HOMEWORK ASSIGNMENT #7

Due Wed. Nov. 23, 2005

1. Recall homework problem 5.2 where we examined LMS in a time-varying system identification application. We will now examine RLS in this application. Refering to the notation and diagram of problem 5.2, assume that

   - $\{x(n)\}$ is generated by an AR-2 model with $[a_1, a_2] = [0.6, -0.1]$ and driving noise $\sigma_v^2 = 0.5$.
   - $\sigma_z^2 = 0.002$.
   - $\mathbf{h}(0) = [-1, 0, 1]^t$.
   - $\mathbf{h}$ and $\mathbf{w}$ have equal length.

   You will need results from the steady-state RLS analysis handout to answer the following questions.

   (a) Assuming $\mathbf{Q} = 10^{-5}\mathbf{I}$, calculate, in Matlab, the optimal RLS forgetting factor λ_{opt} and the corresponding (theoretical) level of RLS steady-state MSE, as well as the optimal LMS stepsize μ_{opt} and the corresponding (theoretical) level of LMS steady-state MSE.

   (b) Simulate both LMS and RLS adaptation of $\mathbf{w}(n)$ using length-2500 sequences, the optimal μ & λ, $\delta = 1$, and initialization $\mathbf{w}(n) = \mathbf{0}$. Generate a plot that superimposes the true parameters $\{\mathbf{h}(n)\}$ and adapted estimates $\{\mathbf{w}(n)\}$ versus n. Generate a second plot that superimposes $J(n)$ for LMS and RLS with the $\lim_{n\to\infty} J(n)$ values calculated in (a), *all in a dB scale*. The results should look something like Fig. 1 and Fig. 2. Finally, average the experimentally-derived steady-state MSE (after convergence) for comparison with theoretical values. Which algorithm converges faster? Which tracks better?

   (c) Repeat (a) & (b) for $\mathbf{Q} = 10^{-5}\mathbf{R}$. Which algorithm converges faster? Which tracks better?

   (d) Repeat (a) & (b) for $\mathbf{Q} = 10^{-5}\mathbf{R}^{-1}$. Which algorithm converges faster? Which tracks better?

2. Prove the following inequalities (which were seen in the steady-state RLS analysis) assuming $\mathbf{R}^{-1}$ exists:

   (a) $\sqrt{\dfrac{\text{tr}(\mathbf{R})^2}{M\,\text{tr}(\mathbf{R}^2)}} \;\leq\; 1$

   (b) $\dfrac{1}{M}\sqrt{\text{tr}(\mathbf{R})\,\text{tr}(\mathbf{R}^{-1})} \;\geq\; 1$

   Hint: Use the Cauchy-Schwarz inequality for vectors, $|\mathbf{x}^H\mathbf{y}|^2 \leq \|\mathbf{x}\|^2\|\mathbf{y}\|^2$, and the fact that the trace of a matrix equals the sum of its eigenvalues.

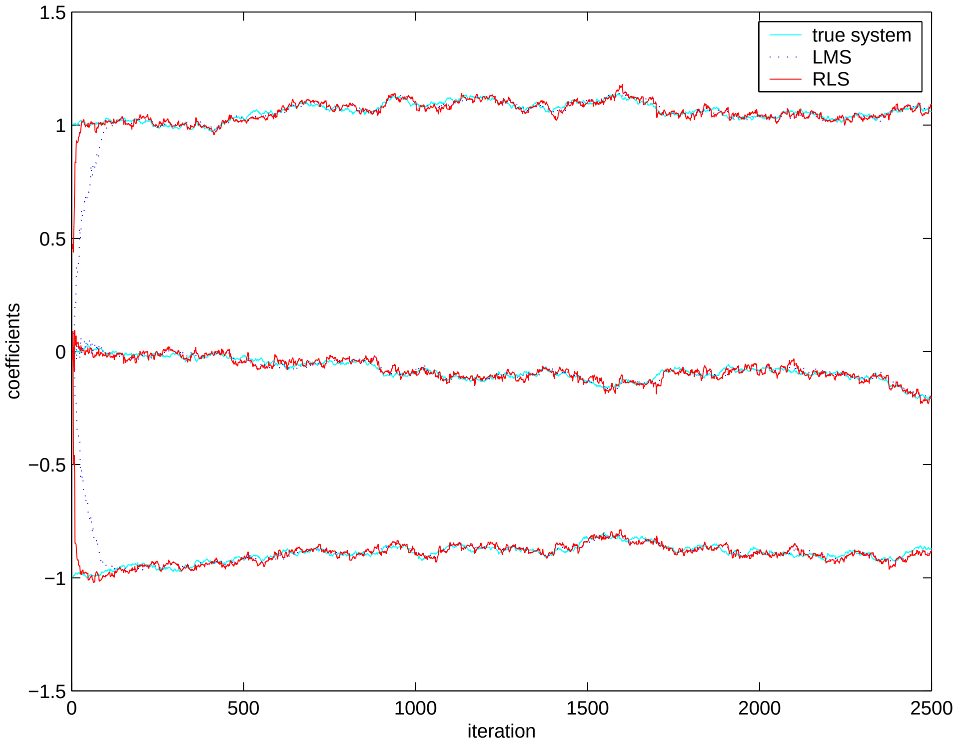

Figure 1: RLS/LMS time-varying system identification.

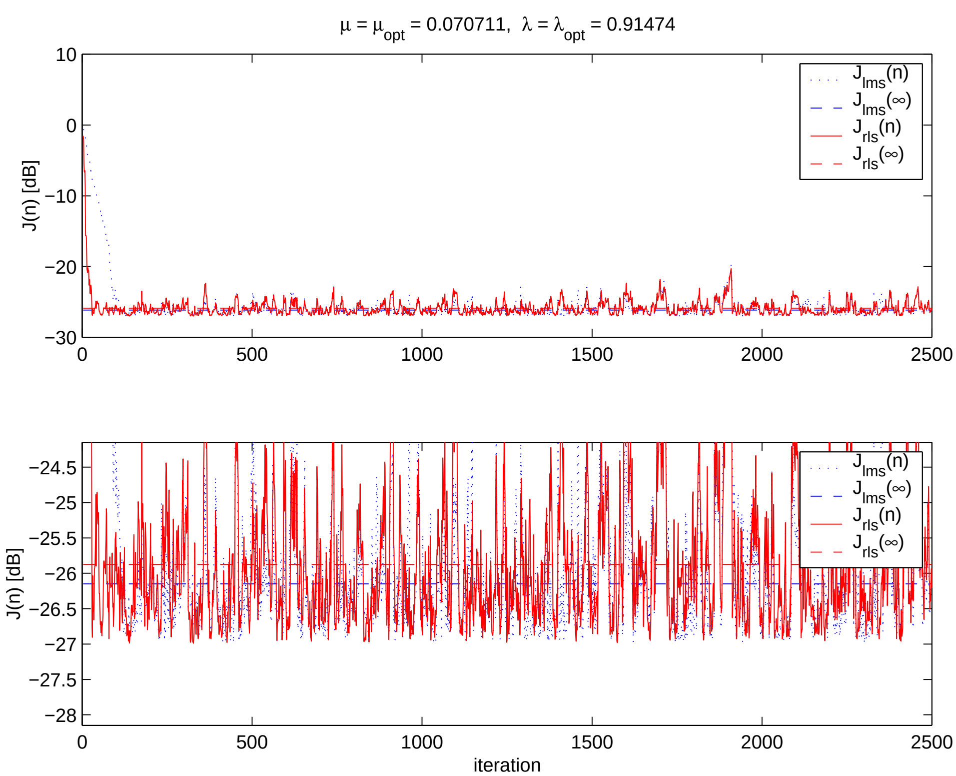

Figure 2: RLS/LMS time-varying system identification.