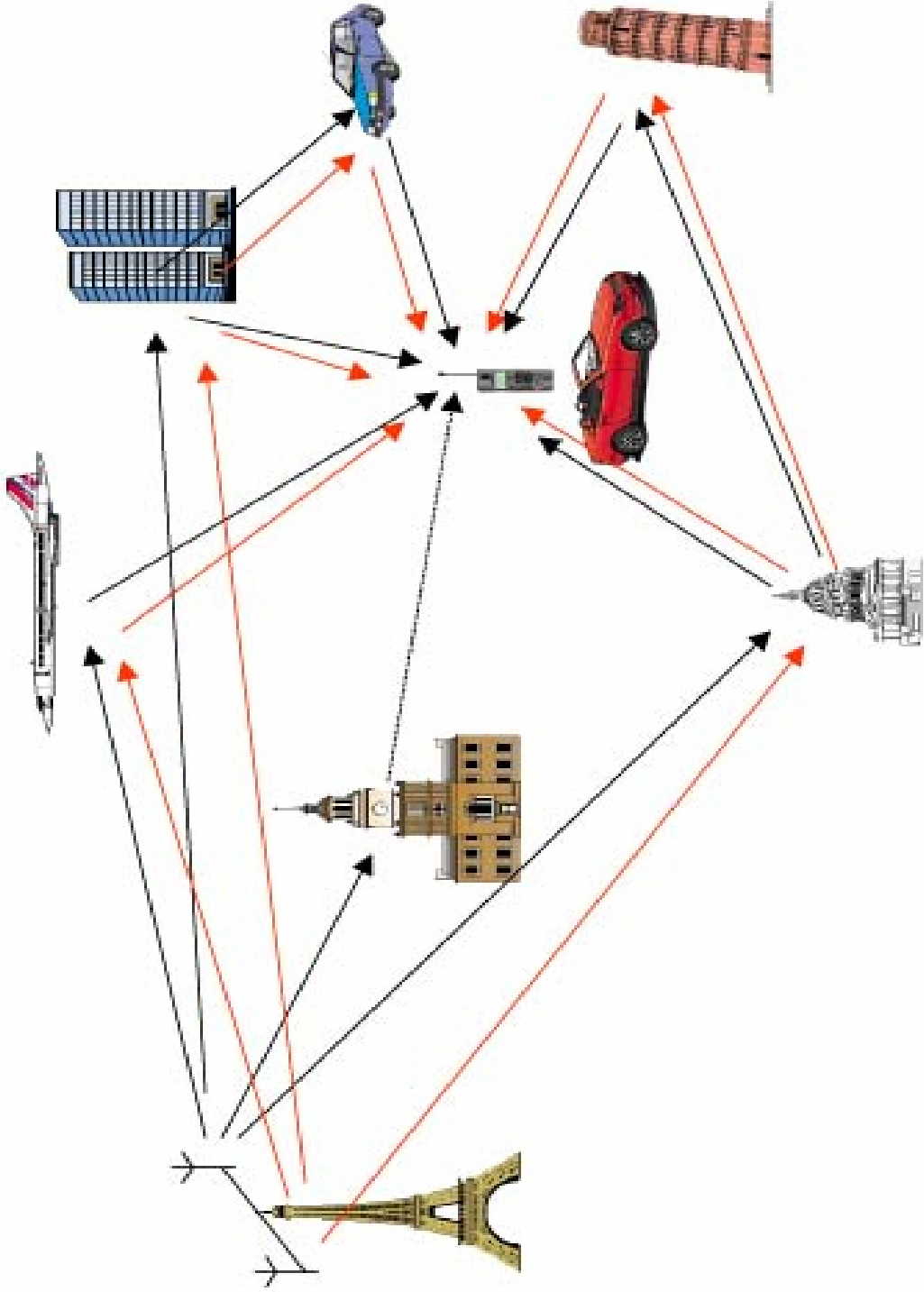

***Beamforming and Combining Strategies
for
MIMO-OFDM
over
Doubly Selective Channels***

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The Propagation Environment



Courtesy: Dr. Youjian Liu, Univ. of Colorado, Boulder.

The Doubly Selective Channel

- Channel modeled as linear time-varying system:

$$r_n = \sum_{l=0}^{N_h-1} h_{n,l} t_{n-l} + v_n$$

- Large delay spread $\Rightarrow N_h \gg 1$
- Rapid time variations modeled by WSSUS Rayleigh fading

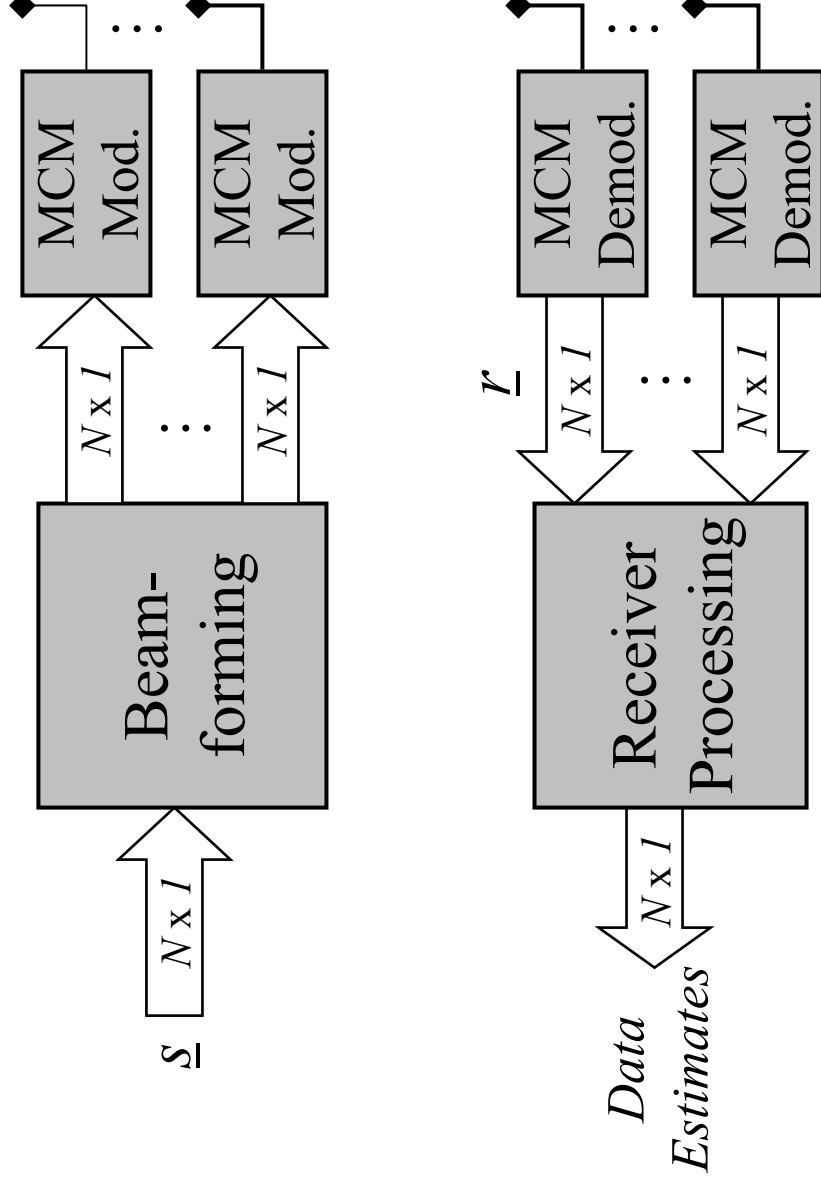
$$E[h_{n_1,l_1} h_{n_2,l_2}^*] = \sigma_{l_1}^2 J_0(2\pi f_d(n_1 - n_2)) \delta_{l_1-l_2}$$

- Popular to use multi-carrier modulation (MCM) on such channels

MIMO - MCM

N sub-carriers

N_t Transmit Antennas

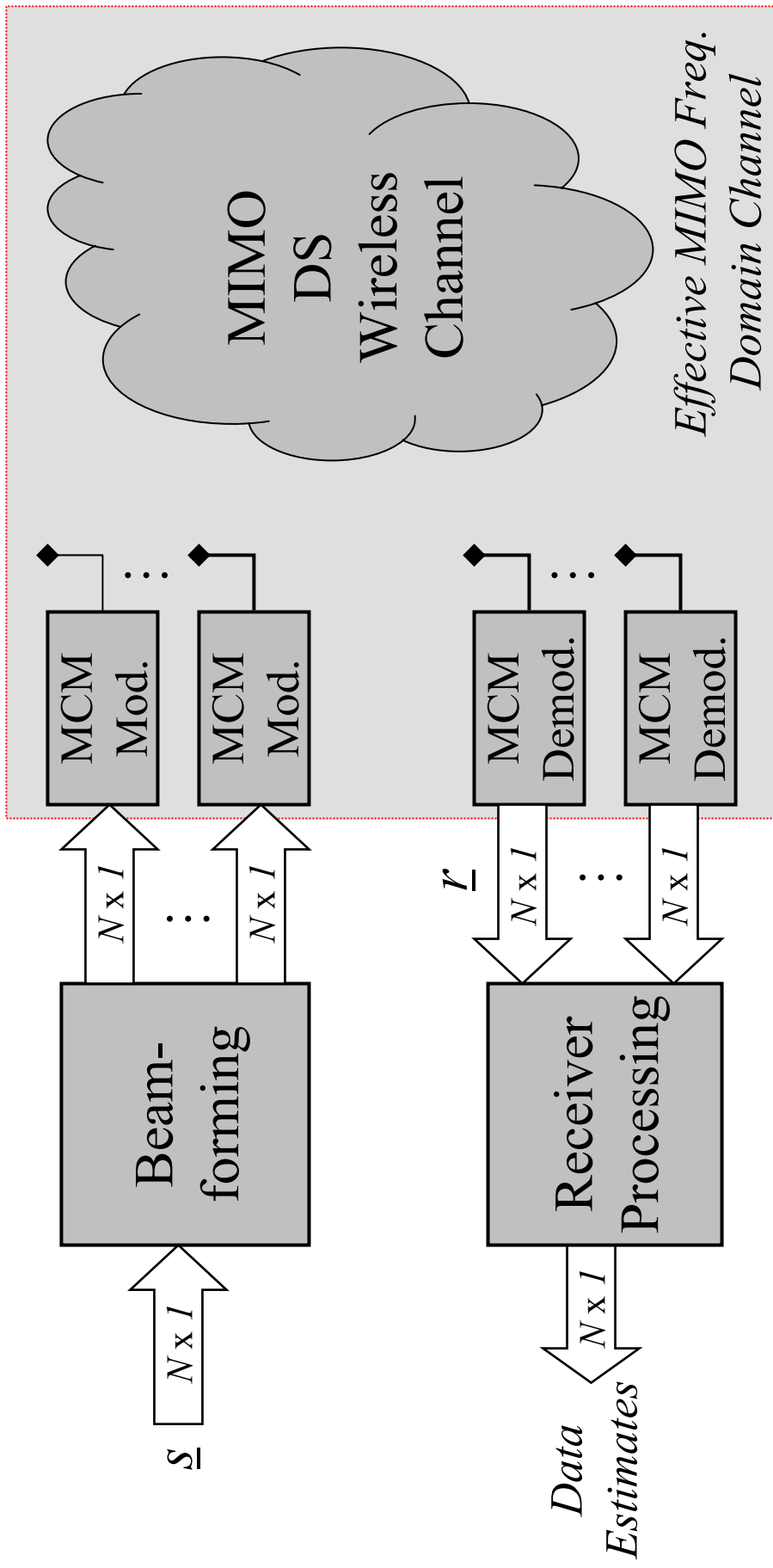


N_r Receive Antennas

MIMO - MCM

N sub-carriers

N_t Transmit Antennas



N_r Receive Antennas

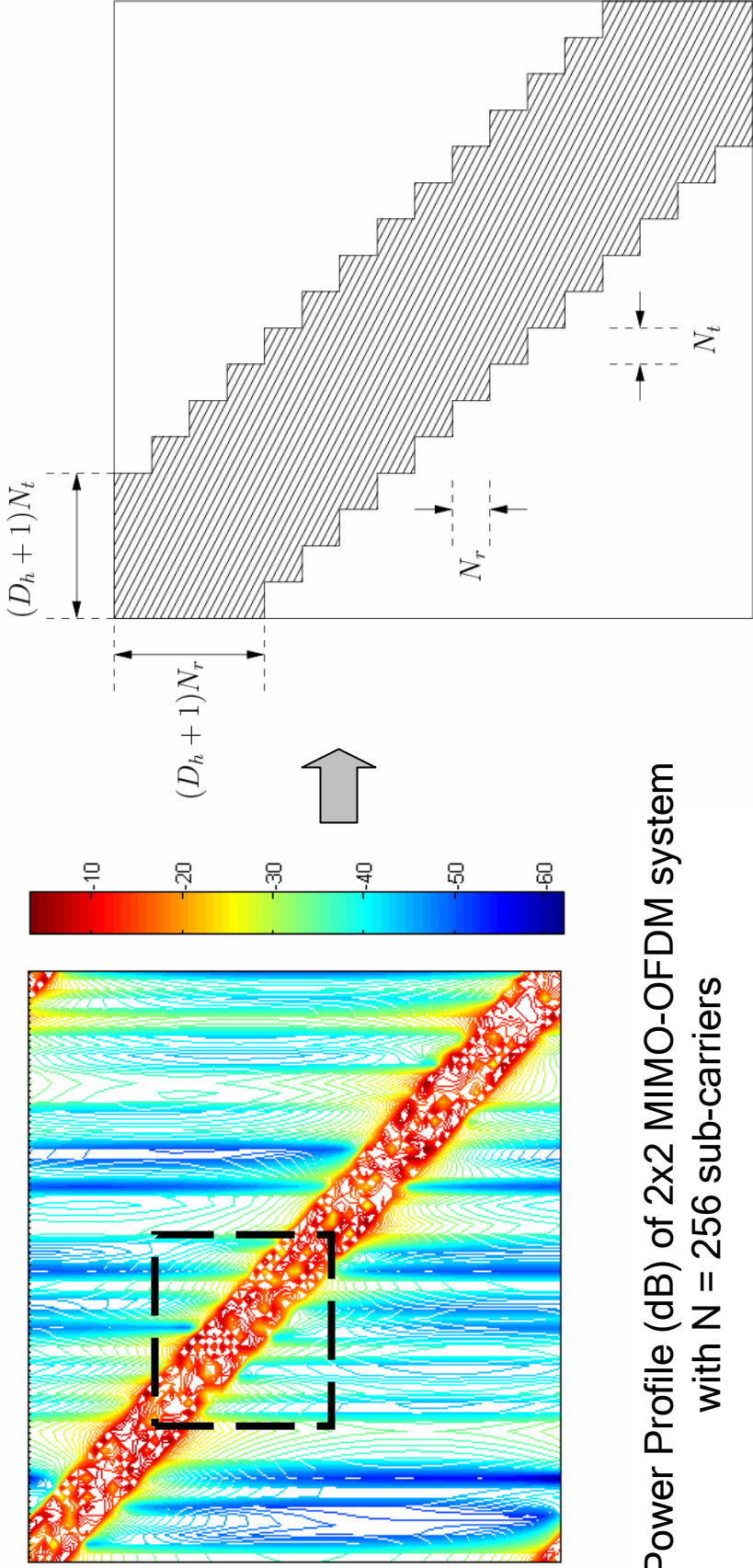
The Challenge

- Delay spread \Rightarrow inter-symbol interference (ISI)
- Doppler spread \Rightarrow inter-carrier interference (ICI)
- Optimal receiver has very high complexity
 - When $N=512$, $N_t = N_r = 2$
 - More than 1×10^6 complex ICI coefficients !!

Can the structure of the MIMO Frequency Domain Channel be exploited?

MIMO - Sub-carrier Coupling Matrix

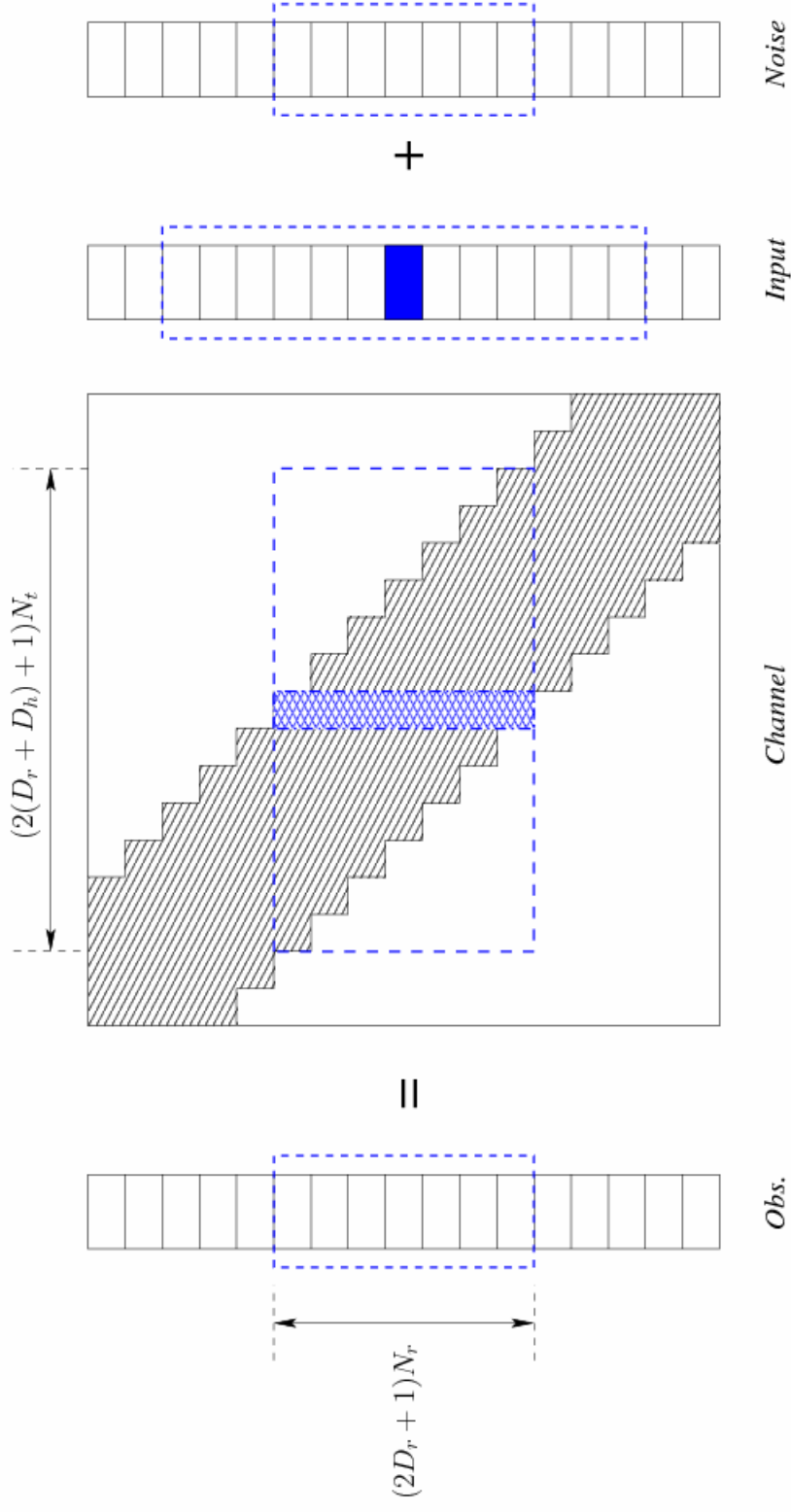
- Use guards / pulse shaping to suppress ISI
- ICI Profile
- D_h = Significant ICI radius



ICI Power Profile (dB) of 2x2 MIMO-OFDM system with $N = 256$ sub-carriers

Local-ICI Processing

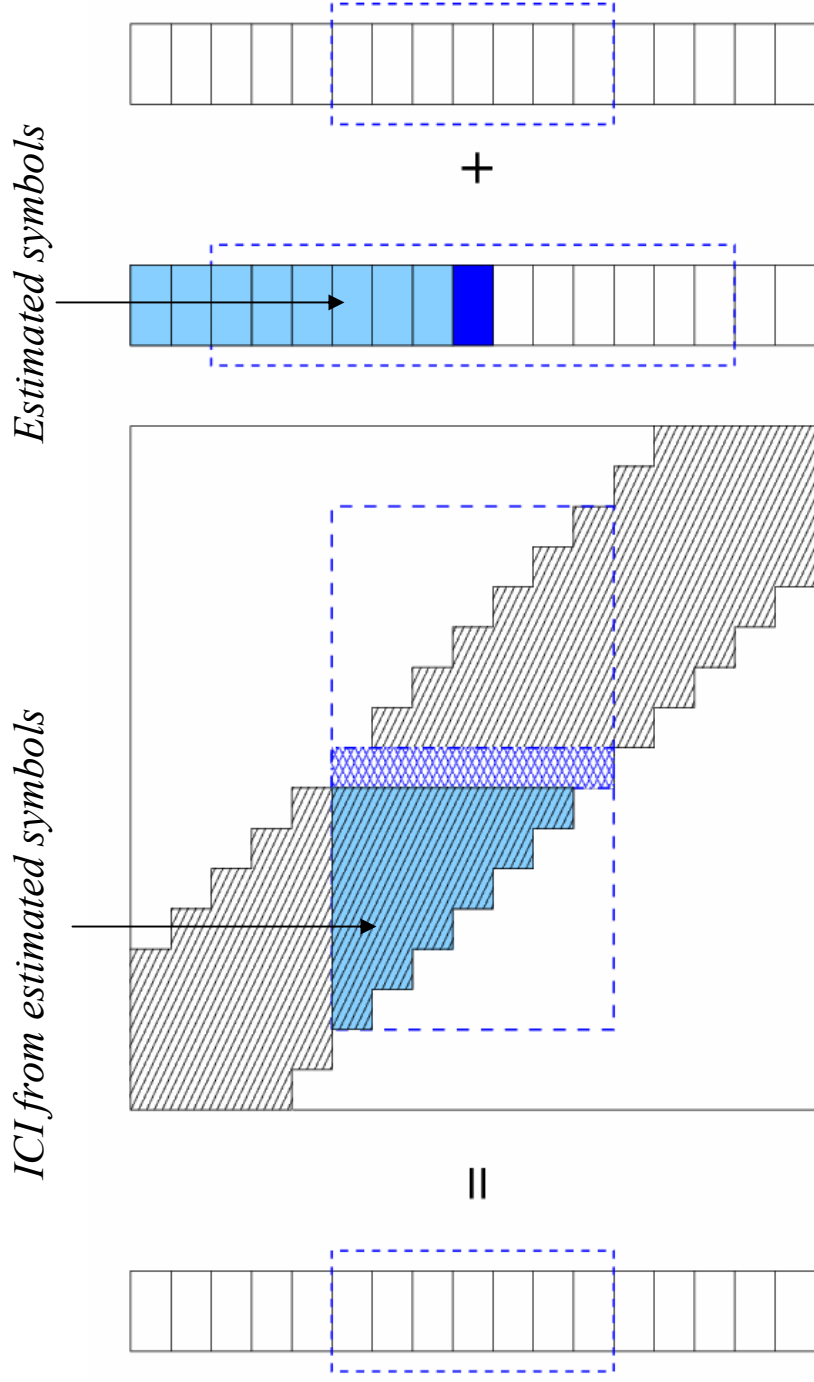
– Consider a local-ICI processing receiver which:



- Local-ICI processing performance/complexity controlled by “ D_r ”

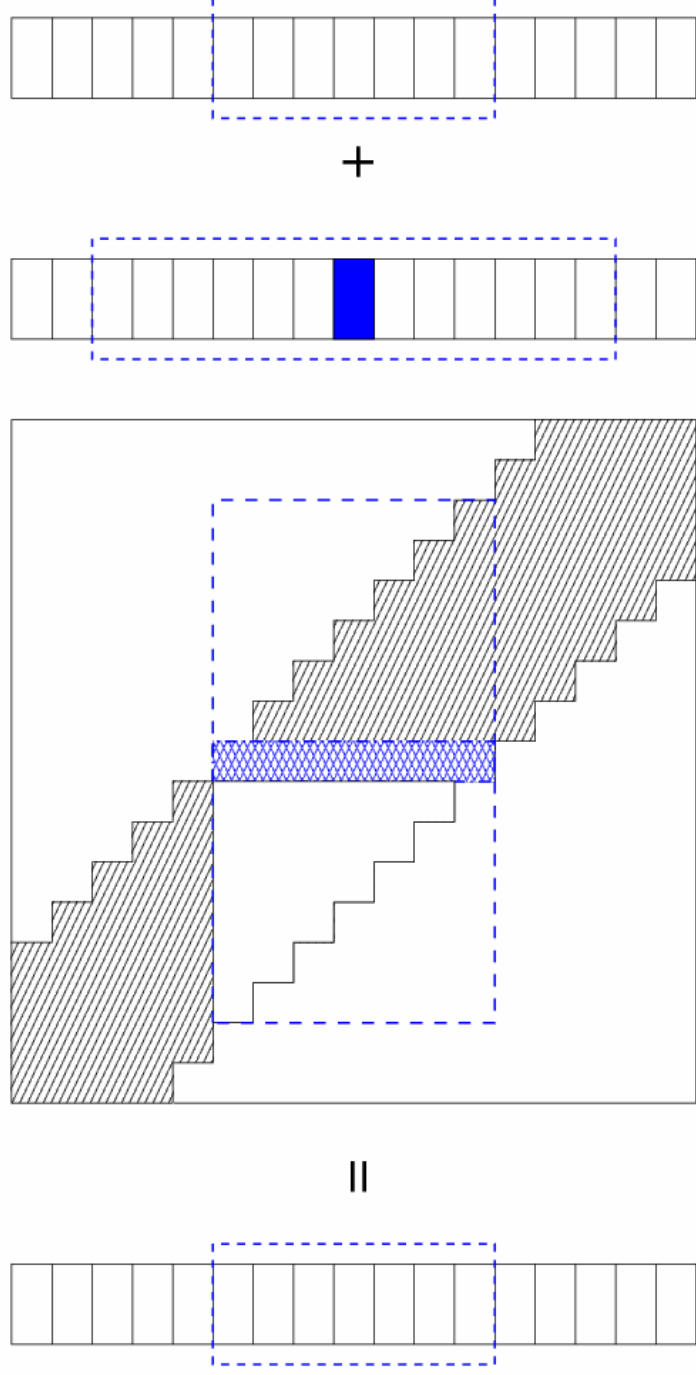
Local-ICI Processing

- Consider a local-ICI processing receiver which:
 - Cancels ICI using previous symbol estimates \Rightarrow Partial-SIC (P-SIC)



Local-ICI Processing

- Consider a local-ICI processing receiver which:
 - Cancels ICI using previous symbol estimates \Rightarrow Partial-SIC (P-SIC)
 - Performs local linear combining after P-SIC
 - Maximizes rate on each sub-carrier



Measuring Performance

- Local-ICI processing applicable to many MCM schemes...
- Choice of MCM scheme affects:
 - Decoding Complexity
 - Spectral Efficiency
 - Diversity Exploitation
- “Practical” performance metric should incorporate effects of all the above
- We measure the achievable ergodic rate when local-ICI processing is used on a generic MIMO-MCM system

Achievable Rate Metric (ARM)

- Assumptions:
 - Coding & Decoding over large blocks of MCM symbols
 - Each sub-carrier coded independently
 - Use of i.i.d. (complex) Gaussian codebooks
- ARM Definition:
$$R = E_{\mathbf{H}} \left[\sum_{k=0}^{N-1} \log(1 + \gamma_k) \right]$$
 - γ_k : SINR for k^{th} sub-carrier using partial SIC and local linear combining.
 - SINR is function of beamforming vectors, local linear combiner and ICI coefficients.
- ARM Advantages:
 - Characterizes trade-off between complexity (D_r) and performance (R)
 - Compares performance of local-ICI processing on various MCM schemes at equal complexity

- Wish to design beamforming vectors for sub-carriers
 - Assume CSI at transmitter
 - Desire good performance in the presence of ICI
 - Assume local-ICI processing receiver

- Consider a MIMO-OFDM system
 - MIMO-OFDM \Rightarrow White Noise
 - MIMO-OFDM \Rightarrow No ISI
 - Equal power across sub-carriers

Beamforming Approaches

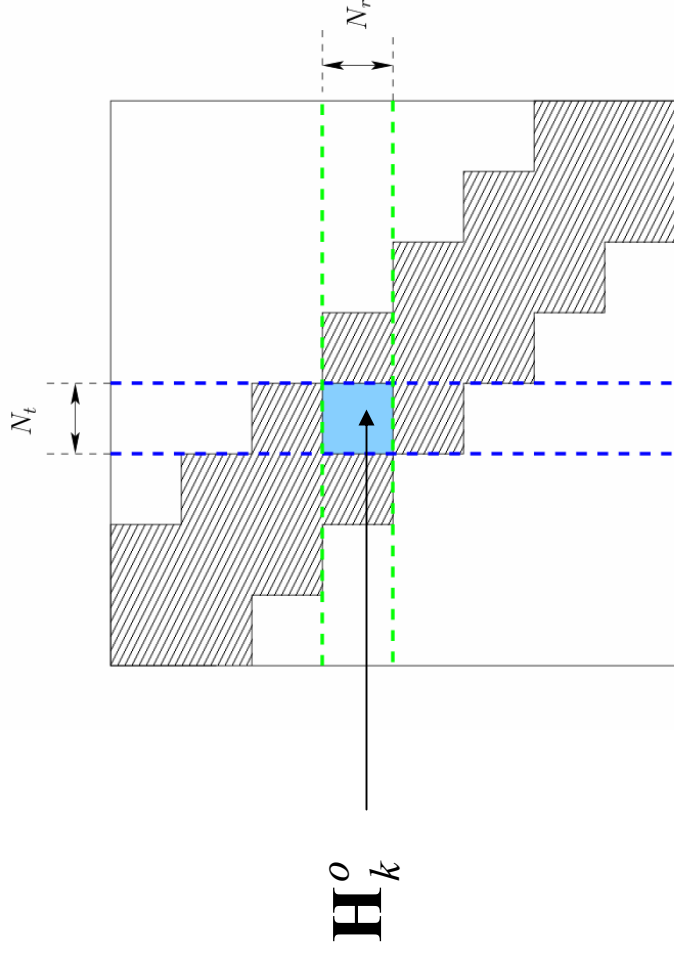
- 0) Traditional Max-SNR beamforming...
 - e.g., [Bolcskei et al, *T.Comm.*, Feb '02]
 - Not designed for ICI

- 1) “Doubly selective” Max-SNR beamforming...
 - Maximize total received energy per sub-carrier

- 2) Approximate Max-ARM beamforming...

Traditional Max-SNR Beamforming

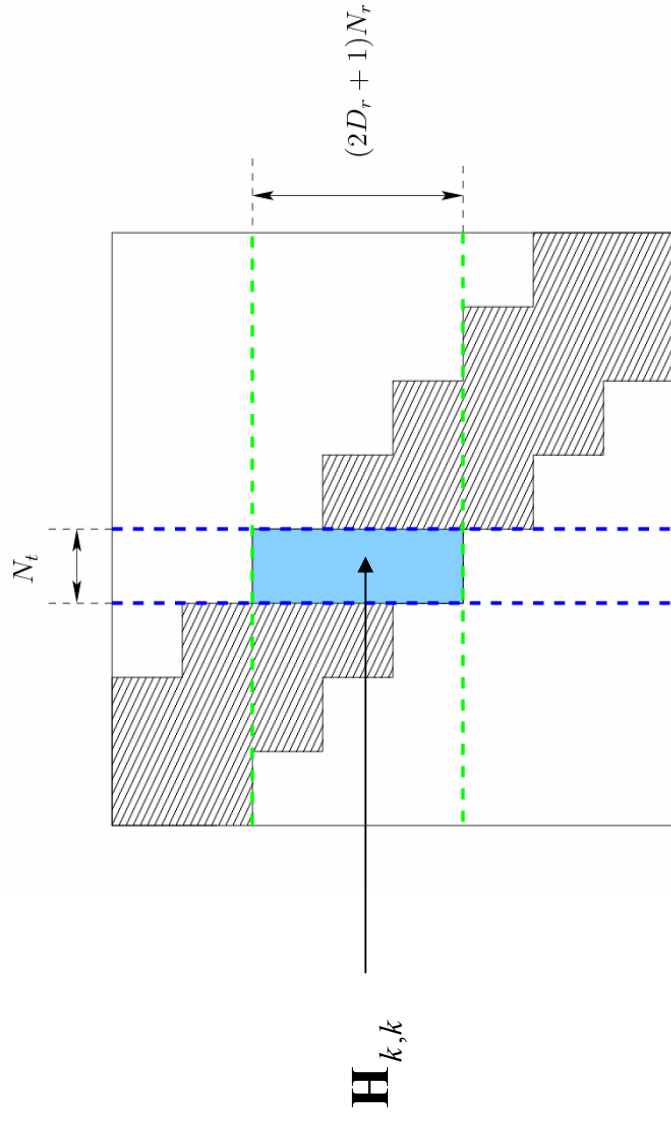
- Maximize SNR for each sub-carrier independently
- “signal” defined as $\mathbf{z}_k^H \mathbf{H}_k^o \mathbf{x}_k S_k$
- Receiver performs antenna combining, but not sub-carrier combining ($D_r=0$)



- Beamforming vector for k^{th} sub-carrier: $\mathbf{x}_k = \nu_* (\mathbf{H}_k^o)^H \mathbf{H}_k^o$

“Doubly-Selective” Max-SNR Beamforming

- Maximize SNR for each sub-carrier independently
- Receiver performs both antenna and sub-carrier combining ($D_r = D_h$)
- “signal” defined as $\mathbf{z}_k^H \mathbf{H}_{k,k} \mathbf{x}_k S_k$



- Beamforming vector for k^{th} sub-carrier: $\mathbf{x}_k = \mathbf{v}^* (\mathbf{H}_{k,k}^H \mathbf{H}_{k,k})$

Max-ARM Beamforming

- Motivation
 - SNR-maximization does not penalize ICI effects on other sub-carriers!
 - Can an ICI penalty be incorporated into beamformer design?
- Ideal solution: Maximize ARM
 - Leads to a difficult optimization problem!

$$R = E_{\mathbf{H}} \left[\sum_{k=0}^{N-1} \log(1 + \gamma_k) \right]$$

$$\gamma_k = \frac{\mathfrak{E}_{sig}(\mathbf{x}_k, \mathbf{z}_k, \mathbf{H})}{\mathfrak{E}_{ici}(\{\mathbf{x}_{k'}\}_{k' \in \mathcal{K}_+(k)}, \mathbf{z}_k, \mathbf{H})} + \mathfrak{E}_{noise}$$

Approximate Max-ARM Beamforming

$$R = E_{\mathbf{H}} \left[\sum_k \log \left(1 + \frac{\mathfrak{E}_{sig}(\mathbf{x}_k, \mathbf{z}_k, \mathbf{H})}{\underbrace{\mathfrak{E}_{ici}(\{\mathbf{x}_{k'}\}_{k' \in \mathcal{K}_+(k)}, \mathbf{z}_k, \mathbf{H}) + \mathfrak{E}_{noise}}_{\gamma_k}} \right) \right]$$

– Iterative optimization:

- Given beamformers $\{\mathbf{x}_k\}$, choose combiners

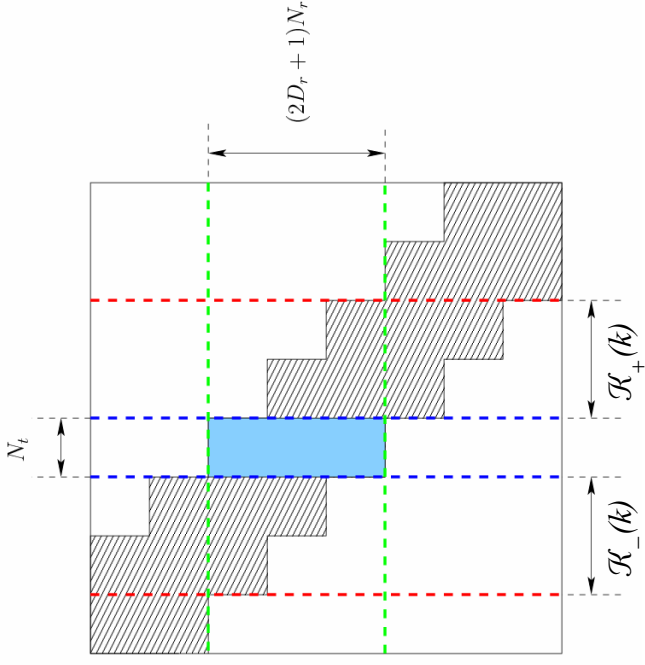
$$\mathbf{z}_k = \max_{\mathbf{z}_k} \gamma_k, \text{ for } k = 0, \dots, N-1$$

– Easy: generalized eigenvalue problem!

- Given combiners $\{\mathbf{z}_k\}$, how to choose beamforming vectors $\{\mathbf{x}_k\}$?
 - Note: Each \mathbf{x}_k affects several γ_k
- Use intuition from ARM for low-SNR and high-SNR regimes...

Intuition from ARM: High-SNR Regime

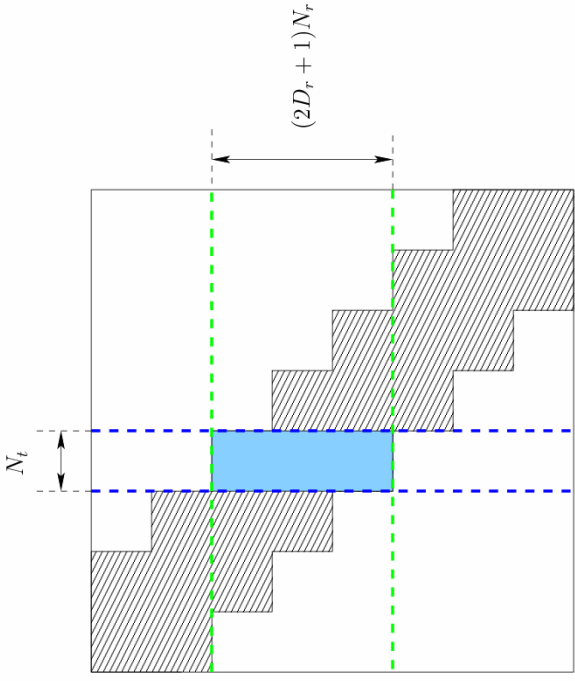
$$R = E_{\mathbf{H}} \left[\sum_k \log \left(1 + \frac{\mathcal{E}_{\text{sig}}(\mathbf{x}_k, \mathbf{z}_k, \mathbf{H})}{\mathcal{E}_{\text{ici}}(\{\mathbf{x}_{k'}\}_{k' \in \mathcal{K}_+(k)}, \mathbf{z}_k, \mathbf{H}) + \mathcal{E}_{\text{noise}}} \right) \right]$$



- When designing \mathbf{x}_k :
 - Residual ICI dominates noise
 - Due to P-SIC, need not consider ICI caused to “future” sub-carriers $\mathcal{K}_+(k)$
 - Maximize signal, minimize ICI caused to “past” sub-carriers $\mathcal{K}_-(k)$

Intuition from ARM: Low-SNR Regime

$$R = E_{\mathbf{H}} \left[\sum_k \log \left(1 + \frac{\mathcal{E}_{\text{sig}}(\mathbf{x}_k, \mathbf{z}_k, \mathbf{H})}{\mathcal{E}_{\text{ici}}(\{\mathbf{x}_{k'}\}_{k' \in \mathcal{K}_+(k)}, \mathbf{z}_k, \mathbf{H}) + \mathcal{E}_{\text{noise}}} \right) \right]$$



- When designing \mathbf{x}_k :
 - Noise dominates residual ICI
 - Need to maximize signal energy

Approximate Max-ARM Beamforming

- With low-SNR and high-SNR intuitions, we propose:

$$\mathbf{x}_k^* = \arg \max_{\|\mathbf{x}_k\|=1} \frac{\mathbf{x}_k^H (\mathbf{H}_{k,k}^H \mathbf{Z}_k^H \mathbf{H}_{k,k}^H) \mathbf{x}_k}{\mathbf{x}_k^H \left(\sum_{k' \in \mathcal{K}_-(k)} \mathbf{H}_{k',k}^H \mathbf{Z}_{k'}^H \mathbf{H}_{k',k}^H + \sigma^2 \mathbf{I} \right) \mathbf{x}_k}, \quad k = 0, \dots, N-1$$

- Note that:
 - At high SNR, we prevent causing ICI to “past” sub-carriers
 - At low SNR, we maximize signal energy

Complexity

| | Traditional Max-SNR | “Doubly Selective” Max-SNR | Approximate Max-ARM |
|---|------------------------|------------------------------------|---|
| Beamformer Design (per OFDM symbol) | $O(NN_t^3)$ | $O(NN_t^3)$ | $O(N_i N N_r^3)$ |
| Combiner Design (per OFDM symbol) | $O(NN_r^3)$ | $O(N(2D_r + 1)^3 N_r^3)$ | $O(N_i N (2D_r + 1)^3 N_r^3)$ |
| Aggregate (per OFDM symbol) | $O(N(N_t^3 + N_r^3))$ | $O(N((2D_r + 1)^3 N_r^3 + N_t^3))$ | $O(N_i N ((2D_r + 1)^3 N_r^3 + N_t^3))$ |

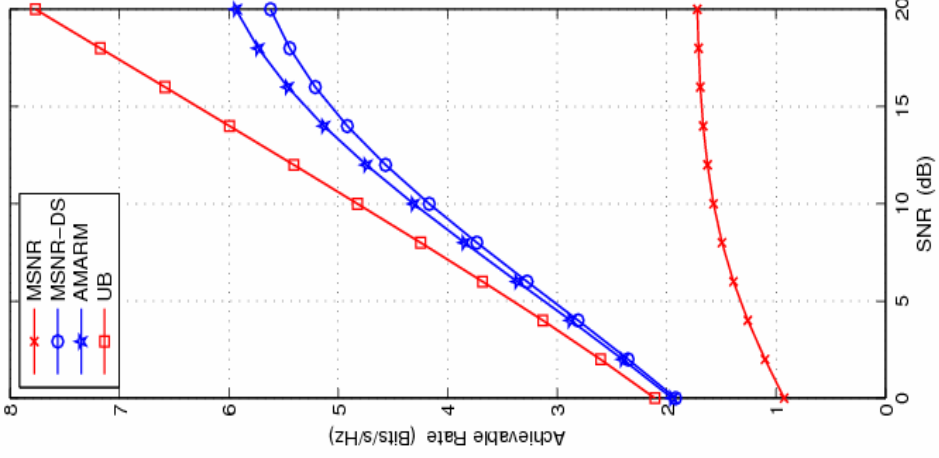
Simulation Setup

- System:
 - ($N_t, N_r = 2, N = 128$) MIMO-OFDM system
 - Bandwidth = 1.5 MHz , Carrier Frequency = 60 GHz
 - Delay spread $10.8\ \mu\text{s}$, or equivalently 16 chips
- Compare with two benchmarks:
 - Traditional Max-SNR beamforming
 - Upper Bound:
 - “Doubly Selective” Max-SNR beamforming
 - Genie-aided “past” and “future” ICI-cancellation at receiver

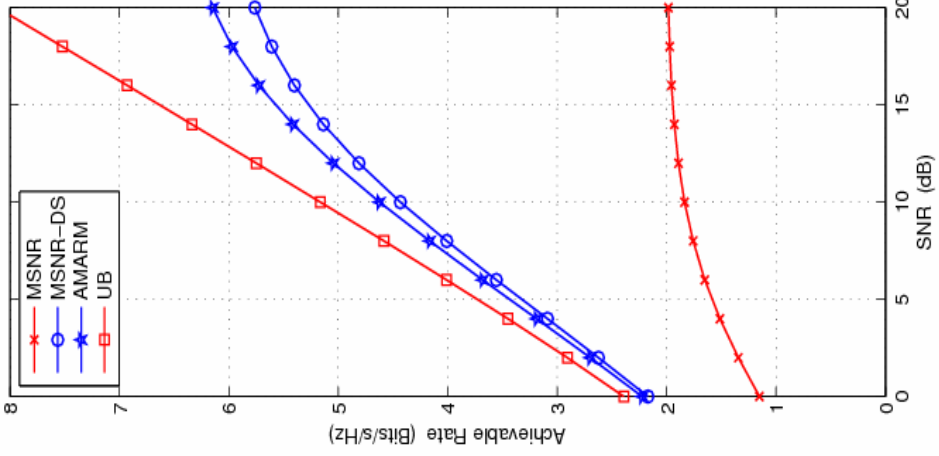
Results - I

$$f_d T_c = 0.008$$

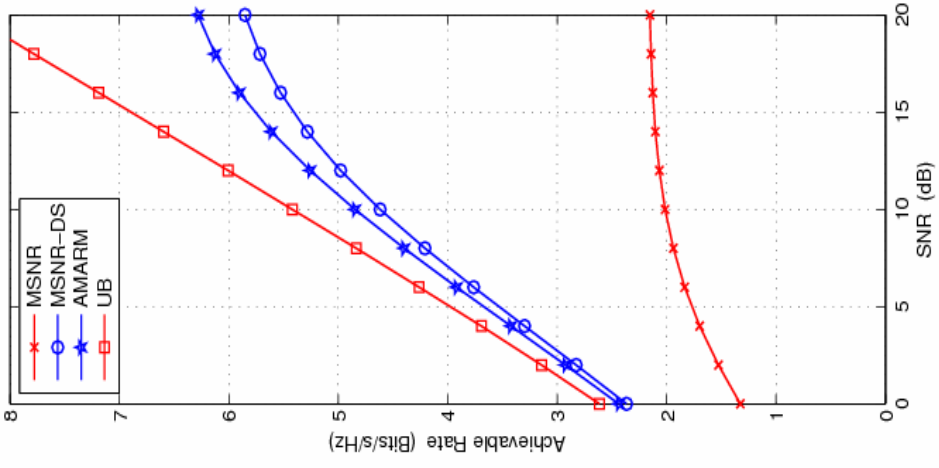
$$N_t = 4$$



$$N_t = 6$$

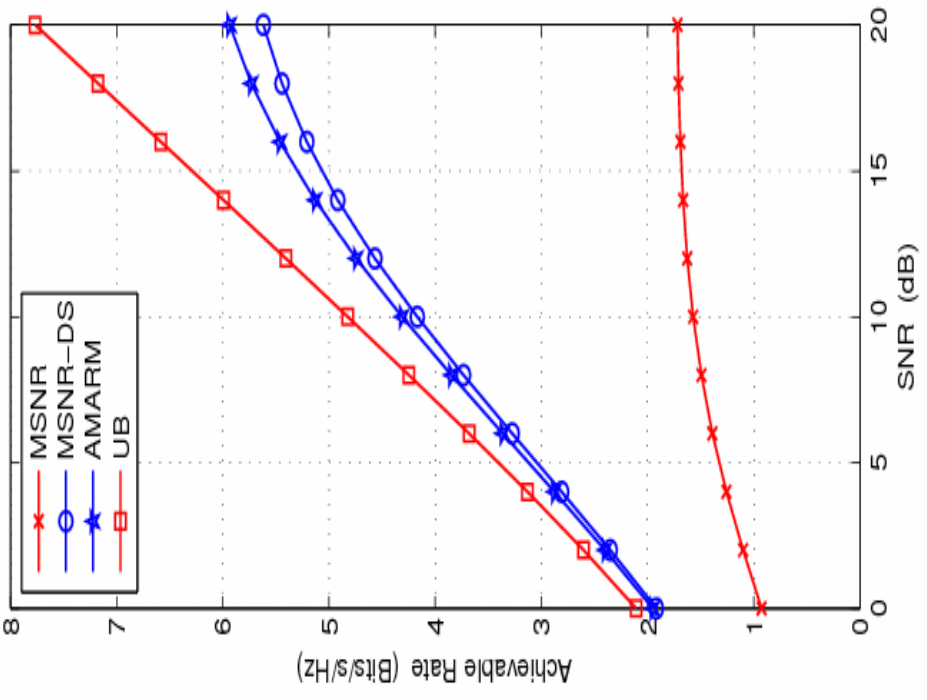


$$N_t = 8$$

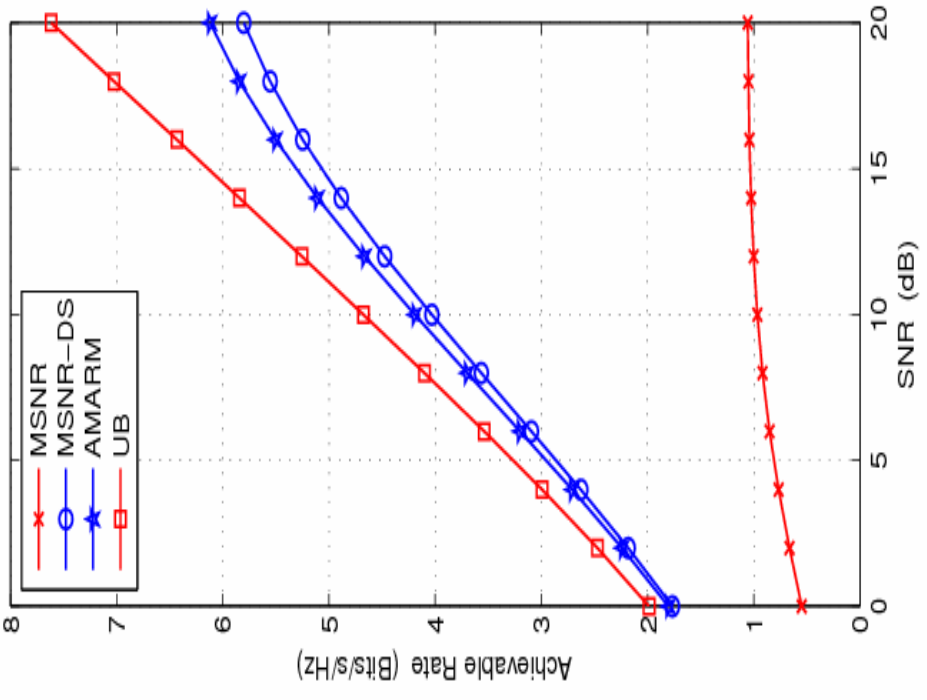


Results - II

$v = 3 \times 69 \text{ km/hr}$
 $f_d T_c = 0.008$
 $D_h = 1$



$v = 3 \times 138 \text{ km/hr}$
 $f_d T_c = 0.016$
 $D_h = 2$

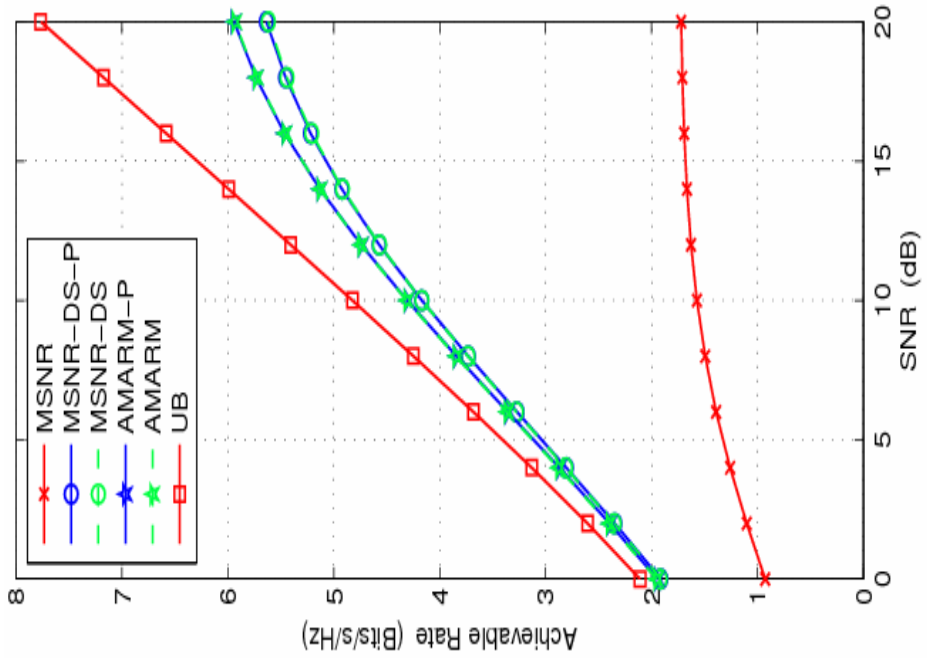


CSI at Transmitter

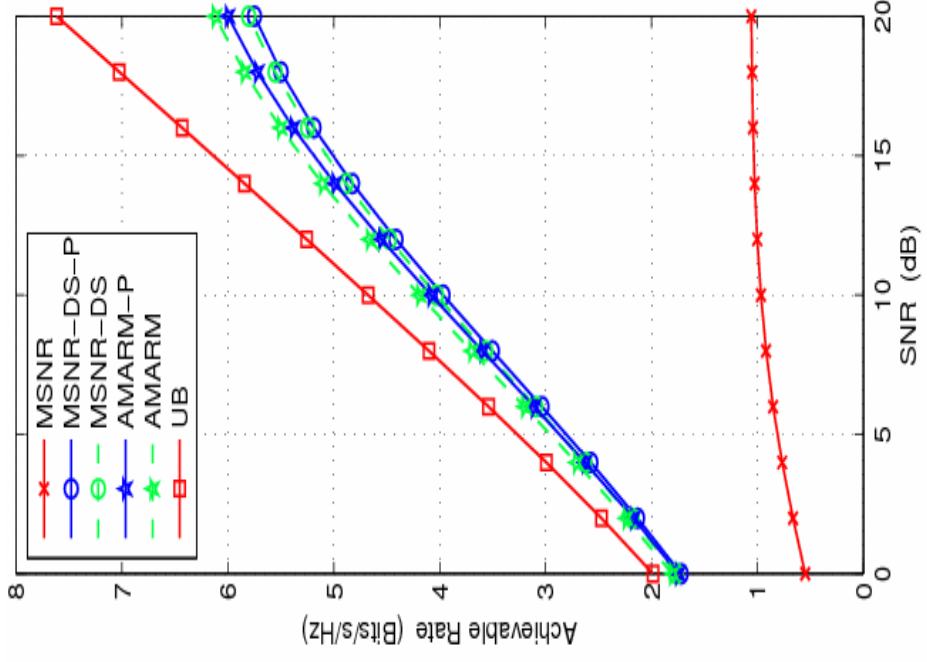
- Practically, perfect transmitter CSI is unrealistic
- Consider Time Division Duplex (TDD) Operation
 - Easy to obtain CSI when in receive mode
 - pilots / decision directed channel estimation
 - This (outdated) CSI can be forward-predicted when in transmit mode
 - For our setup, we use MMSE prediction
 - Use channel correlation from WSSUS Jakes' model

Results - III

$v = 3 \times 69 \text{ km/hr}$
 $f_d T_c = 0.008$
 $D_h = 1$



$v = 3 \times 138 \text{ km/hr}$
 $f_d T_c = 0.016$
 $D_h = 2$



Conclusion

- Considered MIMO-OFDM system over DS channel
- Proposed a low-complexity local-ICI processing receiver
- Derived an ergodic achievable rate expression
- Proposed two beamforming solutions:
 - 1) Max-SNR for DS channels
 - 2) Approximate Max-ARM
- Demonstrated advantages via simulation