Noncoherent Communication over the Doubly Selective Channel via Successive Decoding and Channel Re-Estimation

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Summary:

- We propose a scheme based on successive decoding with channel re-estimation (e.g., [1,2]) for noncoherent communication over the doubly (i.e., time- and frequency-) selective channel.
- We lower-bound its achievable rate and characterizes its high-SNR behavior.
- We verify that, for the doubly selective CE-BEM channel, the pre-log factor of the high-SNR achievable-rate expression coincides with that of the high-SNR ergodic capacity expression from [3].
- We propose a pilot/data power allocation strategy which maximizes a lower bound on the achievable rate.

[1] R. Etkin and D. N. C. Tse, "Degrees of freedom in some underspread MIMO fading channels," *IEEE Trans. Info. Theory*, Apr. 2006.

[2] T. Li and O. M. Collins, "A successive decoding strategy for channels with memory," *IEEE Trans. Info. Theory*, Feb. 2007.

[3] A. P. Kannu and P. Schniter, "On the spectral efficiency of noncoherent doubly selective block-fading channels," *Proc. Allerton 2006*.

Transmission Scheme:

- Uses N_s substreams, where k^{th} substream is denoted $\{s_k(i)\}_{i=1}^{N_b}$. Codeword length N_b is assumed to be large.
- First N_p substreams contain known pilots; remaining $N_s - N_p$ substreams contain data.
- Data substreams are independently encoded, using i.i.d Gaussian codebooks whose rates are chosen in accordance with channel statistics (presumed known).
- Pilot substreams also constructed in accordance with channel statistics.
- Total power constrained to E_{tot} Joules per channel use, E_p of which is allocated to pilots, and the remainder of which is evenly spread across data substreams.

Doubly Selective Channel Model:

$$y_k(i) = \sum_{l=0}^{N_h-1} h_{k,l}(i) s_{k-l}(i) + w_k(i) \quad \text{for} \begin{cases} \text{sample } i = 1,\dots,N_b \\ \text{substream } k = 1,\dots,N \\ N = N_s + N_h - 1 \end{cases}$$

Examples:

1. Time-multiplexing of substreams:
   $k =$ time index, $i =$ block index, $\{h_{k,l}(i)\} =$ time-varying ISI coefs.
2. Frequency-multiplexing of substreams:
   $k =$ subcarrier index, $i =$ symbol index, and $\{h_{k,l}(i)\} =$ ICI coefs.

Collecting $\{h_{k,l}(i)\}_{\forall k,\forall l}$ into $\boldsymbol{h}(i)$, we assume

$$\boldsymbol{h}(i) \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}_h) \quad \text{where} \quad \text{rank}(\boldsymbol{\Sigma}_h) = N_m \quad \& \quad \text{tr}(\boldsymbol{\Sigma}_h) = N_s$$
$$\boldsymbol{w}(i) \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \boldsymbol{I})$$
$$\boldsymbol{h}(i) \perp \boldsymbol{w}(i)$$

Note: $\{y_1(i),\dots,y_k(i)\}$ unaffected by $\{s_{k+1}(i),\dots,s_{N_s}(i)\}$.

Reception Scheme:

1. For each $i \in \{1, \ldots, N_b\}$, compute the MMSE channel estimate from the observations affected only by pilots. Using these channel estimates, decode the 1^{st} data substream.

   Note: Reliable decoding becomes possible with proper rate allocation and long enough code-block.

2. For each $i \in \{1, \ldots, N_b\}$, re-compute the MMSE channel estimate from the observations affected only by pilots and the first data substream. Using these channel estimates, decode the 2^{nd} data substream.

$\vdots$ $\vdots$

$N_s - N_p$. For each $i \in \{1, \ldots, N_b\}$, re-compute the MMSE channel estimate from the observations affected by pilots and all-but-the-last data substream. Using these channel estimates, decode the last data substream.

Achievable-Rate Analysis:

Decoding employs interference cancellation and linear combining, yielding the noisy scalar channel

$$z_k(i) \;=\; s_k(i) + n_k(i) \quad \text{for } i = 1, \ldots, N_b.$$

Though residual interference $n_k(i)$ is non-Gaussian, taking the Gaussian distribution as the "worst-case" yields the achievable-rate lower-bound

$$R_k \;\geq\; \mathrm{E}\Big\{\log\Big(1 + \gamma_{\mathsf{max}}^{(k)}(i)\Big)\Big\}, \qquad \text{for SINR } \gamma^{(k)}(i).$$

Thus, for reliable decoding, substream rates should be chosen as above.

The overall achievable-rate obeys

$$R_{\mathsf{tot}} \;\geq\; \frac{1}{N} \sum_{k=N_p+1}^{N_s} \mathrm{E}\Big\{\log\Big(1 + \gamma_{\mathsf{max}}^{(k)}(i)\Big)\Big\} \text{ nats p.c.u.}$$

High-SNR Regime:

With “well constructed” pilots, the channel estimation error will vanish as the noise power vanishes, so that

$$\lim_{\rho\to\infty} \frac{R_{\text{tot}}(\rho)}{\log(\rho)} = \frac{N_s - N_p}{N} \qquad \text{for SNR } \rho := \frac{E_{\text{tot}}}{N\sigma^2}.$$

Such “well constructed” pilots obey $\operatorname{rank}\big(\boldsymbol{S}_{N_p}(i)\boldsymbol{B}\big) = N_m$, which implies that we need $N_p \geq N_m$,

$$\lim_{\rho\to\infty} \frac{R_{\text{tot}}(\rho)}{\log(\rho)} = \frac{N_s - N_m}{N}.$$

Note: When the channel variation obeys a CE-BEM model:

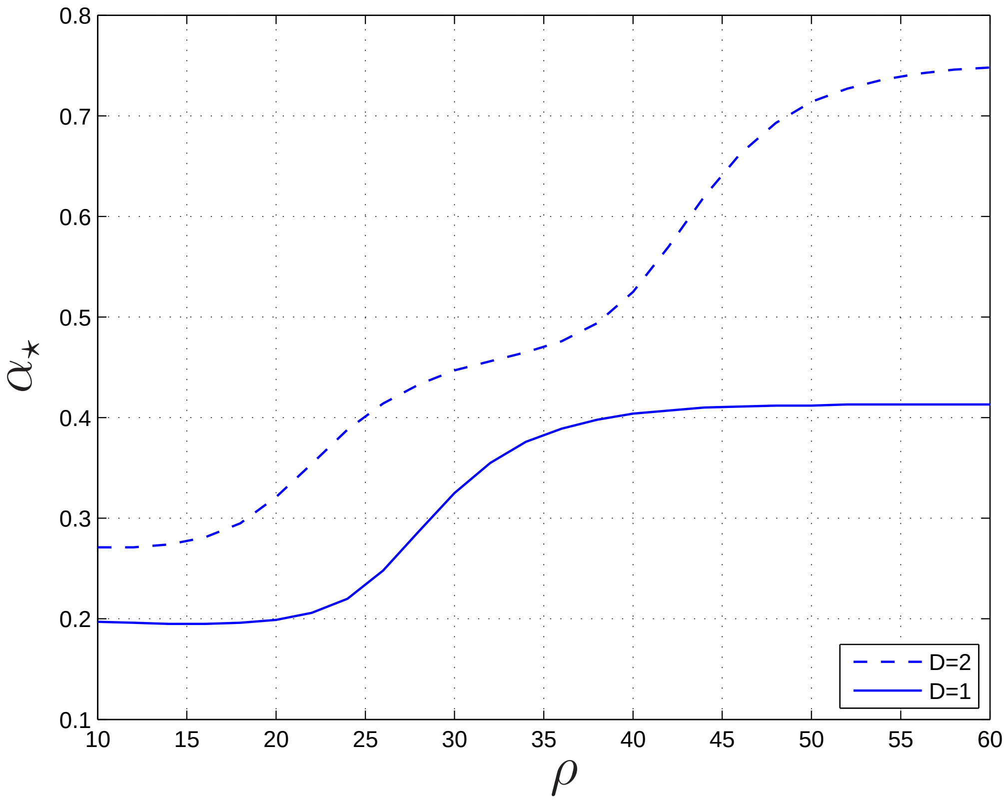
$$\forall\, l, i: \quad h_{k,l}(i) = \frac{1}{\sqrt{N_s}} \sum_{d=-D}^{d=D} \phi_{d,l}(i) e^{j\frac{2\pi}{N}d(k-1)} \quad \text{for } k = 1, \ldots, N$$

where $\{\phi_{d,l}(i)\}$ are i.i.d Gaussian,

the high-SNR noncoherent ergodic capacity expression is known, and its pre-log factor coincides with that above.

Pilot/Data Power Allocation:

Say $E_p = \alpha E_{\text{tot}}$ for $\alpha \in (0,1)$. We describe a scheme to choose α which maximizes an achievable-rate lower-bound.



Above: Doubly selective CE-BEM channel with $N_s = 128$, $N_h = 8$, $D = 1, 2$

Conclusion:

- We proposed a scheme based on successive decoding with channel re-estimation for noncoherent communication over the doubly selective channel.
- We lower-bounded the achievable rate and characterized its behavior at high-SNR.
- We verified that, for the doubly selective CE-BEM channel, the pre-log factor of the high-SNR achievable-rate expression coincides with that of the high-SNR ergodic capacity expression.
- We proposed a pilot/data power allocation strategy which maximizes a lower bound on achievable rate.