On the Achievable Diversity-Multiplexing Tradeoff in Half Duplex Cooperative Channels

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Abstract

In this paper, we propose novel cooperative transmission protocols for coherent flat-fading channels consisting of N (half-duplex and single-antenna) partners and one cell site. In our work, we differentiate between the cooperative relay, broadcast, and multiple-access channels. The proposed protocols are evaluated using Zheng-Tse diversity-multiplexing tradeoff. For the relay channel, we investigate two classes of cooperation schemes; namely, Amplify and Forward (AF) and Decode and Forward (DF). For the first class, we propose a new AF protocol and show it to outperform the space-time coded protocol of Laneman and Wornell without requiring decoding/encoding at the relays. For the class of DF protocols, we develop a dynamic decode and forward (DDF) protocol that achieves the optimal tradeoff for multiplexing gains $0 \leq r \leq 1/N$. Furthermore, with a single relay, the DDF protocol is shown to dominate the class of AF protocols for all multiplexing gains. The superiority of the DDF protocol is shown to be more significant in the cooperative broadcast channel. The situation is reversed in the cooperative multiple-access channel where we propose a new AF protocol that achieves the optimal tradeoff for all multiplexing gains. We also extend some of the proposed protocols to the automatic retransmission request (ARQ) scenario. Particularly, we will show that our single relay ARQ-DDF protocol is optimal for all multiplexing gains. A distinguishing feature of the proposed protocols is that they do not rely on orthogonal subspaces, allowing for a more efficient use of resources.

I. Introduction

Recently, there has been a growing interest in the design and analysis of cooperative transmission protocols for wireless fading channels [1], [2], [3], [4], [5], [6], [7]. In the context of coherent communication, where the channel state information (CSI) is available only at the receiving end, the basic idea is to leverage the antennas available at the other nodes in the network as a source of *virtual* spatial diversity. We use the same setup as considered by Laneman *et al.* in [3]. There, the authors imposed the half-duplex constraint (either transmit or receive, but not both) on the cooperating nodes and proposed several protocols. These protocols were classified as either Amplify and Forward (AF), where the helping node retransmits a scaled version of its soft observation, or Decode and Forward (DF), where the helping node attempts first to decode the information stream and then re-encodes it using (a possibly different) code-book. In all of these protocols, the two partners rely on the use of orthogonal subspaces to repeat each other's signals. Later, Laneman and Wornell extended their DF strategy to the N partners scenario [4]. Other follow-up works have focused on developing practical coding schemes that attempt to exploit the promised information theoretic gains (e.g., [6], [7]).

The use of orthogonal subspaces in cooperative protocols entails a significant price in terms of performance loss for high spectral efficiency scenarios, as observed in [3]. In fact, the authors in [3] pose the following open problem: "*a key area of further research is exploring cooperative diversity protocols in the high spectral efficiency regime.*" This remark motivates our work here. To establish the gain offered by the proposed protocols, we adopt the diversity-multiplexing tradeoff as the measure of performance.

In our work, we consider the cooperative relay, cooperative broadcast (down-link) and cooperative multiple-access (up-link) channels. For the single relay channel, we establish an upper bound on the achievable diversity-multiplexing tradeoff by the class of AF protocols. We then identify a variant within this class that achieves this upper bound. The proposed algorithm is then extended to the general case with $(N-1)$ relays, where it is shown to outperform the space-time coded protocol of Laneman and Worenell without requiring decoding/encoding at the relays. For the class of DF relay protocols, we develop a dynamic decode and forward (DDF) protocol that achieves the optimal tradeoff for multiplexing gains $0 \leq r \leq 1/N$. Furthermore, with a single relay, the DDF protocol is shown to dominate the class of AF protocols for all multiplexing gains. For the cooperative broadcast channel, we present a modified version of the DDF protocol to allow for reliable transmission of the common information and show that the gain offered by the DDF protocol in this scenario is even more significant as compared to the relay channel. For the cooperative multiple-access channel we propose a new AF protocol that achieves the optimal tradeoff for all multiplexing gains.

In [3], Laneman *et al.* also proposed a cooperative scheme (called the incremental relaying protocol), that used limited feedback from the destination (i.e. a single bit per message indicating the success or failure of decoding) to improve the performance. Here, we build a comprehensive framework for Automatic Retransmission reQuest (ARQ) cooperative channels, which encompasses the incremental relaying protocol as a special, and suboptimal, scheme. In particular, we extend the single relay NAF and DDF protocols to incorporate ARQ retransmissions and characterize the significant performance gains they provide. In fact, we will show that our single relay ARQ-DDF protocol is optimal for all multiplexing gains. Due to space limitations, though, a thorough treatment of the ARQ half-duplex cooperative channels is beyond the scope of this paper and the reader is referred to [10] for a complete set of results.

A distinguishing feature of the proposed protocols in this paper is that they do not rely on orthogonal subspaces, allowing for a more efficient use of resources. In fact, using our results one can argue that the inferiority of previously proposed protocols stems from their use of orthogonal subspaces rather than the half-duplex constraint.

II. System Model

We adopt a spatially-white, quasi-static and flat Rayleigh-fading channel model where all the channel gains are assumed to remain constant during one coherence-interval and change independently from one coherence-interval to another. Furthermore, all channel gains are assumed to be unit variance. The additive noise at different nodes are assumed to be zero-mean, mutually-independent, circularly-symmetric and white complex-Gaussian. We also assume that the variances of these noises are proportional to one another such that there is always *fixed* offsets between the different channels signal to noise ratios (SNRs).

All nodes have the same power constraint, have a single antenna and operate synchronously. We assume a coherent communication model implying that the channel gain of a link is only known by the receiving end. Furthermore, no feedback is allowed. Following in the footsteps of [3], we impose the half-duplex constraint on the cooperating partners such that, at any point in time, a node can either transmit or receive, but not both. This is primarily due to the huge power level difference between the incoming and outgoing signals.

Throughout the paper, we assume the use of random Gaussian code-books where a codeword spans the entire coherence-interval of the channel. Furthermore, we assume an asymptotically large codeword size.

We define the SNR of a link, ρ, as

$$\rho \triangleq \frac{E}{\sigma_v^2}, \tag{1}$$

where E denotes the average energy available for transmission of a symbol across the link and σ_v^2 denotes the variance of the noise observed at the receiving end of the link. Now, consider a family of codes $\{C_\rho\}$ indexed by operating SNR ρ, such that the code C_ρ has a rate of $R(\rho)$ bits per channel use (BPCU) and a maximum likelihood (ML) error probability $P_e(\rho)$. For this family, the *multiplexing gain* "r" and the *diversity gain* "d" are defined as

$$r \triangleq \lim_{\rho\to\infty} \frac{R(\rho)}{\log\rho}, \qquad d \triangleq -\lim_{\rho\to\infty} \frac{\log(P_e(\rho))}{\log\rho}. \tag{2}$$

The problem of characterizing the optimal tradeoff between the reliability and throughput of a point-to-point multi-input-multi-output (MIMO) communication system over a coherent quasi-static flat Rayleigh-fading channel was posed and solved by Zheng and Tse in [8]. For a MIMO communication system with M transmit and N receive antennas, they showed that, for any given $r \le \min(M,N)$, the optimal diversity gain $d^*(r)$, is given by the piecewise linear function joining the (r,d) pairs $(k,(M-k)(N-k))$ for $k = 0, ..., \min(M,N)$, provided that the code-length l satisfies $l \ge M + N - 1$.

III. The Half-Duplex Relay Channel

In this section, we consider the cooperative relay scenario where $N-1$ relays help a single source in transmitting its message to the destination. Here, we consider two important classes of relay protocols. The first is the class of Amplify and Forward (AF) relay protocols, where the relays are only allowed to perform linear processing (e.g., addition and multiplication) on their received signals. The second is the class of Decode and Forward (DF) protocols, where the relays are allowed to decode and re-encode the message using (a possibly different) code-book. We emphasize that, a priori, it is not clear whether the DF protocols offer better performance than AF protocols or not (e.g., [3]).

A. Amplify and Forward Protocols

We first consider the single relay scenario (i.e., $N = 2$). For this scenario, we have the following theorem.

Theorem 1: The optimal diversity gain for the cooperative relay scenario with a single AF relay is upper-bounded by

$$d^*(r) \le (1-r) + (1-2r)^+, \tag{3}$$

where $(x)^+$ means $\max\{x, 0\}$.

Proof: Please refer to [9] for a detailed proof. ■

Having Theorem 1 at hand, it now suffices to construct an AF protocol, that achieves this upper-bound in order to establish its optimality. In the proposed AF protocol, which we call the Nonorthogonal Amplify and Forward (NAF) protocol, the source transmits on every symbol interval in the cooperation frame, where a cooperation frame is defined as two consecutive symbol intervals. The relay, on the other hand, transmits only once per

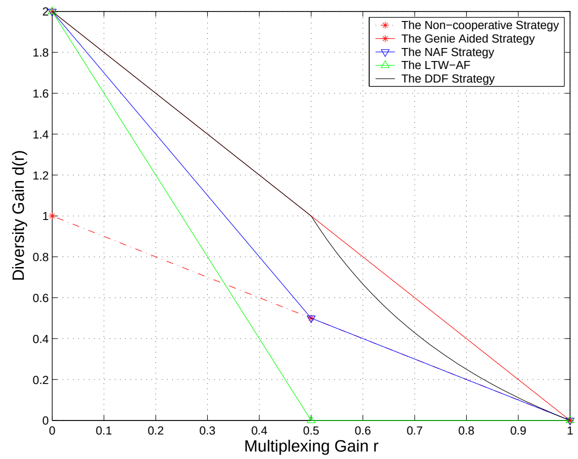

Fig. 1. Diversity-multiplexing tradeoff for the NAF and DDF protocols with one relay.

cooperation frame. It simply repeats the (noisy) signal it observed during the previous symbol interval. It is important to realize that this design is dictated by the half-duplex constraint which implies that the relay, at best, can repeat one symbol in a cooperation frame. We denote the relay's repetition gain as b. Also for cooperation frame k, we denote the information symbols by $\{x_{j,k}\}_{j=1}^{2}$. The signals received by the destination during cooperation frame k are:

$$y_{1,k} = g_1 x_{1,k} + v_{1,k}$$
$$y_{2,k} = g_1 x_{2,k} + g_2 b(h x_{1,k} + w_{1,k}) + v_{2,k}$$

where h, g_1 and g_2 denote the gains of the source-relay, source-destination, and relay-destination channels, respectively. $w_{1,k}$ and $\{v_{j,k}\}_{j=1}^{2}$ denote the noises observed by the relay and the destination during the k^{th} cooperation frame. It is evident that, in this protocol, the relay is not simultaneously transmitting and receiving at any symbol interval. The repetition gain b is chosen to optimize a metric of performance (such as outage probability at the target rate and SNR), subject to relay's average transmission energy constraint. Let us also define a codeword as l (l is taken to be even) consecutive symbol intervals, during which the channel gains are assumed to remain constant. Having definitions (1) and (2), we are now ready to state our result for the NAF protocol.

Theorem 2: The diversity-multiplexing tradeoff achieved by the NAF protocol with a single relay is characterized by:

$$d^*(r) = (1 - r) + (1 - 2r)^+. \tag{4}$$

This curve is shown in Fig. 1.

Proof: Please refer to [9] for a detailed proof. ■

Generalization of the NAF protocol to the case of arbitrary number of relays is rather straight forward. For this purpose, we first define every $N-1$ (i.e., the number of relays) consecutive cooperation frames as a super-frame. Within each super-frame, the relays take turns is relaying source's message in exactly the same way as they did in the case of a single relay. Thus, the destination received signals during a super-frame will be:

$$\begin{aligned} y_{1,1} =& g_1 x_{1,1} + v_{1,1} \\ y_{2,1} =& g_1 x_{2,1} + g_2 b_2 (h_2 x_{1,1} + w_{1,1}) + v_{2,1} \\ &\vdots \\ y_{1,N-1} =& g_1 x_{1,N-1} + v_{1,N-1} \\ y_{2,N-1} =& g_1 x_{2,N-1} + \cdots \\ & g_N b_N (h_N x_{1,N-1} + w_{1,N-1}) + v_{2,N-1} \end{aligned}$$

where the gains of the channels connecting the source to the destination through relay $i, i = 1, ..., N-1$ are denoted by h_{i+1} and g_{i+1}, respectively. Also, $w_{1,i}, i = 1, ...N-1$ represents the noise observed by relay i during cooperation frame i. $y_{j,k}, x_{j,k}$ and $v_{j,k}$ represent the same quantities as before.

Theorem 3: The diversity-multiplexing tradeoff achieved by the NAF protocol with $N-1$ relays is characterized by:

$$d^*(r) = (1-r) + (N-1)(1-2r)^+$$

Proof: Please refer to [9] for a detailed proof. ■

B. Decode and Forward Protocols

In this section we describe the proposed scheme for the DF relay channel, which we call the dynamic decode and forward (DDF) protocol. We first describe and analyze the protocol for the case of a single relay. The generalization to the case of arbitrary number of relays will then follow. Let's define a codeword as l consecutive symbol intervals, during which we assume the channel gains remain unchanged. In the DDF protocol, the source transmits on every symbol interval in the codeword, while the relay listens to the source until it collects sufficient energy to decode the message error-free. Then, it encodes the message using *a different* code-book and transmits it during the rest of the codeword. We denote the signals transmitted by the source and relay as $x_k, k = 1, ..., l$ and $\tilde{x}_k, k = l' + 1, ..., l$, respectively, where l' is the number of symbol intervals the relay has to wait before starting transmission. Using these notations, the destination received signals during a codeword can be written as:

$$\begin{aligned} y_1 =& g_1 x_1 + v_1 \\ &\vdots \\ y_{l'} =& g_1 x_{l'} + v_{l'} \\ y_{l'+1} =& g_1 x_{l'+1} + g_2 \tilde{x}_{l'+1} + v_{l'+1} \\ &\vdots \\ y_l =& g_1 x_l + g_2 \tilde{x}_l + v_l \end{aligned}$$

where g_1 and g_2 respectively denote the gains of the channels connecting the source and relay to the destination and v_k the destination noise of variance σ_v^2. From the description given, it is apparent that relay's waiting time (i.e., l') depends on the instantaneous source-relay channel realization (This is the reason we picked the name *dynamic* decode and forward). In fact, for the relay to be able to decode source's message error-free, l' should be chosen as:

$$l' \triangleq \min\{l, \lceil \frac{lR}{\log_2(1+|h|^2 c\rho)} \rceil\}$$

where R is the the source data rate (in BPCU), h the source-relay channel gain and finally c the ratio of σ_v^2 to σ_w^2 (i.e., relay noise variance). Note that this choice of l', together with an asymptotically large l, guarantees the existence of a code such that relay's error probability is arbitrarily small when $l' < l$. Clearly, $l' = l$ means that the relay does not contribute to the transmission of the message.

Theorem 4: The diversity-multiplexing tradeoff achieved by the DDF protocol with a single relay is characterized by:

$$d^*(r) = \begin{cases} 2(1-r) & \text{if } \frac{1}{2} \geq r \geq 0 \\ (1-r)/r & \text{if } 1 \geq r \geq \frac{1}{2} \end{cases} \quad (5)$$

This curve is shown in Fig. 1.

Proof: Please refer to [9] for a detailed proof. ■

Next, we describe the generalization of the DDF protocol to the case of multiple relays. In this case, the source and relays cooperate in nearly the same manner as in the single relay case. Specifically, the source transmits during the whole codeword while each relay listens until it collects sufficient energy for error-free decoding. Once a relay decodes the message, it uses an independent code-book to re-encode the message, which it then transmits for the rest of the codeword. Note that, since the source-relay channel gains may differ, the relays may require different wait times for decoding. This complicates the protocol, since a given relay's ability to decode the message depends on precise knowledge of the times at which every other relay begins its transmission. Due to space limitation, we do not address this complication here and instead refer the reader to [9].

Theorem 5: The diversity-multiplexing tradeoff achieved by the DDF protocol with $N-1$ relays is characterized by:

$$d(r) = \begin{cases} N(1-r), & \frac{1}{N} \geq r \geq 0, \\ 1 + \frac{(N-1)(1-2r)}{1-r}, & \frac{1}{2} \geq r \geq \frac{1}{N}, \\ \frac{1-r}{r}, & 1 \geq r \geq \frac{1}{2}. \end{cases} \quad (6)$$

Proof: Please refer to [9] for a detailed proof. ■

IV. THE HALF-DUPLEX COOPERATIVE BROADCAST CHANNEL

We now consider the cooperative broadcast (CB) scenario, where a single source broadcasts to N destinations. We assume that $R_c = r_c \log(\rho)$ BPCU is used to encode a common portion of the message (intended for all destinations), while $R_j = r_j \log(\rho)$ BPCU is used to encode the portion of the message intended for the j^{th} destination ($j \in \{1, \ldots, N\}$). The total rate is then $R = R_c + \sum_{j=1}^{N} R_j$ and the multiplexing gain tuple is given by $\mathbf{r} = (r_c, r_1, ..., r_N)$. We define the overall diversity gain d based on the performance of the worst case receiver as $d = \min_j \{d_j\}$, where we require all the receivers to decode the common information. Now, as a first step, one can see that if $r_c = 0$ then the techniques developed for the relay channel can be *exported* to this setting through a proportional time sharing strategy. With this assumption, all the properties of the NAF and DDF protocols, established for the relay channel, carry over to this scenario. The problem becomes slightly more challenging when $r_c > 0$. In fact, it is easy to see that, for a fixed total rate, the highest probability of error corresponds to the case where all destinations are required to decode all the streams. This translates to the following condition (that applies to any cooperation scheme)

$$d(r_c, r_1, r_2, ..., r_N) \geq d(r_c + r_1 + ... + r_N, 0, 0, .., 0).$$

So, we will focus the following discussion on this worst case scenario, i.e., $\mathbf{r} = (r_c, 0, 0, ..., 0), 0 \leq r_c \leq 1$. The first observation is that, in this scenario, the only AF strategy that achieves the full rate extreme point $(r = 1, d = 0)$ is the non-cooperative protocol. Any other AF strategy will require some of the nodes to re-transmit, and therefore not to *listen* during parts of the codeword, which prevents it from achieving full rate. This drawback, however, is avoided in the CB-DDF protocol, where every node can act as a DDF relay for the other nodes, based on its instantaneous channel gain. In fact, as a node starts helping only after it has successfully decoded the message, the CB-DDF protocol achieves the full rate extreme point.

Theorem 6: The diversity-multiplexing tradeoff achieved by the CB-DDF protocol with N destinations is given by:

$$d(r_c) = \begin{cases} N(1-r_c), & \frac{1}{N} \geq r_c \geq 0, \\ 1 + \frac{(N-1)(1-2r_c)}{1-r_c}, & \frac{1}{2} \geq r_c \geq \frac{1}{N}, \\ \frac{1-r_c}{r_c}, & 1 \geq r_c \geq \frac{1}{2}. \end{cases} \tag{7}$$

Proof: Please refer to [9] for a detailed proof. ∎

It is interesting to note that this is exactly the same tradeoff obtained in the relay channel. This implies that requiring all nodes to decode the same message does not entail a price in terms of the achievable tradeoff.

V. The Half-Duplex Cooperative Multiple-Access Channel

In this section, we consider the cooperative multiple-access (CMA) channel, where N sources transmit their independent messages to a common destination. We assume the channel to be symmetric, meaning that all of the sources transmit information at the same rate. In the proposed scheme, which we call the CMA-NAF protocol, the N sources transmit once per cooperation frame, where a cooperation frame is defined by N consecutive symbol intervals. Each source, therefore, gets the chance of transmission once every other N symbol intervals and, when active, transmits a linear combination of the symbol it intends to send and the (noisy) signal it receives from one other source assigned to it. As an example, consider the case of 3 sources, where sources 1, 2 and 3 respectively help sources 3, 1 and 2. Now, if we denote the broadcast and repetition gains of source j by a_j and b_j, respectively, and its intended symbol for transmission by $x_{j,k}$ (k denotes the cooperation frame index), then at startup we will have:

$$\begin{aligned} t_{1,1} &= a_1 x_{1,1} & & \\ t_{2,1} &= a_2 x_{2,1} + b_2 r_{2,1} & r_{2,1} &= h_{12} t_{1,1} + w_{2,1} \\ t_{3,1} &= a_3 x_{3,1} + b_3 r_{3,1} & r_{3,1} &= h_{23} t_{2,1} + w_{3,1} \\ t_{1,2} &= a_1 x_{1,2} + b_1 r_{1,1} & r_{1,1} &= h_{31} t_{3,1} + w_{1,1} \end{aligned}$$

where $t_{j,k}$ and $r_{j,k}$ respectively denote source j's transmitted and received signals during frame k, h_{ji} the gain of the channel connecting sources i and j and $w_{j,k}$ the noise observed by source j during frame k. All of the source noises are assumed to have the same variance σ_w^2. The corresponding signals received by the destination are:

$$\begin{aligned} y_{1,1} &= g_1 t_{1,1} + v_{1,1} \\ y_{2,1} &= g_2 t_{2,1} + v_{2,1} \\ y_{3,1} &= g_3 t_{3,1} + v_{3,1} \\ y_{1,2} &= g_1 t_{1,2} + v_{1,2} \end{aligned}$$

where g_j is the gain of the channel connecting source j to the destination and $v_{j,k}$ the destination noise of variance σ_v^2. Note that, in this protocol, sources do not transmit and receive simultaneously. The broadcast and repetition gains $\{a_j, b_j\}$ are chosen to optimize a metric of performance (such as the outage probability at the target rate and SNR), subject to the sources' average transmission energy constraint. We define L consecutive cooperation frames as a super-frame. It is assumed that, during a super-frame, the sources keep their partners unchanged. However, they switch partners across super-frames according to a scheduling rule. We choose this rule to be the following circular algorithm. In super-frame i, sources with indices $(1, \ldots, N)$ are assigned helpers with indices given by the j^{th} left circular shift of $(1, \ldots, N)$, where $j = \langle i-1 \rangle_{N-1} + 1$. For example, when $N = 3$, in the first super-frame (i.e., $i = 1$), sources $(1, 2, 3)$ are assigned helpers $(2, 3, 1)$, while in the second super-frame (i.e., $i = 2$) they are assigned helpers $(3, 1, 2)$. Since this scheduling algorithm generates $N - 1$ distinct helper configurations, the length of the super-frames is chosen such that a coherence-interval consists of $N-1$ consecutive super-frames. To achieve maximal diversity for a given multiplexing gain, it is required that all codewords span the entire coherence-interval. For this reason, we choose codes of length l given by

$$l = (N-1)L \tag{8}$$

Note that using (2), a multiplexing gain r_j and a diversity gain d_j (associated with the individual ML decoder) can be defined for each of the sources. However, as the sources are transmitting information at the same rate (i.e., symmetric CMA channel), all of these multiplexing gains are equal (i.e., $r = r_j$, all j). Furthermore, as in this protocol only one source is transmitting at any symbol interval, destination's multiplexing gain is also equal to r. That is, the destination receives information at a rate of $R = r \log \rho$. We also define the diversity gain d as $d = \min_j \{d_j\}$

Theorem 7: The optimal diversity-multiplexing tradeoff for the symmetric cooperative multiple-access channel with N partners is characterized by:

$$d^*(r) = N(1-r). \tag{9}$$

Furthermore, this optimal tradeoff curve is achieved by the proposed cooperative protocol.

Proof: Please refer to [9] for a detailed proof. ■

VI. Cooperation Through ARQ

In this section, we only describe two exemplar protocols for the single-antenna, single-relay ARQ channel with one round of retransmission. These are only intended to highlight the basic ideas involved and the significant improvements that can be gained through the use of the ARQ technique in half-duplex cooperative channels. A thorough treatment of the ARQ half-duplex cooperative channels is beyond the scope of this paper and the reader is referred to [10] for a complete set of results. To extend the single relay NAF and DDF protocols to incorporate ARQ retransmissions, we first define l consecutive symbol intervals as a round. In the proposed protocols, which are called the ARQ-NAF and ARQ-DDF protocols, the source starts sending a message by taking $b = Rl$ information bits and encoding them using a code-book of code-length $2l$ (In this setup, we only allow for one round of retransmission and thus, a codeword spans two rounds). During the first round, the source transmits the first l symbols of the codeword and the relay cooperates with it as in the non-ARQ case. At the end of this round, the destination makes an attempt to decode the message. If successful, a positive acknowledgement (ACK) signal is sent back, which causes the source and the relay to abandon transmission of the rest of the codeword and go on to the next message. In case of failure, however, a negative acknowledgement (NACK) signal is sent, in which case the source and the relay start the second round of transmission, during which, the rest of the codeword is sent. As mentioned earlier, we only allow for one round of retransmission. This means that at the end of the second round, the source and the relay go on to the next message, even if the destination is unable to decode the current message error-free. Note that the one-bit signal ACK/NACK is the only feedback allowed in our system model. Furthermore, it is

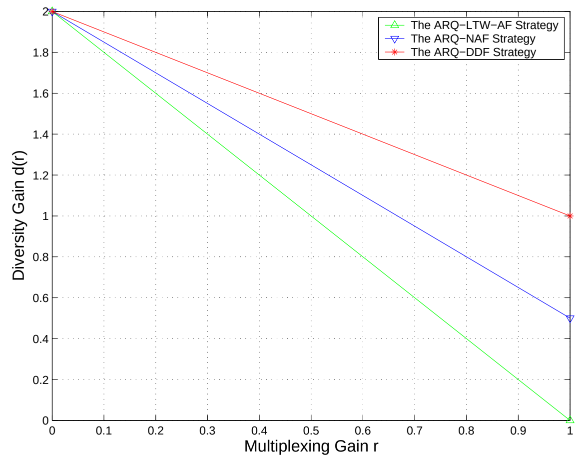

Fig. 2. Diversity-multiplexing tradeoff for the single relay ARQ-NAF, ARQ-DDF (both with one round of retransmission) and LTW incremental relaying protocols.

sent back to the source and the relay through error-free and zero-delay channels. It is also important to note that for ARQ protocols, the source *effective* data rate R_e is *not* constant. In fact, R is the *nominal*, and not the effective data rate. However, for sufficiently large SNR's, R_e merges to R and becomes essentially constant [11]. In analyzing these protocols, we assume that during the transmission of a message, the channel gains remain constant (the long-term static scenario in [11]). Also, we assume a negligible probability of undetected error (This assumption is justified by the use of powerful CRC outer codes).

Theorem 8: The diversity-multiplexing tradeoff achieved by the single relay ARQ-NAF protocol with one round of retransmission is characterized by

$$d^*(r) = 2 - 3r/2 \quad \text{for} \quad 1 \geq r \geq 0. \tag{10}$$

This curve is shown in Fig. 2.

Proof: Please refer to [10] for a detailed proof. ■

Theorem 9: The optimal diversity-multiplexing tradeoff for the single relay ARQ channel with one round of retransmission is characterized by

$$d^*(r) = 2(1 - r/2) \quad \text{for} \quad 1 \geq r \geq 0. \tag{11}$$

Furthermore, this optimal tradeoff curve is achieved by the proposed single relay ARQ-DDF protocol. This curve is shown in Fig. 2.

Proof: Please refer to [10] for a detailed proof. ■

Comparing tradeoff curves of the ARQ-NAF and ARQ-DDF protocols (one round of retransmission) with those of the NAF and DDF protocols reveals an interesting phenomenon. The tradeoff curves of the ARQ based protocols are stretched versions (with a factor of two) of those of the non-ARQ based protocols [11]. The reason for this phenomenon is that messages which are decoded erroneously always have two rounds of transmission and thus a data rate of $R/2$. Fig. 2 also shows the significant gains achieved by our ARQ based cooperative protocols compared to Laneman *et al.*'s incremental relaying protocol, specially at high multiplexing gains (e.g. at a multiplexing gain of $r = 1$, the ARQ-DDF protocol gives a diversity gain of $d = 1$, while the incremental relaying protocol provides no diversity gain $d = 0$).

VII. Conclusion

In this paper, we considered the design of cooperative protocols for a system consisting of a number of (half-duplex and single-antenna) partners and a cell site. In particular, we considered the cooperative relay, broadcast and multiple-access channels and proposed efficient (optimal in some cases) protocols. We also studied the single relay ARQ channel and proposed two protocols for it. The superiority of the proposed protocols' diversity-multiplexing curves, compared to those of the protocols that rely on the use of orthogonal subspaces, such as the LTW protocol, is a result of allowing the source to transmit continuously.

Finally, it is instructive to contrast the tradeoff curves of the proposed NAF relay, DDF relay and CMA-NAF protocols. From Fig. 1, one can see that for multiplexing gains greater that 0.5, the diversity gain achieved by the proposed NAF relay protocol is identical to that of the non-cooperative protocol. This is due to the fact that the *cooperative* link provided by the relay can not support multiplexing rates greater than 0.5, as a result of the half duplex constraint. Hence, for these range of multiplexing gains, there is only one link from the source to the destination, and thus, the tradeoff curve is identical to that of a point-to-point system with one transmit and one receive antenna. Fig. 1 also reveals that while the DDF relay protocol achieves the optimal diversity gain for multiplexing gains less than 0.5, it too suffers from the half-duplex constraint for multiplexing gains greater than 0.5. In the case of CMA-NAF protocol, however, this problem was avoided by exploiting the availability of an independent information streams per source. This implies that, under the half duplex constraint, cooperative multiple-access schemes inspired by the relay channel formulation ignore a potential source for performance improvement (i.e., the distributed nature of the information across the different nodes).

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