Iterative Equalization for Single Carrier Cyclic Prefix in Doubly-Dispersive Channels

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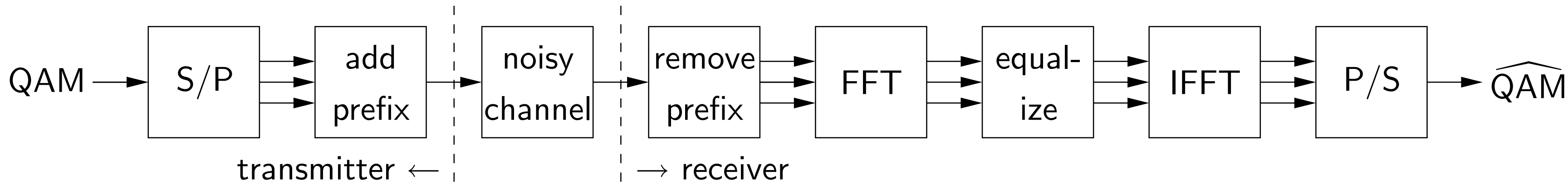
Background:

- First, consider communication over time-dispersive channels.
- Options:
  1. Single-Carrier Modulation with Time-Domain Equalization
     - $\mathcal{O}(N_h)$ operations/symbol for chan length N_h.
     - low peak-to-average-power ratio (PAPR).
  2. Multi-Carrier Modulation with Freq-Domain Equalization
     - $\mathcal{O}(\log N)$ operations/symbol for block length N.
     - high PAPR.
     - "OFDM."
  3. Single-Carrier Modulation with Freq-Domain Equalization
     - $\mathcal{O}(\log N)$ operations/symbol for block length N.
     - low PAPR.
     - "single carrier cyclic prefix (**SCCP**)."

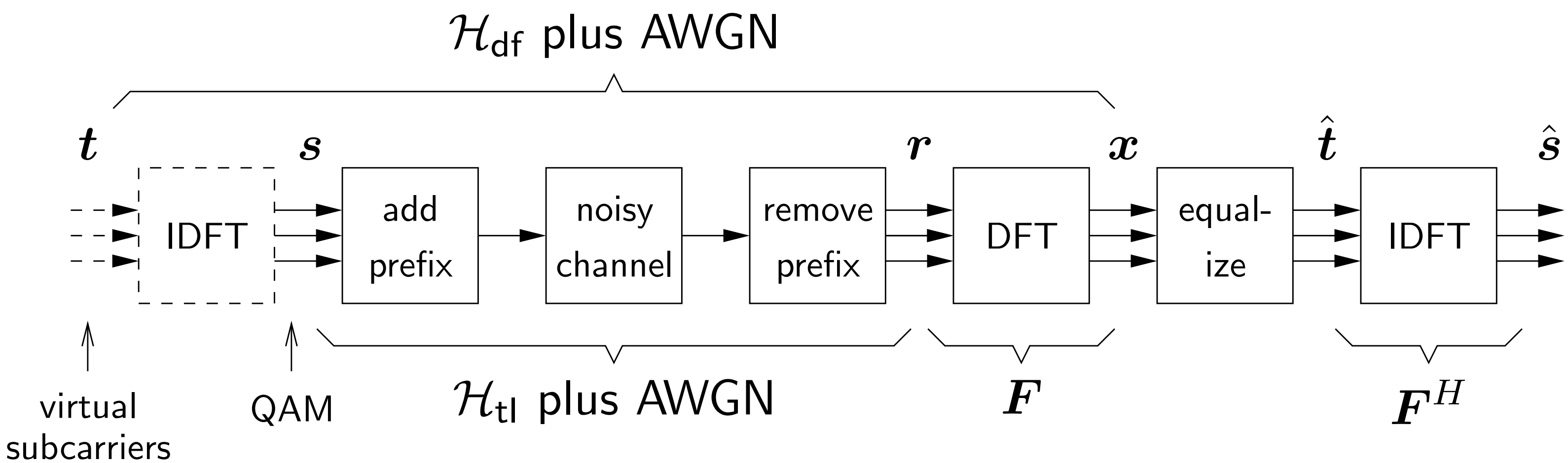
Single-Carrier Cyclic Prefix (SCCP):



- SCCP is like OFDM with both FFTs at the receiver.
- Freq-domain equalization requires only one mult-per-symbol if:
  1. cyclic prefix length $>$ channel delay spread,
  2. channel time-invariant over the FFT-block interval.
- Our final goal, however, is communication over *time-dispersive and frequency-dispersive* channels.

How can we handle SCCP with significant channel variation over the block interval?

System Model:



$$\boldsymbol{r} = \mathcal{H}_{\text{tl}}\boldsymbol{s} + \boldsymbol{\nu}$$

$$\boldsymbol{x} = \underbrace{\boldsymbol{F}\mathcal{H}_{\text{tl}}\boldsymbol{F}^H}_{\mathcal{H}_{\text{df}}}\underbrace{\boldsymbol{F}\boldsymbol{s}}_{\boldsymbol{t}} + \underbrace{\boldsymbol{F}\boldsymbol{\nu}}_{\boldsymbol{w}}$$

where $\mathcal{H}_{\text{tl}}$ = circular-convolution matrix,

$\mathcal{H}_{\text{df}}$ = "virtual-subcarrier" coupling matrix.

$\rightsquigarrow$ $\mathcal{H}_{\text{df}}$ diagonal iff channel is LTI and prefix-length is adequate.

Virtual-Subcarrier Coupling Matrix $\mathcal{H}_{\mathsf{df}}$:

$$\mathcal{H}_{\mathsf{df}} = \begin{pmatrix} h_{\mathsf{df}}(0,0) & h_{\mathsf{df}}(-1,1) & \dots & h_{\mathsf{df}}(1-N,N-1) \\ h_{\mathsf{df}}(1,0) & h_{\mathsf{df}}(0,1) & \dots & h_{\mathsf{df}}(2-N,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ h_{\mathsf{df}}(N-1,0) & h_{\mathsf{df}}(N-2,1) & \dots & h_{\mathsf{df}}(0,N-1) \end{pmatrix}$$

$$h_{\mathsf{df}}(\nu,k) := \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{N-1} h_{\mathsf{tl}}(n,l)e^{-j\frac{2\pi}{N}n\nu}e^{-j\frac{2\pi}{N}lk}$$

$=$ response at carrier $k+\nu$ to an impulse applied at carrier k

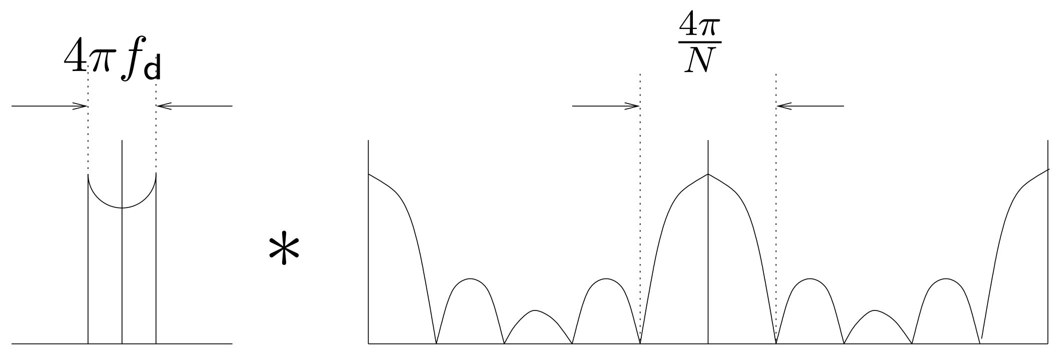
$h_{\mathsf{tl}}(n,l) :=$ response at time n to an impulse applied at time $n-l$

Inter-Carrier Interference Mechanism:

Doppler Spread meets Finite Block Length:

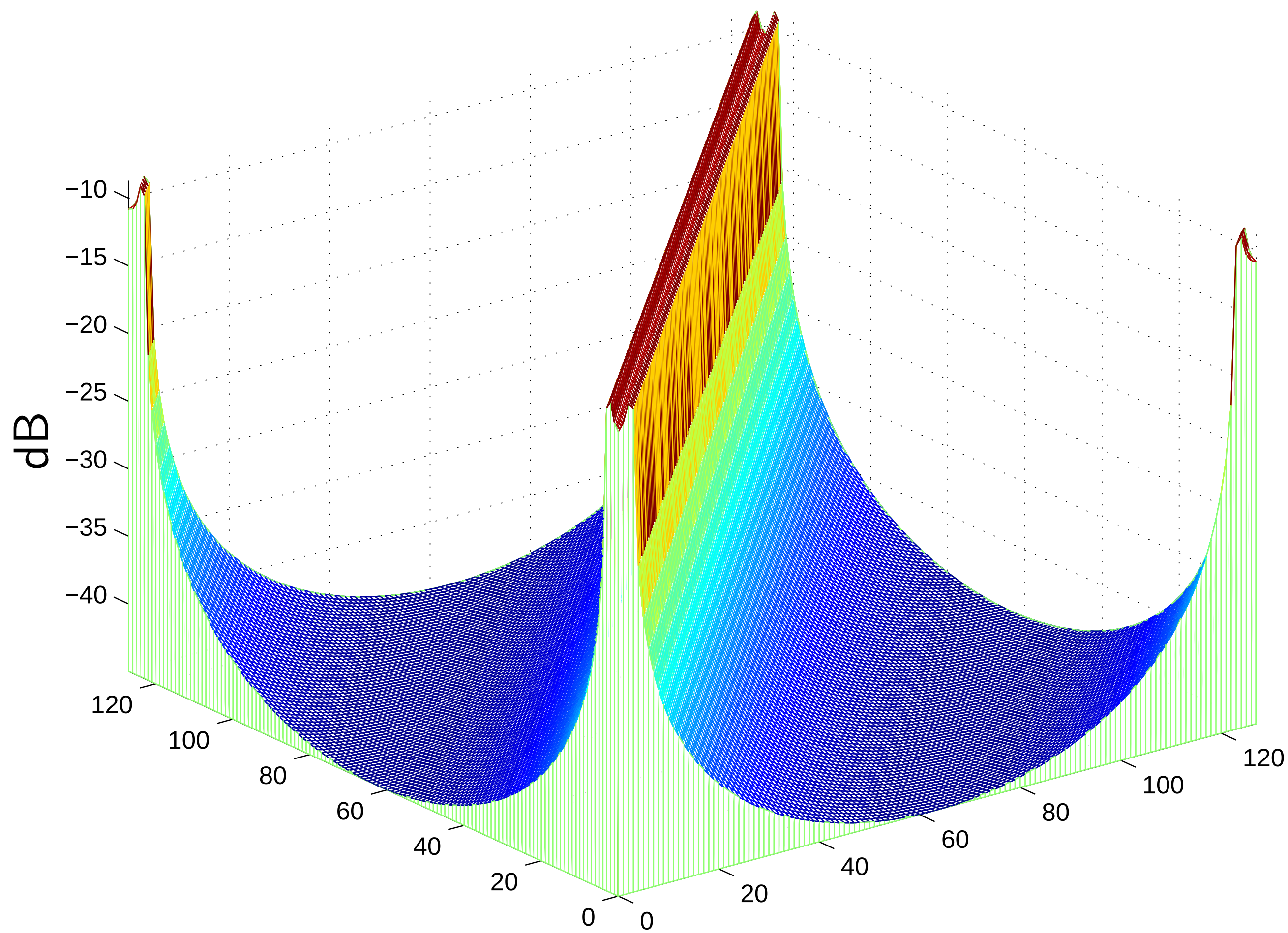
$$\mathrm{E}\{|h_{\mathsf{df}}(\nu,k)|^2\} = \overbrace{\left(\frac{I_{[0,2\pi f_{\mathsf{d}})}(|\phi|)\ \sum_l \sigma_l^2}{\sqrt{(2\pi f_{\mathsf{d}})^2 - \phi^2}}\right)}^{\text{assuming WSSUS Rayleigh}} * \overbrace{\left(\frac{\sin(\phi N/2)}{N\sin(\phi/2)}\right)^2}^{\text{block length } N}\Bigg|_{\phi = \frac{2\pi}{N}\nu}$$

$=$ Samples of

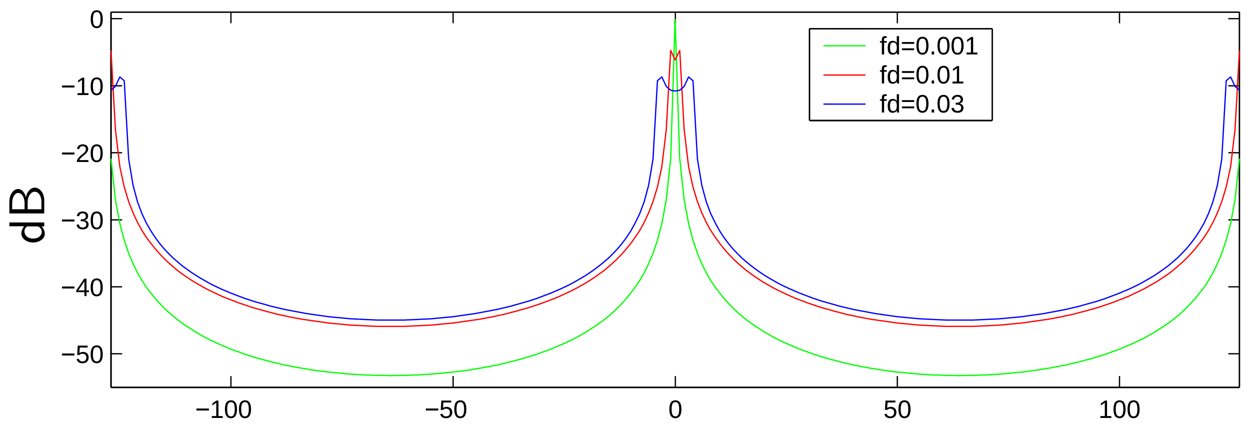


Note: *Zero Doppler spread* $\Rightarrow$ *Sample at sinc nulls* $\Rightarrow$ *Zero ICI*

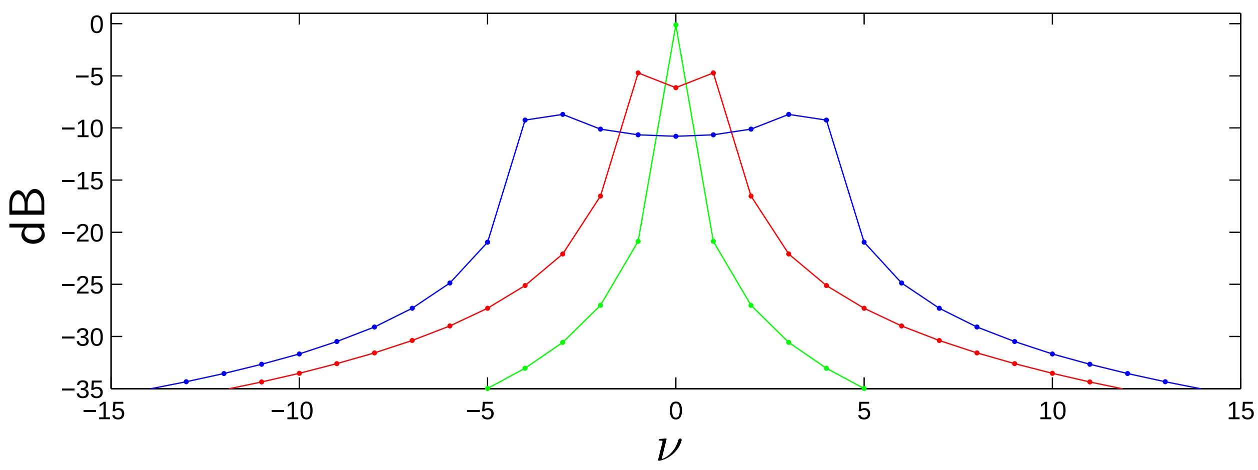
Rayleigh $\mathrm{E}\{|h_{\mathsf{df}}(\nu,k)|^2\}$ for $N = 128$ and $f_{\mathsf{d}} = 0.03$:



Rayleigh $\mathrm{E}\{|h_{\mathsf{df}}(\nu,\cdot)|^2\}$ for $N = 128$ and various f_{d}:



(zoomed)



SCCP Equalization/Detection:

Objective: Recover finite-alphabet vector $\boldsymbol{s}$ from $\boxed{\boldsymbol{x} = \mathcal{H}_{\mathsf{df}} \boldsymbol{F}\boldsymbol{s} + \boldsymbol{w}}$.

Classical Strategies:

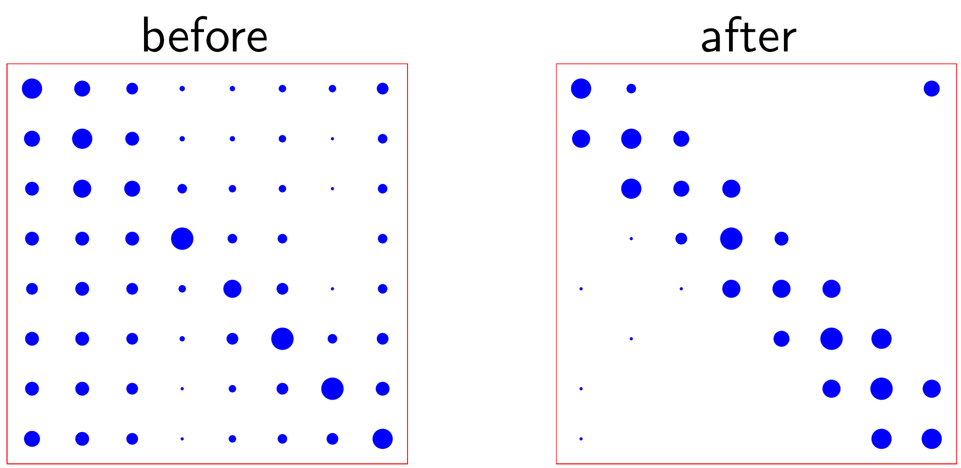
- ZF, LS: $\hat{\boldsymbol{s}}_{\mathsf{zf}} = \text{slice}\Big[\boldsymbol{F}^H \mathcal{H}_{\mathsf{df}}^{-1}\boldsymbol{x}\Big]$
- MMSE: $\hat{\boldsymbol{s}}_{\mathsf{mmse}} = \text{slice}\Big[\boldsymbol{F}^H \mathcal{H}_{\mathsf{df}}^H\big(\mathcal{H}_{\mathsf{df}}\mathcal{H}_{\mathsf{df}}^H + \sigma_w^2\boldsymbol{I}\big)^{-1}\boldsymbol{x}\Big]$
- MLSD: $\hat{\boldsymbol{s}}_{\mathsf{mlsd}} = \arg\max_{\boldsymbol{s}} \|\boldsymbol{x} - \mathcal{H}_{\mathsf{df}}\boldsymbol{F}\boldsymbol{s}\|^2$

With LTV channel: $\rightsquigarrow$ Equalization requires $\geq O(N^3)$ operations

$\rightsquigarrow$ Low-complexity advantage of SCCP is lost!

Linear Pre-Processing to Simplify Detection:

- Use linear pre-processing to simplify detection.
  - Want to make $\mathcal{H}_{\text{df}}$ sparse
  - ICI-response "shortening"
  - Reminiscent of ISI-shortening for single-carrier MLSD
- Time-domain windowing = Doppler-domain convolution!



Max-SINR Window Coefficients:

- Say we allow $2D$ diagonals of controlled ICI.

- Max-SINR window coefficients $\boldsymbol{b}_\star$ are

$$\boldsymbol{b}_\star \;=\; \text{gen-evec}_{\text{max}}\Big(\boldsymbol{A} \odot \boldsymbol{R}^*,\;\; \mathrm{diag}(\boldsymbol{R} + \sigma^2 \boldsymbol{I}) - \boldsymbol{A} \odot \boldsymbol{R}^*\Big)$$

where, for WSSUS Rayleigh fading,

$$[\boldsymbol{A}]_{m,n} \;=\; \frac{\sin\big(\frac{\pi}{N}(2D+1)(n-m)\big)}{N \sin\big(\frac{\pi}{N}(n-m)\big)}$$

$$[\boldsymbol{R}]_{n,m} \;=\; J_0\big(2\pi f_{\mathsf{d}}(n-m)\big) \sum_{l=0}^{N_h-1} \sigma_l^2$$

- Note that $\boldsymbol{b}_\star$ is a function of $\Big\{D, N, f_{\mathsf{d}}, \frac{\sum \sigma_l^2}{\sigma^2}\Big\}$

Windowed-System Model:

- Apply windowing before first receiver DFT:

$$
\begin{aligned}
\breve{\boldsymbol{x}} &= \boldsymbol{F}\,\mathcal{D}(\boldsymbol{b})\boldsymbol{r} \\
&= \underbrace{\boldsymbol{F}\,\mathcal{D}(\boldsymbol{b})\boldsymbol{F}^H}_{\mathcal{C}(\boldsymbol{\beta})}\ \underbrace{\boldsymbol{F}\boldsymbol{r}}_{\boldsymbol{x}} \qquad \text{for } \ \boldsymbol{\beta} = \boldsymbol{F}\boldsymbol{b}/\sqrt{N} \\
&= \underbrace{\mathcal{C}(\boldsymbol{\beta})\breve{\mathcal{H}}_{\mathsf{df}}}_{\text{nearly banded}}\ \underbrace{\boldsymbol{F}\boldsymbol{s}}_{\boldsymbol{t}} + \mathcal{C}(\boldsymbol{\beta})\boldsymbol{w}
\end{aligned}
$$

- *Goal:*

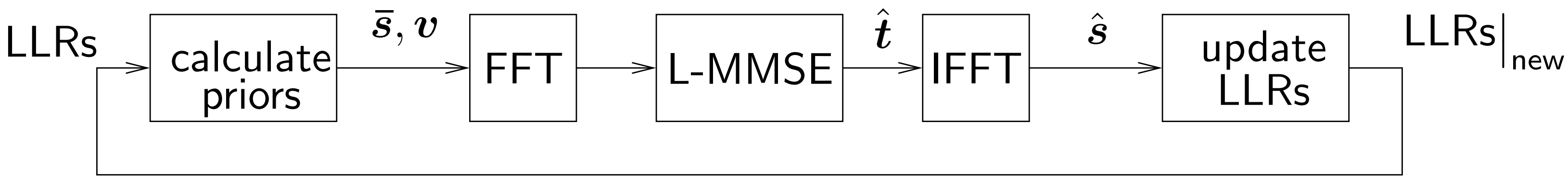
  Estimate finite-alphabet $\{s_0, \ldots, s_{N-1}\}$ given $\breve{\mathcal{H}}_{\mathsf{df}}$, $\boldsymbol{\beta}$, and $\breve{\boldsymbol{x}}$.

- *Approach:*

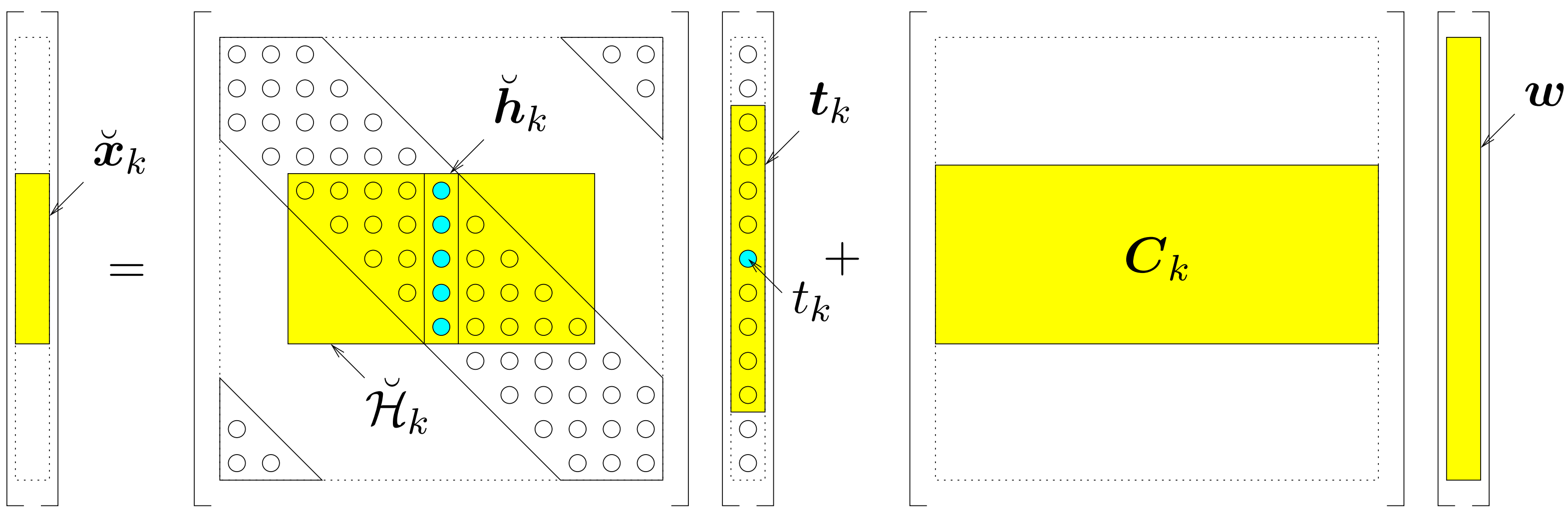
  Leverage sparse $\breve{\mathcal{H}}_{\mathsf{df}}$ to estimate $\boldsymbol{t}$, then relate $\boldsymbol{t} \rightarrow \boldsymbol{s}$.

Iterative MMSE Estimation:

Block Iteration:



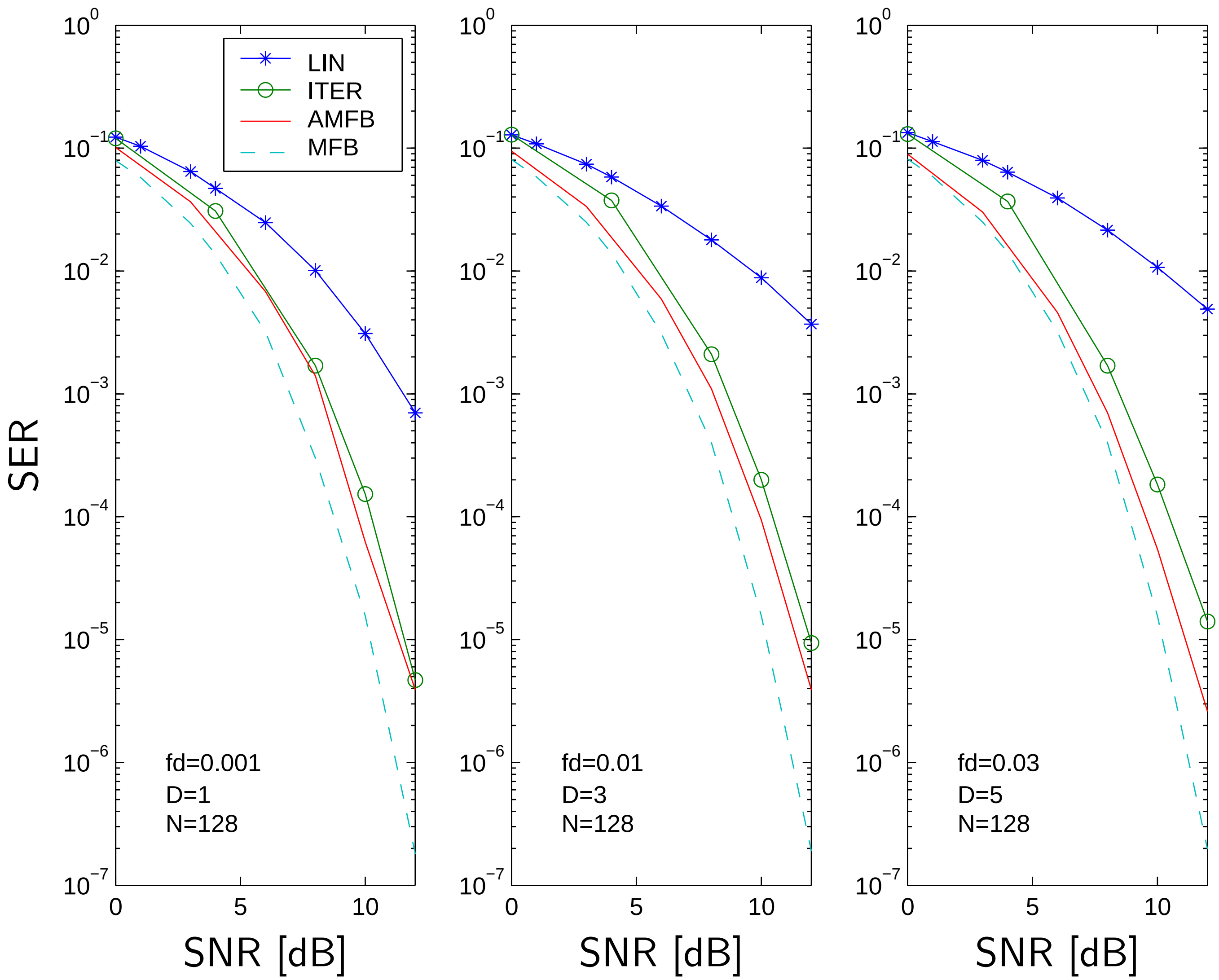
L-MMSE step for each k:



Algorithm requiring $\mathcal{O}(D^2 \log N)$ operations/symbol:

$$
\begin{array}{l}
L^{(0)}(s_k) = 0 \;\forall k \\
\texttt{for } i = 0 \ldots, \\
\quad \texttt{for } k = 0 \ldots N-1, \\
\quad\quad \bar{s}_k^{(i+1)} = \tanh(L^{(i+1)}(s_k)/2) \\
\quad\quad v_k^{(i+1)} = 1 - (\bar{s}_k^{(i+1)})^2 \\
\quad \texttt{end} \\
\quad \bar{\boldsymbol{t}}^{(i)} = \boldsymbol{F}\bar{\boldsymbol{s}}^{(i)} \\
\quad \texttt{for } k = 0 \ldots N-1, \\
\quad\quad \boldsymbol{g}_k^{(i)} = \big(\breve{\mathcal{H}}_k \boldsymbol{F}\,\mathcal{D}(\boldsymbol{v}^{(i)})\boldsymbol{F}^H \breve{\mathcal{H}}_k^H + \sigma^2 \boldsymbol{C}_k \boldsymbol{C}_k^H\big)^{-1} \breve{\mathcal{H}}_k \boldsymbol{F}\,\mathcal{D}(\boldsymbol{v}^{(i)})\boldsymbol{F}^H \boldsymbol{i}_k \\
\quad\quad \hat{t}_k^{(i)} = \bar{t}_k^{(i)} + \boldsymbol{g}_k^{(i)H}(\boldsymbol{x}_k - \breve{\mathcal{H}}_k \bar{\boldsymbol{t}}^{(i)}) \\
\quad \texttt{end} \\
\quad \boldsymbol{Q}^{(i)} = \boldsymbol{F}^H \Big(\sum_{k=0}^{N-1} \breve{\mathcal{H}}_k^H \boldsymbol{g}_k^{(i)} \boldsymbol{i}_k^H\Big)\boldsymbol{F} \\
\quad \boldsymbol{P}^{(i)} = \boldsymbol{F}^H \Big(\sum_{k=0}^{N-1} \boldsymbol{C}_k^H \boldsymbol{g}_k^{(i)} \boldsymbol{i}_k^H\Big)\boldsymbol{F} \\
\quad \hat{\boldsymbol{s}}^{(i)} = \boldsymbol{F}^H \hat{\boldsymbol{t}}^{(i)} \\
\quad \texttt{for } k = 0 \ldots N-1, \\
\quad\quad L^{(i+1)}(s_k) = L^{(i)}(s_k) + 4\,\dfrac{\mathrm{Re}\{Q_{k,k}^{(i)}(\hat{s}_k^{(i)} - \bar{s}_k^{(i)})\} + |Q_{k,k}^{(i)}|^2 \bar{s}_k^{(i)}}{\boldsymbol{q}_k^{(i)H}\,\mathcal{D}(\boldsymbol{v}^{(i)})\boldsymbol{q}_k^{(i)} - |Q_{k,k}^{(i)}|^2 v_k^{(i)} + \sigma^2 \|\boldsymbol{p}_k^{(i)}\|^2} \\
\quad \texttt{end} \\
\texttt{end}
\end{array}
$$

Uncoded-SER versus SNR:



Observations:

- Classical (Joint Linear) MMSE:
  - $\mathcal{O}(N^2)$ operations/symbol.
  - Worst performance.
- Iterative MMSE:
  - $\mathcal{O}(\log N)$ operations/symbol.
  - ~ 2dB from MFB
  - Easily combined with decoding algorithm (i.e., turbo eq).
- Approximate MFB:
  - Uses sparse $\breve{\mathcal{H}}_{\text{df}}$ with perfect interference cancellation.
- MFB:
  - Uses true $\breve{\mathcal{H}}_{\text{df}}$ with perfect interference cancellation.

Summary:

- SCCP reception complicated by time-selectivity.
- Proposed a two-stage SCCP receiver for doubly-selective channels:
  1. SINR-optimal windowing,
  2. Iterative MMSE estimation.
- Like classical SCCP receivers, requires $\mathcal{O}(\log N)$ operations/symbol.
- Uncoded error rate is ~ 2dB from MFB.
- Soft decoding can be easily incorporated.