Linear and Decision Feedback Equalization Structures for Asynchronous DS-CDMA under ICI

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Abstract: This paper formulates an asynchronous short-code DS-CDMA system operating under inter-chip interference (ICI) as a generalization of the fractionally-spaced equalization problem. The model permits straightforward derivation of finite-length MMSE chip-rate linear equalization structures as well as multiuser versions of the DFE. The performance of various equalization structures is compared and observations made.

1 Introduction

First, we develop a model for an asynchronous DS-CDMA system that operates under the presence of inter-chip interference. In uplink transmission (i.e., mobile to base station) the chip-rate signals asynchronously generated by individual mobiles each see the effect of a different chip-rate FIR channel. In the downlink channel (i.e., base station to mobile), the user signals are generated synchronously and propagate through the same ICI channel on their way to a given mobile.

Chip-rate linear filters can be employed at the receivers to estimate the transmitted symbol (or "bit") sequence of each user. In addition, symbol estimation may be assisted by a decision feedback equalizer (DFE) operating at the symbol rate. In this report we derive expressions for the equalizer coefficient vectors that minimize the mean square symbol estimation error. Since, optimally, each user's DFE relies on the symbol decisions of all other users, we refer to this decision feedback structure as the *multiuser DFE*. The performance of various implementations of the multiuser DFE are compared in a simulation study that focuses on the uplink and downlink scenarios separately.

2 System Model

First, we define the following CDMA system quantities:

- spreading gain: L,
- number of active users: K,
- symbol sequence of ℓ^{th} user: $\{s_n^\ell\}$,
- code vector of ℓ^{th} user: $\mathbf{c}^\ell \in \mathbb{C}^L$,
- additive noise/interference process: $\{w_k\}$

Using these quantities, we will derive two system models: one pertaining to the CDMA uplink, in which the channel (and delay) between each user and the base station may be different, and the other pertaining to the downlink, where a given user sees the (synchronized) broadcast of all the other users' data codes through the same channel.

2.1 Uplink Model

Given the set of ICI channel vectors $\{\mathbf{h}^\ell\}$, where the ℓ^{th} user's channel[1] takes the form $\mathbf{h}^\ell := \left(h_0^\ell, \ldots, h_{N_h-1}^\ell\right)^t \in \mathbb{C}^{N_h}$, we can formulate the chip-rate output contribution from the ℓ^{th} user at time n as:

$$\mathbf{x}^\ell(n) \;=\; \mathbf{H}^\ell \mathbf{C}^\ell \mathbf{s}^\ell(n), \tag{1}$$

with channel convolution matrix $\mathbf{H}^\ell$, code matrix $\mathbf{C}^\ell$, and symbol vector

$$\mathbf{s}^\ell(n) \;:=\; \left(s_n^\ell, s_{n-1}^\ell, \ldots, s_{n-N_s+1}^\ell\right)^t. \tag{2}$$

Anticipating that the first stage of our receiver will use a length-N_f linear equalizer, symbol estimates will be derived from a length-N_f data window. Hence, the convolution matrix $\mathbf{H}^\ell$ will have dimensions $N_f \times N_f + N_h - 1$ and will take on the following structure:

$$\mathbf{H}^\ell \;=\; \begin{pmatrix} h_0^\ell & h_1^\ell & \ldots & h_{N_h-1}^\ell & & & & \\ & h_0^\ell & h_1^\ell & \ldots & h_{N_h-1}^\ell & & & \\ & & & \ddots & & \ddots & & \\ & & & & h_0^\ell & h_1^\ell & \ldots & h_{N_h-1}^\ell \end{pmatrix}. \tag{3}$$

The size of $\mathbf{H}^\ell$ and the Sylvester structure of $\mathbf{C}^\ell$:

$$\mathbf{C}^\ell \;=\; \begin{pmatrix} \mathbf{c}^\ell & & & \\ & \mathbf{c}^\ell & & \\ & & \ddots & \\ & & & \mathbf{c}^\ell \end{pmatrix} \tag{4}$$

mandate that $\mathbf{C}^\ell$ have dimensions $N_f + N_h - 1 \times \lceil \frac{N_f + N_h - 1}{L} \rceil$. This implies that

$$N_s \;:=\; \left\lceil \frac{N_f + N_h - 1}{L} \right\rceil \tag{5}$$

symbols from the ℓ^{th} user will contribute to the length-N_f observation through $\mathbf{x}^\ell(n)$.

Note that there is no upper bound imposed on N_h, allowing for chip-rate channels longer than one symbol duration. This suggests one way of incorporating asynchronicity (i.e., user-specific time delay) into the system model. Say that $\{\tau^\ell\}$ specifies the users delays in chip intervals, so that $0 \le \tau^\ell < L$ for each ℓ. Then τ^ℓ can be modeled by prepending $\lceil \tau^\ell \rceil$ zeros onto the channel vector $\mathbf{h}^\ell$, and incorporating[2] the fractional delay $\tau^\ell - \lceil \tau^\ell \rceil$ into the definition of the sampled

[1] For simplicity, and without loss of generality, we assume that all users have FIR channel responses of equal length N_h. This may require zero-padding the shorter channels.

[2] When the chip pulse shape is rectangular, a fractional chip delay is of no consequence. When the chip pulse shape is bandlimited, the DFT may be used to circularly resample an adequately-sampled version of the code waveform.

code vector $\mathbf{c}^\ell$. Since we prefer to treat all channels as having length N_h, we set N_h as the length of the longest delayed channel, and append zeros onto those channels with shorter length.

Since N_f+N_h-1 might not be an integer multiple of L, the rightmost column of $\mathbf{C}^\ell$ might not contain a full code vector $\mathbf{c}^\ell$ as suggested by the matrix construction in (4). A straightforward dimensional analysis shows that the last code vector will be shortened to length $L - N_t$, where

$$0 \leq N_t := LN_s - (N_f + N_h - 1) \leq L - 1. \tag{6}$$

The received vector $\mathbf{r}(n)$ has contributions from all K users as well as from a (chip-rate) additive Gaussian noise process $\{w_k\}$. The noise process is used to model background noise and other interference, as discussed below.

There are a number of options in modelling inter-cell interference (i.e., the interference caused by users outside the cell of interest). One method involves choosing K large enough to represent all internal and external users, resulting in $K \gg L$. Another method chooses K to represent the number of internal users, in which case $K \leq L$, and assumes that the intra-cell interference is Gaussian (and perhaps white) [1]. This first method has the advantage of accurately modelling the structure of the intra-cell interference, but has the disadvantage of resulting in potentially large matrix quantities. The second method results in more tractable system models, but ignores potentially important characteristics of the interference structure[3]. One might then consider a compromise, where a small number of strong out-of-cell interferers are included among the K users, and the combined interference from the (generally) large number of weaker out-of-cell interferers is approximated as Gaussian. In this case we expect that models of realistically-loaded systems will have $K > L$. Whatever model is chosen, it is important to recall that the out-of-cell interference in a well loaded system accounts for approximately one third of the total interference power [2]. When using the Gaussian model, we expect choosing an SNR = 6dB for fully loaded systems and SNR = 9dB for half-loaded systems (w.r.t. energy-per-symbol) operating under perfect power control [1].

Collecting the noise samples into the vector $\mathbf{w}(n)$, we can formulate the length-N_f received vector as follows:

$$\begin{aligned}
\mathbf{r}(n) &= \mathbf{w}(n) + \sum_{\ell=1}^{K} \mathbf{x}^\ell(n) \\
&= \mathbf{w}(n) + \sum_{\ell=1}^{K} \mathbf{H}^\ell \mathbf{C}^\ell \mathbf{s}^\ell(n) \\
&= \mathbf{w}(n) + \begin{pmatrix} \mathbf{H}^1\mathbf{C}^1 & \mathbf{H}^2\mathbf{C}^2 & \cdots & \mathbf{H}^K\mathbf{C}^K \end{pmatrix} \begin{pmatrix} \mathbf{s}^1(n) \\ \mathbf{s}^2(n) \\ \vdots \\ \mathbf{s}^K(n) \end{pmatrix}.
\end{aligned} \tag{7}$$

For the development of the DFE in Section 4, it will be convenient to re-order the source vector in (7) with respect to the timing index. This implies that the channel-code matrix in (7) must undergo a corresponding re-ordering of columns. Hence, with the time-ordered source vector $\mathbf{s}(n)$:

$$\mathbf{s}(n) = \left(s_n^1, \ldots, s_n^K \;\; s_{n-1}^1, \ldots s_{n-1}^K \;\; \cdots \;\; s_{n-N_s-1}^1, \ldots, s_{n-N_s+1}^K\right)^t \in \mathbb{C}^{KN_s}, \tag{8}$$

[3] For example, the Gaussian approximation does not take into account the potentially constant-modulus nature of external user interference.

the received signal model takes the form

$$\mathbf{r}(n) \;=\; \mathbf{w}(n) + \mathcal{H}\mathbf{s}(n), \tag{9}$$

where it follows from (3), (4), and (7) that the multiuser channel matrix $\mathcal{H}$ is structured as follows:

$$\mathcal{H} \;=\; \begin{pmatrix} \uparrow & & \uparrow & & & & & & \\ \mathbf{b}_0^1 & \cdots & \mathbf{b}_0^K & & & & & & \\ \downarrow & & \downarrow & \uparrow & & \uparrow & & & \\ & & & \mathbf{b}_1^1 & \cdots & \mathbf{b}_1^K & & & \\ & & & \downarrow & & \downarrow & \ddots & \uparrow & & \uparrow \\ & & & & & & & \mathbf{b}_{N_s-1}^1 & \cdots & \mathbf{b}_{N_s-1}^K \\ & & & & & & & \downarrow & & \downarrow \end{pmatrix}. \tag{10}$$

Each consecutive block $\{\mathbf{b}_n^1 \cdots \mathbf{b}_n^K\}$ in (10) starts L rows down from the previous block. The length $L+N_h-1$ vectors $\mathbf{b}_n^\ell$ represent (typically) truncated versions of the chip-rate convolution $\mathbf{b}^\ell := \mathbf{h}^\ell \star \mathbf{c}^\ell$. The truncation is unavoidable when the length of $\mathbf{b}^\ell$ is greater than L. Because different blocks $\{\mathbf{b}_n^1 \cdots \mathbf{b}_n^K\}$ (especially those at the edges) may be truncated differently, the channel matrix $\mathcal{H}$ does not, in general, have a Sylvester structure. However, under orthogonal codes, L users of equal power, and the absence of both ICI and delay, $\mathcal{H}$ becomes orthogonal (as noted in [3]).

Q: *Does $\mathcal{H}$ have some sort of "generalized" Sylvester structure?*

Q: *Though asynchronicity or ICI seem to prevent SPE, can we derive physically-meaningful single-user identifiability conditions on $\mathcal{H}$?*

At this point we note that the formulation of the DS-CDMA received signal in (9) is a generalization of the single-user T/L-spaced equalization problem. Specifically, (9) reduces to the equalization problem when $K = 1$ and $\mathbf{c}^1$ is a column of the identity matrix $\mathbf{I}_L$. The column choice corresponds to the equalizer's choice of "decimation phase." In the equalization context, $\mathcal{H}$ is known to have a Sylvester structure.

2.2 Downlink Model

We propose a downlink model that is a simplification of the uplink model. Specifically, there is only one channel (and delay) between the K users' signals transmitted at the base station and any given mobile. This means that we can decompose the multiuser channel matrix into the product

$$\mathcal{H} \;=\; \mathbf{H}^\ell \begin{pmatrix} \mathbf{c}^1 \cdots \mathbf{c}^K & & & \\ & \mathbf{c}^1 \cdots \mathbf{c}^K & & \\ & & \ddots & \\ & & & \mathbf{c}^1 \cdots \mathbf{c}^K \end{pmatrix}. \tag{11}$$

where $\mathbf{H}^\ell$ is the ICI channel convolution matrix (as in (3)) from the base station to the ℓ^{th} mobile, and where the code vectors in the rightmost block $\{\mathbf{c}^1 \cdots \mathbf{c}^K\}$ of (11) might be truncated to length $L - N_t$ (as in (4)).

2.3 Extension to Fractionally-Sampled or Antenna Array Sytems

The received signal model (9) assumes a receiver having one sensor which samples at the chip rate. Certain systems, however, sample at a multiple of the chip rate and/or use multiple sensors. Though more expensive to build, these systems are known for numerous performance advantages [4]. Below we show that our CDMA model is easily extendible to these systems.

First, let us take the case of P sensors. If the response mapping the ℓ^{th} user's signal to the p^{th} sensor is given by the length N_h vector $\mathbf{h}^{p,\ell}$, then we can construct the channel matrix $\mathbf{H}^\ell$ as follows:

$$\mathbf{H}^\ell = \begin{pmatrix} \mathbf{H}^{1,\ell} \\ \vdots \\ \mathbf{H}^{P,\ell} \end{pmatrix}$$

where $\mathbf{H}^{p,\ell}$ has the Toeplitz form of (3) and $\mathbf{H}^\ell$ has a Sylvester form. This implies that the received vector $\mathbf{r}(n)$ will be a stacked version of the chip-rate responses from the P sensors, and that $\mathbf{w}(n)$ will be a stacked version of the noise samples at the P sensors.

The case of temporal oversampling may be treated in exactly the same way if we define $\mathbf{h}^{p,\ell}$ as the p^{th} chip-rate subchannel of a system that samples at P times the chip rate. (See [9] for an overview of the multichannel view of fractionally sampled systems.) Again, $\mathbf{H}^\ell$, $\mathbf{r}(n)$, and $\mathbf{w}(n)$ can be constructed by stacking contributions from the P subchannels.

3 Linear Equalizers

A linear chip-rate equalizer $\mathbf{f}$ can be used to estimate a particular user's symbol sequence from chip-rate samples of the received sequence. With observations $\mathbf{r}(n)$ from (9), the symbol-rate soft estimates y_n take the form

$$\begin{aligned} y_n &= \mathbf{f}^t\mathbf{r}(n) && (12) \\ &= \mathbf{f}^t\mathcal{H}\mathbf{s}(n) + \mathbf{f}^t\mathbf{w}(n). && (13) \end{aligned}$$

In order to recover the ℓ^{th} user's symbols at some fixed delay ν $(0 \geq \nu \geq N_s - 1)$, we desire $y_n \approx s_{n-\nu}$, or, in other words, a multiuser system response that obeys

$$\mathbf{q}^{\ell,\nu} := \mathcal{H}^t\mathbf{f}^{\ell,\nu} \approx \mathbf{e}_\delta\big|_{\delta=\nu K+\ell-1}, \qquad (14)$$

where $\mathbf{e}_\delta$ denotes a vector with a one in the δ^{th} position and zeros elsewhere. Note that δ acts to select the particular combination of system delay *and* user.

In its most general form, as above, the linear equalizer does not incorporate knowledge of the desired user's code (or timing) into its structure. In contrast, we might consider applying the desired user's spreading sequence to the input signal before chip-rate equalization. In order for such schemes to work properly, however, the de-spreading sequence must be properly synchronized. Since we are unwilling to assume knowledge of the desired user's timing, we focus on the general linear equalizer of (12).

If we collect a set of equalizers, each designed to estimate a different user's symbols (e.g., at a common delay ν as in Figure 1), into the matrix $\mathbf{F}^\nu := \big(\mathbf{f}^{1,\nu}, \mathbf{f}^{2,\nu}, \ldots, \mathbf{f}^{L,\nu}\big)$, then $\mathbf{F}^\nu$ is analogous to a *multiuser detector*. Particular constructions of $\{\mathbf{f}^\ell\}$ bear similarity to well-known multiuser detectors, such as the decorrelating detector [5, 6] and the MMSE detector [7, 8]. Another well-known design, where $\mathbf{f}^\ell = \mathbf{c}^\ell$ for each ℓ (under perfect synchronization), is referred to as the *matched filter* (MF) receiver. Even in the absence of ICI, asynchronism, and noise, the MF receiver is suboptimal when the code vectors are not orthogonal.

Below we derive two well-known equalization structures: the zero-forcing (ZF) equalizer and the minimum mean-squared error (MMSE) equalizer. We restrict our scope to finite-length equalizers. The expressions that result are, in general, not directly useful for implementation since their calculation involves knowledge of joint statistics (between transmitted symbols and received chips) that are not usually present at the receiver. They *are* useful, though, as performance benchmarks and in predicting the asymptotic performance of commonly-used adaptive algorithms (such as LMS and CMA).

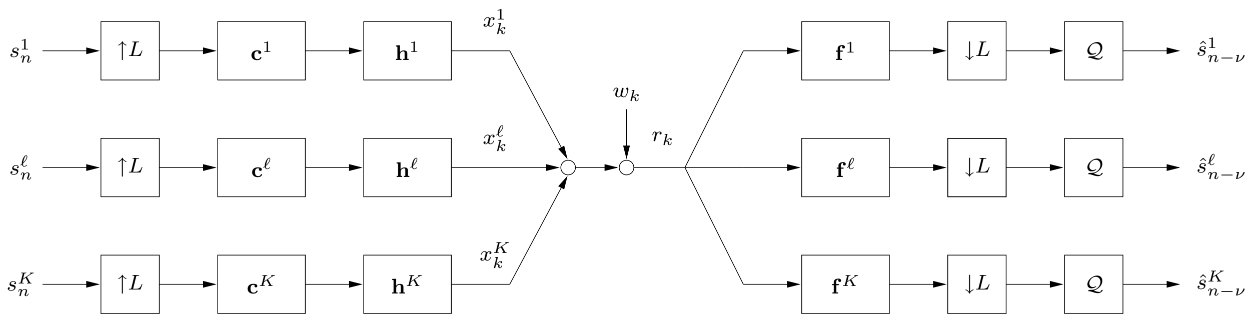

Figure 1: Block diagram of MU-LE system showing channel and receiver.

3.1 Zero-forcing Equalization

Zero-forcing (ZF) equalizers for user ℓ and system delay ν, when they exist, are defined by

$$\left\{\mathbf{f} \ : \ \mathcal{H}^t \mathbf{f} = \mathbf{e}_{\nu K+\ell-1}\right\}. \tag{15}$$

The necessary and sufficient condition for the existence of ZF equalizers for *all* users $1 \leq \ell \leq K$ and for *all* delays $0 \leq \nu \leq N_s - 1$ is that $\mathcal{H}$ must be full column rank. This will be referred to as *strong perfect equalization* (SPE). In the case of a chip-rate single-sensor receiver, the following dimensional requirement is necessary for the possibility of full column rank $\mathcal{H}$:

$$N_f \geq K \left\lceil \frac{N_f + N_h - 1}{L} \right\rceil. \tag{16}$$

The requirement (16) has an interesting implication when $K = L$. We can see this by writing

$$\left\lceil \frac{N_f + N_h - 1}{L} \right\rceil \quad = \quad \frac{N_f + N_h - 1}{L} + \frac{N_t}{L} \tag{17}$$

which in the case $K = L$ leads to the dimensional requirement

$$N_f \geq N_f + N_h - 1 + N_t \quad \Leftrightarrow \quad N_h \leq 1 - N_t \quad \Leftrightarrow \quad \begin{cases} N_h = 1 \\ N_t = 0 \end{cases}. \tag{18}$$

Thus, *when $K = L$, strong perfect equalization of non-trivial or asynchronous channels is impossible with a finite-length equalizer.* This is analogous to the inability of a single-user T-spaced FIR equalizer to perfectly equalize a non-trivial finite length channel. In fact, the requirement (18) *implies* the T-spaced equalization result when $L = K = 1$.

When $K < L$, the following condition is sufficient to guarantee (16):

$$N_f \geq \left\lceil \frac{K}{L-K}(N_h - 2 + L) \right\rceil. \tag{19}$$

Note, however, that SPE is not necessary for the perfect demodulation of all users. In fact, for ZF demodulation of the ℓ^{th} user, (15) requires only that there exists some delay $\nu_\ell : 0 \leq \nu_\ell \leq N_s - 1$ such that

$$\mathbf{e}_{\nu_\ell K+\ell-1} \quad \in \quad \mathrm{col}(\mathcal{H}^t). \tag{20}$$

Even when feasible, however, ZF equalization may result in severe amplification of the additive noise $\mathbf{w}(n)$. In addition, calculation of the ZF equalizer requires perfect knowledge of the spreading codes, ICI channels, and delays of all users. For these reasons, the ZF equalizer is not considered practical. The minimum mean-square error (MMSE) equalizer, presented in the next section, addresses both of these issues.

3.2 MMSE Equalization

The minimum mean-square equalizer for the estimation of the ℓ^{th} user's symbols at system delay ν minimizes the quadratic cost

$$J_{\text{mse}}^{\ell,\nu} \;:=\; \mathrm{E}\big\{|e_n^{\ell,\nu}|^2\big\}, \quad \text{where} \quad e_n^{\ell,\nu} \;:=\; y_n - s_{n-\nu}^{\ell}. \tag{21}$$

Assuming that the noise and symbol sequences are each zero-mean and uncorrelated, as well as uncorrelated with each other, (21) can be written:

$$J_{\text{mse}}^{\ell,\nu}(\mathbf{f}) \;=\; \mathbf{f}^H \underbrace{(\sigma_s^2\mathcal{H}^*\mathcal{H}^t + \sigma_w^2\mathbf{I})}_{\mathbf{R}_{r,r}^*}\mathbf{f} - 2\mathbf{f}^t \underbrace{\mathcal{H}^*\mathbf{e}_{\nu K+\ell-1}}_{\boldsymbol{\rho}_{\ell,\nu}^*} + 1, \tag{22}$$

where $\sigma_s^2 = \mathrm{E}\big\{|s_n|^2\big\}$, $\sigma_w^2 = \mathrm{E}\big\{|w_k|^2\big\}$, and $(\cdot)^*$ denotes complex conjugation. For notational simplicity, we will assume that $\sigma_s^2 = 1$. The matrix $\mathbf{R}_{r,r} := \mathrm{E}\{\mathbf{r}(n)\mathbf{r}^H(n)\}$ is the received signal autocorrelation matrix, and the vector $\boldsymbol{\rho}_{\ell,\nu} := \mathrm{E}\{\mathbf{r}(n)(s_{n-\nu}^{\ell})^*\}$ is the cross-correlation between the received signal and the ℓ^{th} user's ν-delayed source sequence. From (22) it follows that the MMSE equalizer for the $\{\ell,\nu\}$ user/delay combination is

$$\begin{aligned}
\mathbf{f}_m^{\ell,\nu} \;&=\; (\mathbf{R}_{r,r}^{\dagger}\boldsymbol{\rho}_{\ell,\nu})^* && (23)\\
&=\; \big((\mathcal{H}\mathcal{H}^H + \sigma_w^2\mathbf{I})^{\dagger}\mathcal{H}\mathbf{e}_{\nu K+\ell-1}\big)^* && (24)
\end{aligned}$$

where $(\cdot)^{\dagger}$ denotes the pseudo-inverse. Then, from (22) and (24), it can be shown that the MMSE at $\{\ell,\nu\}$ is

$$\begin{aligned}
\mathcal{E}_m^{\ell,\nu} \;&=\; 1 - \boldsymbol{\rho}_{\ell,\nu}^H\mathbf{R}_{r,r}^{\dagger}\boldsymbol{\rho}_{\ell,\nu} && (25)\\
&=\; 1 - \mathbf{e}_{\nu K+\ell-1}^t\mathcal{H}^H(\mathcal{H}\mathcal{H}^H + \sigma_w^2\mathbf{I})^{\dagger}\mathcal{H}\mathbf{e}_{\nu K+\ell-1}. && (26)
\end{aligned}$$

We observe that when $\mathcal{H}$ is full column rank, $\mathcal{E}_m^{\ell,\nu} \to 0$ as $\sigma_w^2 \to 0$ for all $\{\ell,\nu\}$. In this limiting case, the MMSE equalizer takes the form of the minimum-norm ZF equalizer.

The MMSE equalizer is known to be the asymptotic mean solution of the trained LMS adaptive equalization algorithm. In addition, it often well-characterizes the mean solution of CMA when used for adaptive blind equalization [9].

4 Multiuser DFE

In single-user systems, a decision-feedback equalizer (DFE) uses previous symbol estimates for interference cancellation of ISI corrupted data transmission. When the symbol estimates are correct, they have the advantage of not being corrupted by additive channel noise, giving the DFE a performance advantage over linear structures. When the symbol estimates are incorrect, there is the danger of error propagation leading to catastrophic performance.

Historically, [7] presented the first MMSE-based linear and DFE structures for asynchronous DS-CDMA operating in the absence of ICI. The decorrelating MU-DFE was proposed in [10] for synchronous DS-CDMA operating in the absence of ICI. The asynchronous case was analyzed in [11], resulting in a decorrelating MU-DFE with IIR forward filters. Both linear and DFE versions of ZF and MMSE detectors were developed in [12] for block data transmission of DS-CDMA under ICI. The author does not know of existing literature describing the design of the finite-length MMSE MU-DFE for DS-CDMA under ICI.

Figure 2 illustrates the MU-DFE presented in this section. In single-user case, causality mandates that the DFE operate only on past decisions. In multi-user case, a given user faces not only the unavailability of his own current/future decisions, but the unavailability of other users' current/future decisions, as well as the unavailability of out-of-cell users' past/current/future decisions. This can be mitigated in part by staggering the decision instants in the implementation, so that the current decisions of in-cell stronger users become available to weaker users in time for their decisions [10] and by allowing soft handoff schemes whereby a receiver may demodulate strong out-of-cell interferers. Proper design of the stronger users' forward equalizers takes their lack of noiseless current symbol estimates into account.

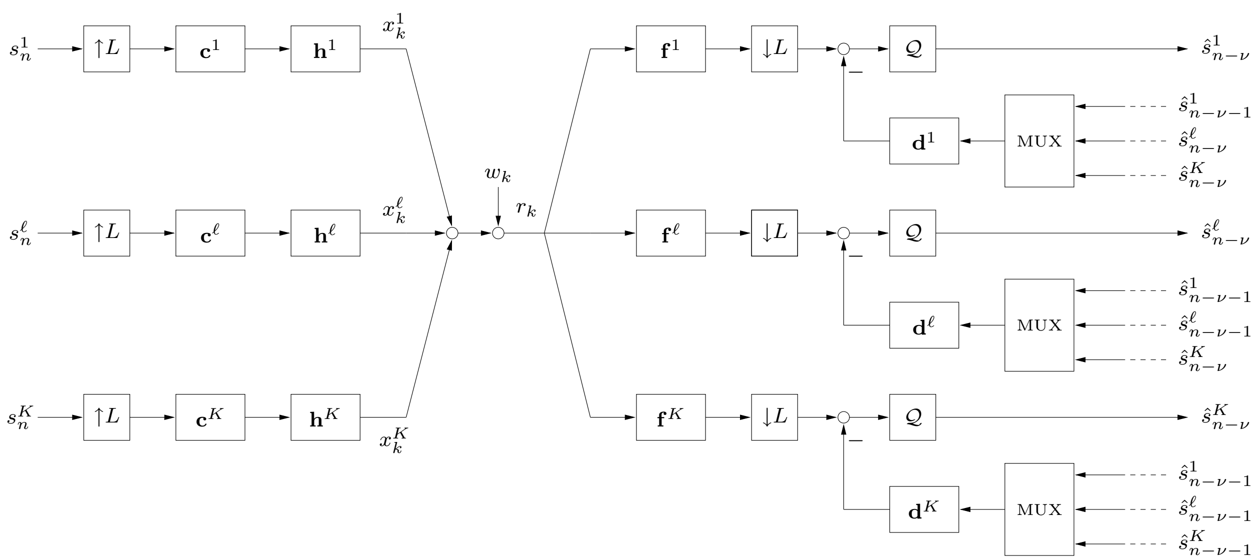

Figure 2: Block diagram of MU-DFE system showing channel and receiver models. Note staggering of DFE inputs.

4.1 MMSE MU-DFE

Our derivation of the MMSE MU-DFE assumes the absence of symbol errors. Under this assumption, it is well known that the soft decision errors become linear in the filter coefficients.

As in (14), let

$$\mathbf{q}^{\ell,\nu} \;:=\; \mathcal{H}^t \mathbf{f}^{\ell,\nu} \;\in\; \mathbb{C}^{KN_s} \tag{27}$$

represent the response vector characterizing the cascade of the multiuser channel with the forward equalizer $\mathbf{f}^{\ell,\nu}$. Using $\mathbf{d}^{\ell,\nu}$ to represent the corresponding feedback coefficient vector, the soft errors can be formulated as follows:

$$e_n^{\ell,\nu} \;=\; \mathbf{s}^t(n)\big(\mathbf{e}_{\nu K+\ell-1} - \bar{\mathcal{H}}^t \mathbf{f}^{\ell,\nu} + \bar{\mathbf{d}}^{\ell,\nu}\big) + \mathbf{w}^t(n)\mathbf{f}^{\ell,\nu}, \tag{28}$$

where $\bar{\mathcal{H}}$ and $\bar{\mathbf{d}}^{\ell,\nu}$ are zero-padded versions of $\mathcal{H}$ and $\mathbf{d}^{\ell,\nu}$ extended long enough to capture any additional symbols used by the DFE. Their dimensions will be given shortly.

As described earlier, we propose a MU-DFE that staggers the decision implementations. Assuming the presence of symbol estimates for all K users, the technique is defined as follows: the users are ordered[4] $\{1,\dots,K\}$ and decisions on the set of "current" symbols $\{s_{n-\nu}^{\ell} : 1 \le \ell \le K\}$ are made in the order $\ell = \{K, K-1, \dots, 1\}$ within the current symbol interval. Using N_a to denote the maximum symbol duration used in feedback estimation of $\{s_{n-\nu}^{\ell}\}$, it follows that the length of the ℓ^{th} feedback filter $\mathbf{d}^{\ell,\nu}$ is

$$N_d \;=\; KN_a - \ell. \tag{29}$$

(See Figure 3.) This implies that the quantities in (28) have the construction

$$\bar{\mathcal{H}} \;=\; \Big(\; \underbrace{\mathcal{H}}_{KN_s} \quad \underbrace{\mathbf{0}}_{\max\{0,K(N_a+\nu-N_s)\}} \;\Big), \tag{30}$$

$$\bar{\mathbf{d}}^{\ell,\nu} \;=\; \Big(\; \underbrace{\mathbf{0}}_{K\nu+\ell} \quad \underbrace{(\mathbf{d}^{\ell,\nu})^t}_{N_d} \quad \underbrace{\mathbf{0}}_{\max\{0,K(N_s-N_a-\nu)\}} \;\Big)^t. \tag{31}$$

This staggered MU-DFE is formulated with the source vector ordering of (8) in mind: for *any* $\{\ell,\nu\}$, decision feedback estimation of $s_{n-\nu}^{\ell}$ is accomplished using hard estimates of the N_d elements to the right of $s_{n-\nu}^{\ell}$ in $\mathbf{s}(n)$. For example, with $K=3$, $\nu=1$, and $\ell=2$,

$$\mathbf{s}(n) \;=\; \big(s_n^1,\, s_n^2,\, s_n^3,\, s_{n-1}^1,\, \underbrace{s_{n-1}^2}_{s_{n-\nu}^{\ell}},\, \underbrace{s_{n-1}^3,\, s_{n-2}^1,\, s_{n-2}^2,\, s_{n-2}^3, \dots,\, s_{n-N_a}^3}_{\text{quantized versions used in estimate of } s_{n-\nu}^{\ell}}\big)^t.$$

It should now be evident that the MU-DFE design problem has been posed in a form compatible with single-user DFE design. Hence, we can apply standard single-user DFE techniques in the design and analysis of the MU-DFE.

It has been shown that the MMSE design of the forward filter can be decoupled from the design of the feedback filter [13] so long as the dimensions of the latter are known. The mechanics are as follows: Since the feedback filter is able to exactly cancel N_d coefficients of the forward system response $\bar{\mathcal{H}}^t \mathbf{f}^{\ell,\nu}$, the forward filter is designed to minimize the component of the system response which the $\mathbf{d}^{\ell,\nu}$ is unable to cancel (e.g., the precursor) in combination with minimizing the noise power. Using the standard finite-length DFE design procedure [14], the MMSE feedback coefficients are given by

$$\mathbf{d}_m^{\ell,\nu} \;=\; (\mathbf{V}^{\ell,\nu})^t \mathcal{H}^t \mathbf{f}_m^{\ell,\nu} \tag{32}$$

[4]As suggested in [10], it makes sense to order the users in increasing power, so that the most noise-free information is available for demodulation of the weakest users.

where we define

$$\mathbf{V}^{\ell,\nu} \;:=\; \begin{pmatrix} \mathbf{0} \\ \mathbf{I} \\ \mathbf{0} \end{pmatrix} \begin{matrix} \updownarrow K\nu+\ell \\ \updownarrow N_d = KN_a-\ell \\ \updownarrow \max\{0, K(N_s-N_a-\nu)\} \end{matrix}, \tag{33}$$

and the MMSE forward filter coefficients are given by

$$\mathbf{f}_m^{\ell,\nu} \;=\; \big((\mathcal{H}\mathbf{P}^{\ell,\nu}\mathcal{H}^H + \sigma_w^2\mathbf{I})^{\dagger}\mathcal{H}\mathbf{e}_{\nu K+\ell-1}\big)^* \tag{34}$$

where we define

$$\mathbf{P}^{\ell,\nu} \;:=\; \mathbf{I} - \mathbf{V}^{\ell,\nu}(\mathbf{V}^{\ell,\nu})^t \tag{35}$$

$$= \begin{pmatrix} \mathbf{I}_{K\nu+\ell} & & \\ & \mathbf{0}_{N_d} & \\ & & \mathbf{I}_{\max\{0,K(N_s-N_a-\nu)\}} \end{pmatrix}. \tag{36}$$

In the absence of decision errors, the MMSE resulting from the use of (32) and (34) is

$$\mathcal{E}_m^{\ell,\nu} \;=\; 1 - \mathbf{e}_{\nu K+\ell-1}^t\mathcal{H}^H(\mathcal{H}\mathbf{P}^{\ell,\nu}\mathcal{H}^H + \sigma_w^2\mathbf{I})^{\dagger}\mathcal{H}\mathbf{e}_{\nu K+\ell-1}. \tag{37}$$

Note that the design of the forward filters incorporates (where necessary) the unavailability of noise-free estimates of other users' current decisions. For synchronous CDMA in the absence of noise and ICI (but with possibly non-orthogonal codes), the MMSE design is analogous to the decorrelating MUD of [10], whereby the forward equalizers $\{\mathbf{f}^\ell\}$ ensure decorrelation between the ℓ^{th} user and the set of users $\{k : k > \ell\}$.

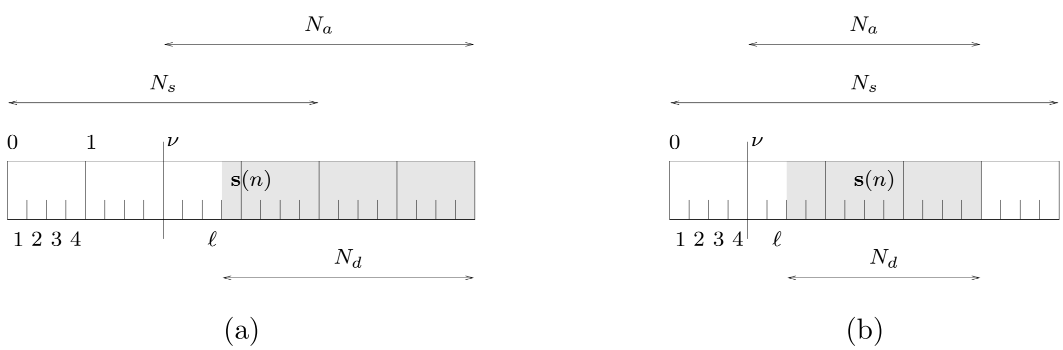

Figure 3: Two examples of MU-DFE problem setup, showing symbol-rate quantities above blocks and chip-rate quantities below blocks: (a) $K = 4$, $N_s = 4$, $N_a = 4$, $\nu = 2$, and $\ell = 3$, (b) $K = 4$, $N_s = 5$, $N_a = 3$, $\nu = 1$, and $\ell = 2$. Shaded areas show portions of multiuser source vector whose estimations are used in decision feedback of $s_{n-\nu}^\ell$.

We have not discussed optimization of $\mathcal{E}_m^{\ell,\nu}$ with respect to ν. As formulated above, all users are estimated at the same delay ν. This is in fact suboptimal, because users are not guaranteed to achieve the lowest MMSE at the same delay. The DFE design becomes quite tedious, though, when a different ν is allowed for each user ℓ. Still, the average performance loss involved in uniform delay selection should be investigated, at least through simulation.

4.2 MMSE Design of Suboptimal DFE Structures

The MU-DFE of Section 4.1 requires decisions on all K users, which are generally not available at the mobile (i.e., in downlink transmission), and may not even be available at the base station (i.e., in uplink transmission), especially when K is used to model out-of-cell interferers. In addition, it was suggested that the users be (adaptively) ordered according to their power levels, which may pose more receiver complexity than is desired. Thus we consider the design of suboptimal DFE structures which do not impose the constraints mentioned above. As we shall see, the MMSE design of various suboptimal structures can be formulated with minor modifications to the technique in Section 4.1.

4.2.1 Group Ordering

The power-ordering suggested in Section 4.1 can be relaxed to a group classification scheme [11], whereby users are collected into two or more power-ordered groups and decisions within the strongest groups are made earliest. Though the relationship between user power and decision order *within* a group is lost, it still pays to stagger the decisions and use all available previously-made decisions on the current symbols (from users within the group as well as from users belonging to stronger groups). Figures 5 and 6 suggest that this scheme performs nearly as well as full user power-ordering.

A relaxation of the group ordering scheme is the case of only one group, i.e., feedback of all available staggered decisions without an attempt at user power ordering. Figures 5 and 6 suggest that, though this scheme is suboptimal to those that perform some degree of power ordering, it performs much better than any scheme which does not incorporate staggered decisions.

4.2.2 Strictly Causal Feedback

The DFE structures previously described employ a staggered decision implementation, allowing the use of previously-made noise-free estimates of other users' *current* symbols (as well as past symbols). We might also consider an "un-staggered" scheme, whereby only *past* decisions are available to any given user's DFE. We will refer to this scheme as the *strictly causal* MU-DFE. The MMSE design of the strictly causal structure can be obtained through a redefinition of the $\mathbf{V}^{\ell,\nu}$ in (33) to

$$\mathbf{V}^{\ell,\nu} \quad := \quad \begin{pmatrix} \mathbf{0} \\ \mathbf{J}^{\ell} \\ \mathbf{0} \end{pmatrix} \begin{matrix} \updownarrow K\nu+\ell \\ \updownarrow KN_a-\ell \\ \updownarrow \max\{0, K(N_s-N_a-\nu)\} \end{matrix} , \qquad (38)$$

where $\mathbf{J}^{\ell}$ is the matrix

$$\mathbf{J}^{\ell} \quad := \quad \begin{pmatrix} \mathbf{0} \\ \mathbf{I} \end{pmatrix} \begin{matrix} \updownarrow K-\ell \\ \updownarrow N_d \end{matrix} . \qquad (39)$$

As before, N_d refers to the number of taps in the feedback filter, implying that $N_d = K(N_a - 1)$. As in (35), $\mathbf{P}^{\ell,\nu} = \mathbf{I} - \mathbf{V}^{\ell,\nu}(\mathbf{V}^{\ell,\nu})^t$, though the form is no longer the same as that suggested by (36). With the redefinitions above, the MSE optimal $\mathbf{d}^{\ell,\nu}$ and $\mathbf{f}^{\ell,\nu}$ are given by (32) and (34), respectively, and the corresponding MMSE is given by $\mathcal{E}_m^{\ell,\nu}$ in (37). See Figure 4(a) for an illustrative example of the strictly-causal DFE setup.

Figures 5 and 6 suggest that the performance of the strictly-causal DFE is significantly worse than the staggered-decision structures (with or without power-ordering) described in previous sections.

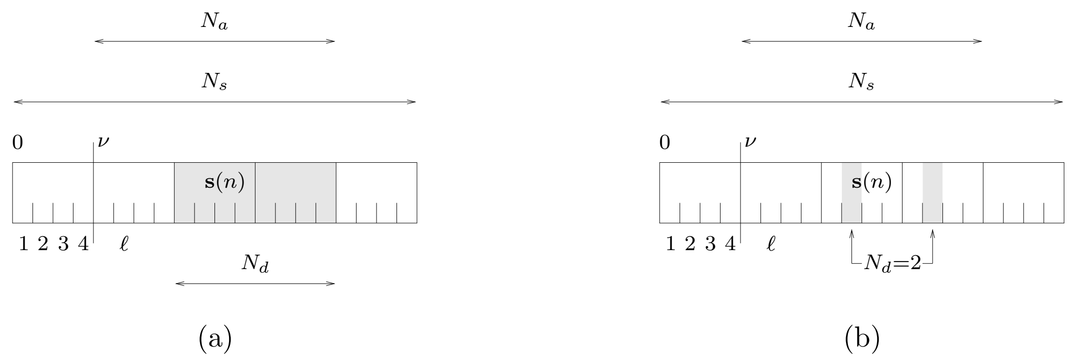

Figure 4: Examples of suboptimal DFE structures: (a) strictly-causal DFE, and (b) single-user DFE. Symbol-rate quantities shown above blocks and chip-rate quantities below blocks: $K = 4$, $N_s = 5$, $N_a = 2$, $\nu = 1$, and $\ell = 2$. Shaded areas show portions of multiuser source vector whose estimations are used in decision feedback of $s^{\ell}_{n-\nu}$.

4.2.3 Single-User Feedback

The downlink scenario presents a situation in which implementation constraints (e.g., cost and power) limit demodulation to only a single user. In such a situation, decision estimates from other users are, most likely, unavailable in the demodulation of the given user. The MMSE design of the suboptimal DFE structure that uses only a given user's past decisions follows directly from the use of the $\mathbf{V}^{\ell,\nu}$ given in (38), but with $\mathbf{J}^{\ell}$ constructed as follows:

$$\mathbf{J}^{\ell} \quad := \quad \begin{pmatrix} \underbrace{0\ldots0}_{K-1}\,1 & & & & \\ & \underbrace{0\ldots0}_{K-1}\,1 & & & \\ & & \ddots & & \\ & & & \underbrace{0\ldots0}_{K-1}\,1 & \\ & & & & \underbrace{0\ldots0}_{K-\ell} \end{pmatrix}^{t}. \tag{40}$$

The number of taps in the feedback filter is now given by $N_d = N_a - 1$. Thus, N_d is expected to be $\approx 1/K$ as long as in the multiuser feedback case. As with the strictly-causal design, $\mathbf{d}^{\ell,\nu}$ $\mathbf{f}^{\ell,\nu}$, and $\mathcal{E}_m^{\ell,\nu}$ are given in (32), (34), and (37), respectively. See Figure 4(b) for an illustrative example of the single-user DFE setup.

Figures 5 and 6 suggest that the performance of the single-user DFE is only slightly better than the performance of the linear equalizer and is clearly the worst of the suboptimal DFE designs presented thus far.

4.2.4 Other Schemes

Realistic DFE implementations may be a combinations and/or variations on the schemes above. For example, the set of K users may include out-of-cell interferers for which symbol estimates will not be available, so not all users will be involved in the feedback process. Then, it may be that the users whose decisions are available will be group-ordered as described earlier.

As an example, say that the K users in $\mathbf{s}(n)$ are ordered so that previous decisions on the first K' users will be available for decision feedback. Then $\mathbf{V}^{\ell,\nu}$ appears as in (38), and $\mathbf{J}^{\ell}$ is given by

$$\mathbf{J}^{\ell} \quad := \quad \begin{pmatrix} \mathbf{I}_{K'-\ell}\ \mathbf{0}_{K'-\ell\times K-K'} & & & \\ & \mathbf{I}_{K'}\ \mathbf{0}_{K'\times K-K'} & & \\ & & \ddots & \\ & & & \mathbf{I}_{K'}\ \mathbf{0}_{K'\times K-K'} \end{pmatrix}^{t}. \tag{41}$$

In this case, the length-KN_s vector $\bar{\mathbf{d}}^{\ell,\nu}$ has the construction

$$\big(\underbrace{0\ldots0}_{K\nu+\ell},\, d_0^{\ell,\nu}\ldots d_{K'-\ell-1}^{\ell,\nu},\, \underbrace{0\ldots0}_{K-K'},\, d_{K'-\ell}^{\ell,\nu}\ldots d_{2K'-\ell-1}^{\ell,\nu},\, \underbrace{0\ldots0}_{K-K'},\, \cdots,\, d_{K'-\ell}^{\ell,\nu}\ldots d_{2K'-\ell-1}^{\ell,\nu},\, \underbrace{0\ldots0}_{\substack{K-K'+\\ \max\{0,K(N_s-N_a-\nu)\}}}\ \big)^{t}.$$

5 Performance Comparisons

To compare the performance of the various linear and decision feedback structures presented earlier, we constructed simulation studies tailored to the uplink and downlink scenarios. Both setups used a fully loaded system with $K = L = 8$ users, Walsh-Hadamard codes, and 3-chip ICI channels whose impulse response was the sum of $(0,1,0)^t$ with a Gaussian random vector of standard deviation 0.2. The user powers were chosen from a log-normal distribution with standard deviation of 7.5 dB, a typical assumption associated with shadow fading in a suburban environment [15]. In the uplink study, the user signals arrived asynchronously at randomly chosen multiples of the chip interval, whereas in the downlink study they arrived synchronously. In the uplink study, the forward equalizer length $N_f = 2L$ was chosen to incorporate this asynchronism, whereas in the downlink scheme $N_f = L$ limits computation complexity. Both scenarios assumed a decision feedback history of symbol duration $N_a = 3$.

Figures 5 and 6 were based on results averaged over thousands of such system realizations. The solid lines were calculated by (1) finding the set of system delays $\{\nu_i\}$ optimal for average user performance at the i^{th} system realization, (2) calculating user-averaged MMSE $\{\mathcal{E}_i\}$ at these $\{\nu_i\}$, and (3) averaging the $\{\mathcal{E}_i\}$ over the multitude of system realizations. The dotted lines above and below show the performance of the worst and best user (respectively) at the previously calculated $\{\nu_i\}$ averaged over the many system realizations.

From the simulation studies, we note that the availability of noise-free estimates of other users' current and past symbols leads to significant implementation gain, even in a low-SNR environment. Though the high-SNR results lead to interesting observations about the performance as external interference approaches zero, they are academic in nature, since the CDMA environment is expected to have high levels of interference from neighboring cells, perhaps on the order of 5-10 dB.

6 Conclusions

In conclusion, we make several observations about the multiuser scenario and consider the implications. First, we noted that concept of system delay in the single-user equalization scenario is extended to the concept of delay-and-user choice in the multiuser equalization scenario. In the single user case we are interested in finding the system delay giving best (or nearly best) performance, and so we tolerate (if not encourage) disappearance of "bad" minima associated with blind equalization schemes such as CMA. If, however, we desire to use CMA at the base-station to acquire/track weak users, the disappearance of CMA minima becomes a problem.

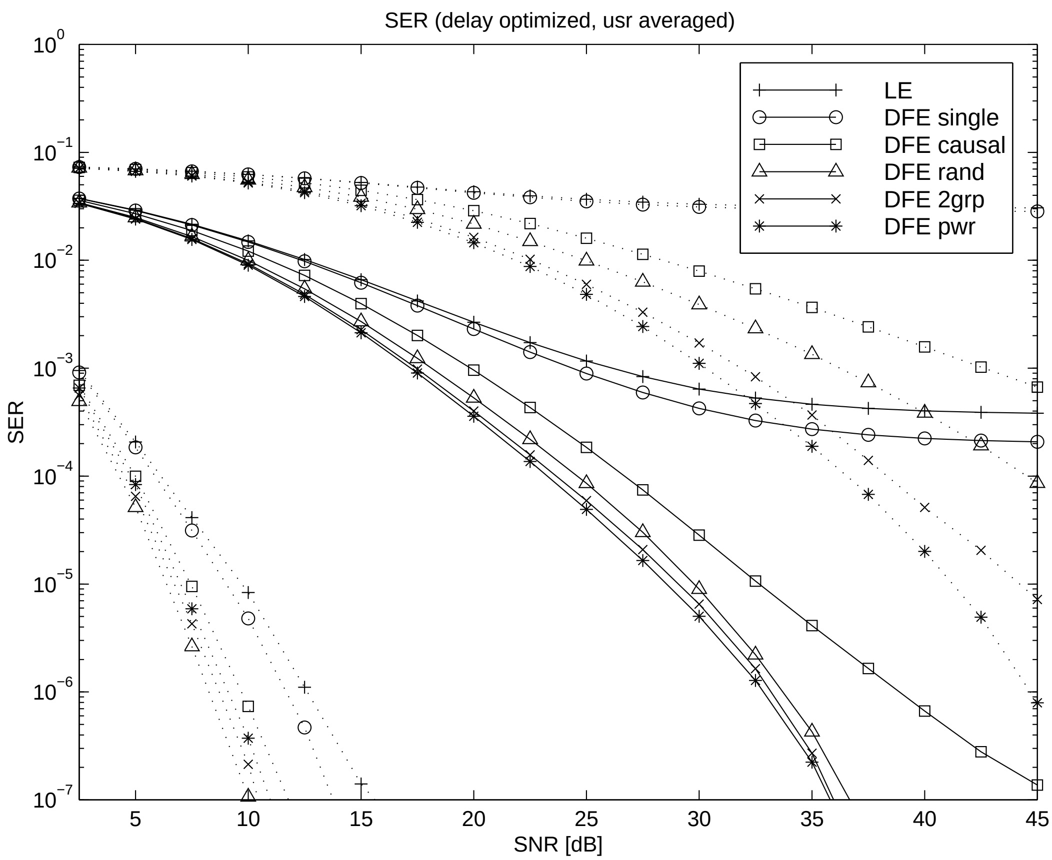

Figure 5: Comparison of six multiuser equalization schemes on uplink: (i) LE, (ii) single-user DFE, (iii) strictly-causal DFE, (iv) randomly-ordered DFE, (v) two-group DFE, and (vi) power-ordered DFE.

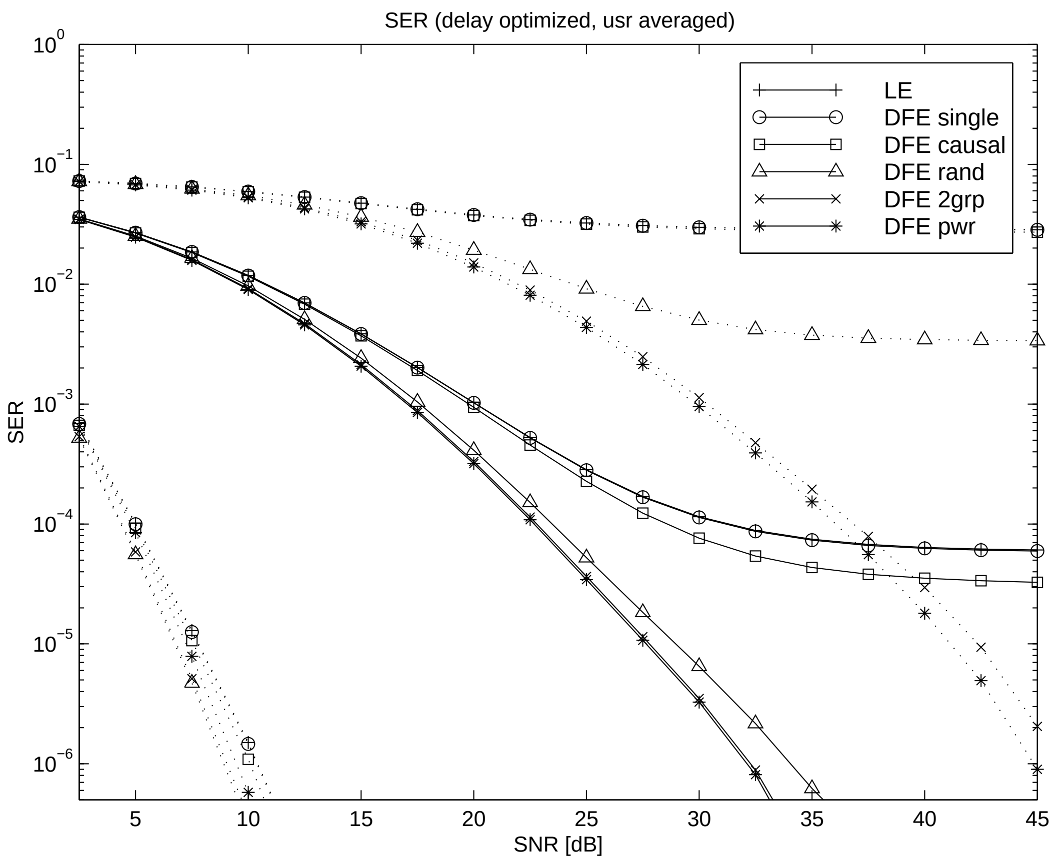

Figure 6: Comparison of six multiuser equalization schemes on downlink: (i) LE, (ii) single-user DFE, (iii) strictly-causal DFE, (iv) randomly-ordered DFE, (v) two-group DFE, and (vi) power-ordered DFE.

Single-user results imply that high levels of noise increase the tendency for "weak" local minima to disappear [16], and we expect that typical levels of inter-cell interference are sufficient to excite these phenomena. Even if the minima don't disappear, recent work on CMA's regions of convergence [17] indicates that it may be hard to initialize CMA so that it converges to the minima of weak users. With these challenges in mind, the author has developed a CMA initialization scheme that appears successful in avoiding these problems. The results will be the subject of a forthcoming report.

Next, consider the effect of channel delay in single user communications. It is well known that fractionally-spaced receivers are nearly insensitive to timing offset. In the multiuser case, however, relative changes in time delay between different mobiles can radically change the interference structure at the base station. This problem becomes worse as the number of mobiles increase and as the chip rate increases. What are the advantages of fractionally-chip-spaced receivers?

Another problem faced at the base station could result from the reassignment of user codes to new users at different (and unknown) locations in the cell. The resulting step changes in interference structure could pose challenges to stochastic-gradient based adaptive equalization algorithms such as LMS and CMA. Tracking speed, then, becomes an important issue. Such environments also have the potential to excite error propagation in the DFE structures proposed earlier. In such cases, alternate DFE structures (e.g., [18]) might prove useful.

The likelihood of a quickly time-varying interference environment also motivates reconsideration of the "blind DFE initialization" problem statement: for a given blind algorithm, find a fixed equalizer initialization which results in global convergence for a reasonably wide class of channels. In the multi-access scenario, we cannot afford to re-initialize all equalizers to a fixed (highly non-optimal) setting each time the interference structure significantly changes because this could severely affect the demodulation of all users.

One last consideration has to do with implementation cost at the mobile versus the base station. Since the mobiles are cost and power limited, it does not make sense to suggest computationally complex equalization structures for them. In fact, the use of L multipliers (as in our downlink simulation) is a significant computational increase over the binary de-spreading operation used in existing systems. But, as noted earlier, successful de-spreading involves knowledge of user timings which we did not assume: the ability to synchronize is made more difficult by severe ICI environments, and needs to be better understood.

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