathcal{E}_{noise}}$$

Approximate Max-ARM Beamforming

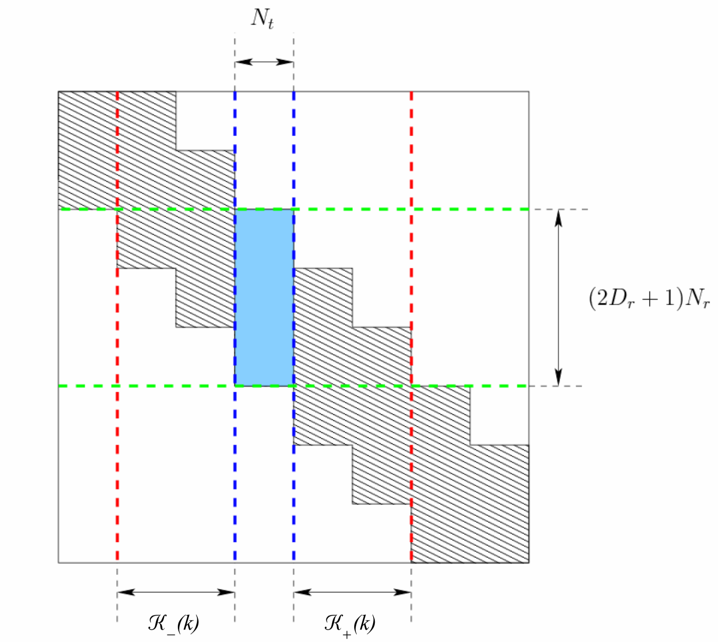
$$R = E_{\mathbf{H}}\left[\sum_k \log\left(1 + \frac{\mathcal{E}_{sig}(\mathbf{x}_k, \mathbf{z}_k, \mathbf{H})}{\mathcal{E}_{ici}(\{\mathbf{x}_{k'}\}_{k' \in \mathcal{K}_+(k)}, \mathbf{z}_k, \mathbf{H}) + \mathcal{E}_{noise}}\right)\right]$$

with the fraction marked by $\underbrace{}_{\gamma_k}$

- Iterative optimization:
  - Given beamformers $\{\mathbf{x}_k\}$, choose combiners
    $$\mathbf{z}_k = \max_{\mathbf{z}_k} \gamma_k, \text{ for } k = 0, \ldots, N\text{-}1$$
    - Easy: generalized eigenvalue problem!
  - Given combiners $\{\mathbf{z}_k\}$, how to choose beamforming vectors $\{\mathbf{x}_k\}$?
    - Note: Each $\mathbf{x}_k$ affects several γ_k
  - Use intuition from ARM for low-SNR and high-SNR regimes…

Intuition from ARM: High-SNR Regime

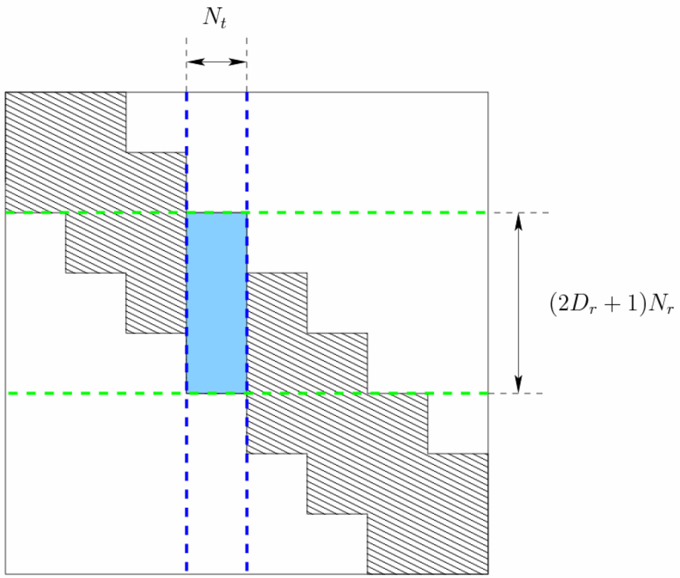
$$R = E_{\mathbf{H}}\left[\sum_k \log(1+\frac{\mathcal{E}_{sig}(\mathbf{x}_k,\mathbf{z}_k,\mathbf{H})}{\mathcal{E}_{ici}(\{\mathbf{x}_{k'}\}_{k'\in\mathcal{K}_+(k)},\mathbf{z}_k,\mathbf{H})+\mathcal{E}_{noise}})\right]$$



- When designing $\mathbf{x}_k$:
  - Residual ICI dominates noise
  - Due to P-SIC, need not consider ICI caused to “future” sub-carriers $\mathcal{K}_+(k)$
  - Maximize signal, minimize ICI caused to “past” sub-carriers $\mathcal{K}_-(k)$

Intuition from ARM: Low-SNR Regime

$$R = E_{\mathbf{H}}\left[\sum_k \log(1+\frac{\mathcal{E}_{sig}(\mathbf{x}_k,\mathbf{z}_k,\mathbf{H})}{\mathcal{E}_{ici}(\{\mathbf{x}_{k'}\}_{k'\in\mathcal{K}_+(k)},\mathbf{z}_k,\mathbf{H})+\mathcal{E}_{noise}})\right]$$



- When designing $\mathbf{x}_k$:
  - Noise dominates residual ICI
  - Need to maximize signal energy

Approximate Max-ARM Beamforming

– With low-SNR and high-SNR intuitions, we propose:

$$\mathbf{x}_k^* = \underset{\|\mathbf{x}_k\|=1}{\arg\max} \frac{\mathbf{x}_k^H(\mathbf{H}_{k,k}^H \mathbf{z}_k \mathbf{z}_k^H \mathbf{H}_{k,k})\mathbf{x}_k}{\mathbf{x}_k^H(\sum_{k' \in \mathcal{K}\_(k)} \mathbf{H}_{k',k}^H \mathbf{z}_{k'} \mathbf{z}_{k'}^H \mathbf{H}_{k',k} + \sigma^2 \mathbf{I})\mathbf{x}_k}, \quad k = 0, \cdots, N-1$$

– Note that:

- At high SNR, prevent causing ICI to “past” sub-carriers
- At low SNR, maximize signal energy

Complexity

	Traditional Max-SNR	"Doubly Selective" Max-SNR	Approximate Max-ARM
Beamformer Design (per OFDM symbol)	$O(NN_t^3)$	$O(NN_t^3)$	$O(N_i NN_t^3)$
Combiner Design (per OFDM symbol)	$O(NN_r^3)$	$O(N(2D_r+1)^3 N_r^3)$	$O(N_i N(2D_r+1)^3 N_r^3)$
Aggregate (per OFDM symbol)	$O(N(N_t^3+N_r^3))$	$O(N((2D_r+1)^3 N_r^3+N_t^3))$	$O(N_i N((2D_r+1)^3 N_r^3+N_t^3))$

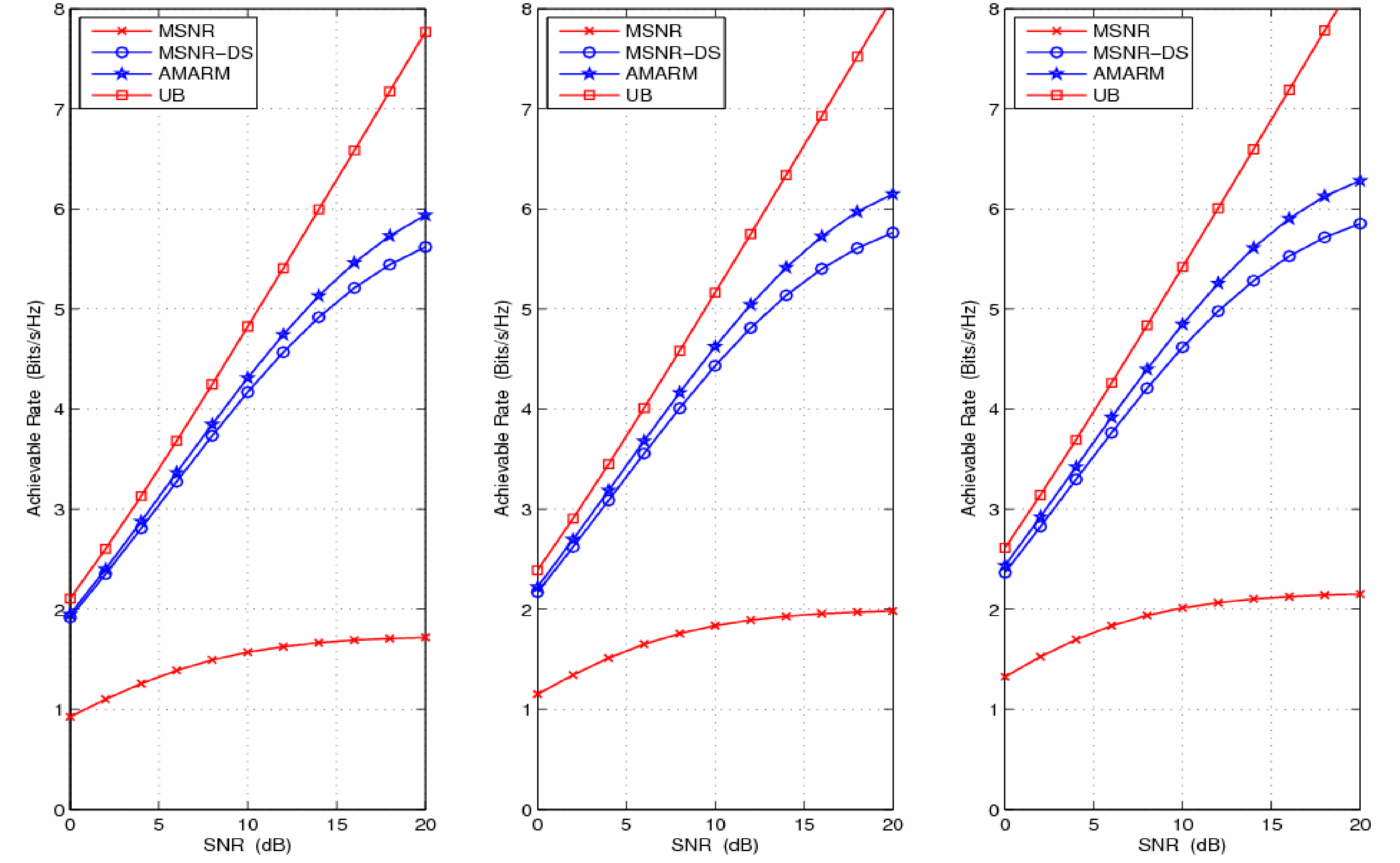
Simulation Setup

- System:
  - (N_t , $N_r = 2$, $N = 128$) MIMO-OFDM system
  - Bandwidth = $1.5\ MHz$, Carrier Frequency = $60\ GHz$
  - Delay spread $10.8\ \mu s$, or equivalently 16 chips

- Compare with two benchmarks:
  - Traditional Max-SNR beamforming
  - Upper Bound:
    - “Doubly Selective” Max-SNR beamforming
    - Genie-aided perfect “past” and “future” ICI-cancellation at receiver

Results - I

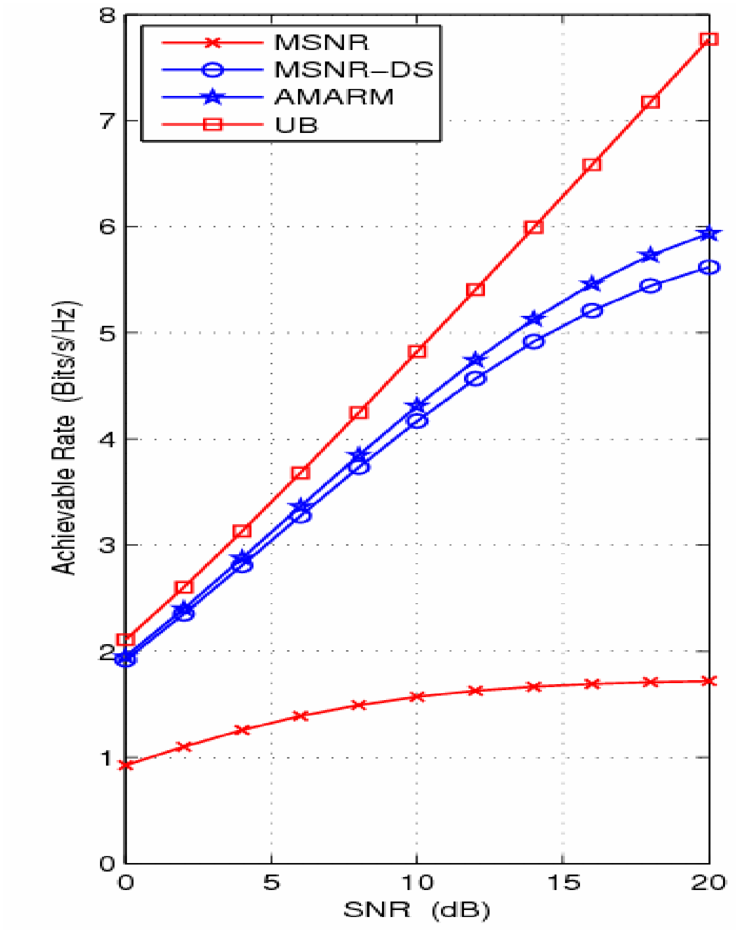
$f_d T_c = 0.008$



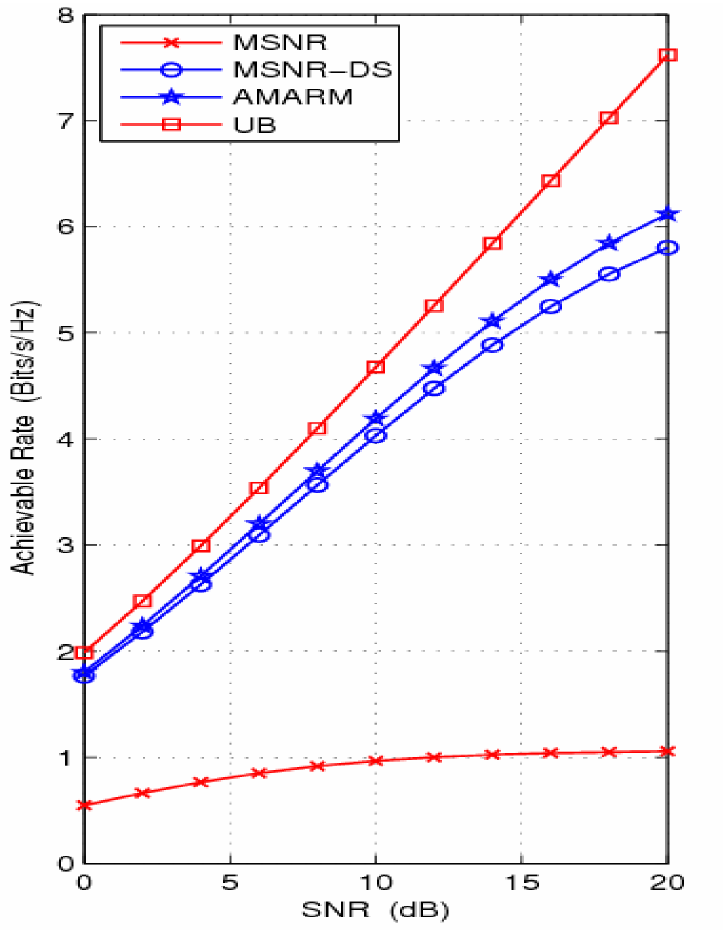
$N_t = 4$ $N_t = 6$ $N_t = 8$

Results - II

$v = 3$ x $69\ km/hr$
$f_d T_c = 0.008$
$D_h=1$



$v = 3$ x $138\ km/hr$
$f_d T_c = 0.016$
$D_h=2$



CSI at Transmitter

- Practically, perfect transmitter CSI is impossible

- Consider Time Division Duplex Operation

  - Easy to obtain CSI when in receive mode
    - pilots / decision directed channel estimation

  - This (outdated) CSI can be forward-predicted when in transmit mode

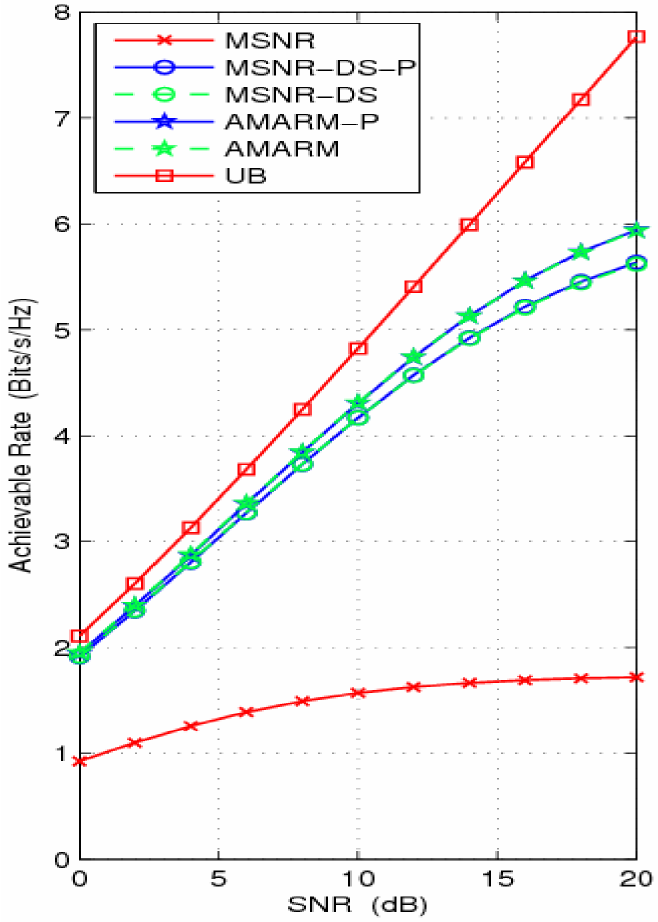
  - For our setup, we use MMSE prediction
    - Use channel correlation from WSSUS Jakes' model

Results - III

$v = 3$ x $69\ km/hr$

$f_d T_c = 0.008$

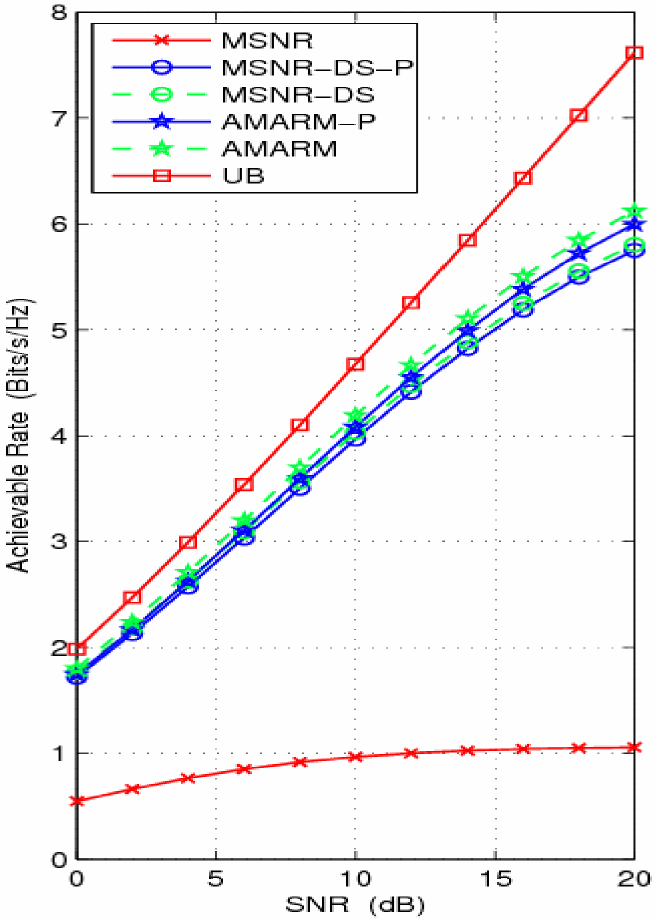
$D_h=1$



$v = 3$ x $138\ km/hr$

$f_d T_c =0.016$

$D_h=2$



Conclusion

- For MCM over DS channels:
  - Considered low-complexity receiver processing
  - Derived an ARM
  - Showed the versatility of our ARM
    - Characterized Rate / Complexity Trade-off
    - Complexity-constrained performance evaluation