ECE-894a **Adaptive Filtering** **Autumn 2005**

Homework #6 Oct. 26, 2005

HOMEWORK ASSIGNMENT #6

Due Wed. Nov. 2, 2005 (in class)

1. In this problem we simulate the two variable stepsize algorithms discussed in class in the parameter identification configuration. In all simulations, use the same setup, parameters, and initializations as you did in problem 2 of the previous homework, except use a sequence length of 10000.

   (a) The first algorithm is "adaptive gain LMS":

$$\begin{aligned} e(n) &= d(n) - \boldsymbol{w}^H(n)\boldsymbol{u}(n) \\ \boldsymbol{w}(n+1) &= \boldsymbol{w}(n) + \mu(n)\boldsymbol{u}(n)e^*(n) \\ \mu(n+1) &= \Big[\mu(n) + \alpha \mathrm{Re}\{\boldsymbol{\psi}^H(n)\boldsymbol{u}(n)e^*(n)\}\Big]_0^{\mu_{\max}} \\ \boldsymbol{\psi}(n+1) &= \boldsymbol{\psi}(n) - \mu(n)\boldsymbol{u}(n)\boldsymbol{u}^H(n)\boldsymbol{\psi}(n) + \boldsymbol{u}(n)e^*(n) \end{aligned}$$

   The brackets above indicate that $\mu(n)$ is limited to $0 \le \mu(n) \le \mu_{\max}$. Recall that this algorithm was formulated to find μ that minimizes steady-state error.

   Assume that $\boldsymbol{\psi}(n) = \mathbf{0}$ and $\mu(0) = 0$. Choose values for α and $\mu_{\max}$ that lead to reasonably fast convergence without stability or tracking problems. Make a parameter trajectory plot like the left plot in Fig. 4 of homework assignment #5, a stepsize trajectory plot, and a plot of $\mathrm{E}\{|e(n)|^2|\boldsymbol{w}(n), \boldsymbol{h}(n)\}$. How does the average post-convergence value of $\mu(n)$ compare to μ_{opt}?

   (b) The second algorithm is a "variable stepsize LMS":

$$\begin{aligned} e(n) &= d(n) - \boldsymbol{w}^H(n)\boldsymbol{u}(n) \\ g_k(n) &= u_k(n)e^*(n) \\ \mu_k(n) &= \Big[\mu_k(n-1) + \rho\, \mathrm{sign}\{g_k(n)\}\, \mathrm{sign}\{g_k(n-1)\}\Big]_0^{\mu_{\max}} \\ w_k(n+1) &= w_k(n) + \mu_k(n)g_k(n) \end{aligned}$$

   The subscript k refers to the k^{th} index in the corresponding vector. This algorithm was motivated by the observation that the sign of the gradient terms $g_k(n)$ are consistent during the transient phase, and alternate equally between two values in the steady-state phase. For complex arguments $g \in \mathbb{C}$, we define

$$\mathrm{sign}\big(g\big) := \mathrm{sign}(\mathrm{Re}(g)) + j\,\mathrm{sign}(\mathrm{Im}(b)).$$

   Assume $\mu_k(0) = 0$ for every k. Choose values for ρ and $\mu_{\max}$ that lead to reasonably fast convergence with stability or tracking problems. Make a parameter trajectory plot like the left plot in Fig. 4 of homework assignment #5, a stepsize trajectory plot, and a plot of $\mathrm{E}\{|e(n)|^2|\boldsymbol{w}(n), \boldsymbol{h}(n)\}$. Does this algorithm behave as well as the previous one? What seem to be advantages/disadvantages?

2. In this problem we will investigate the behavior of various modifications of LMS in the linear channel equalization application, illustrated below in Fig. 1. Throughout, use the measured microwave channel response `chan.mat` available on the course webpage (a downsampled truncated version of Channel #2 from the SPIB database[1]), a unit-variance i.i.d. 64-QAM source $\{s(n)\}$, i.i.d. additive white Gaussian noise $\{w(n)\}$ with variance $\sigma_n^2 = 0.01$, $N_f = 8$ equalizer taps, $\Delta = 5$, and equalizer initialization at the origin. You can create the source using the supplied `make_qam.m` file.

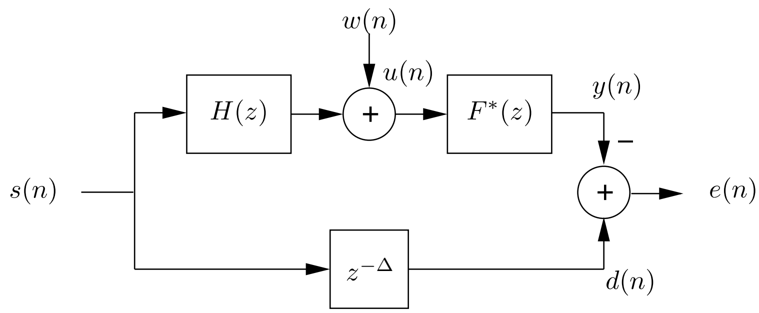

Figure 1: Linear Channel Equalization

Calculate learning curves (i.e., MSE versus iteration) using $\mathrm{E}\{|e(n)|^2|\boldsymbol{f}(n)\}$ rather than the instantaneous error $|e(n)|^2$. To simulate the *average* performance of an adaptive algorithm, it is necessary to average the results of many simulations. In all plots, average the results of at least 100 simulations, and plot over the first 250 iterations.

(a) Why did I specify $\Delta = 5$?

(b) Compare SE-LMS to LMS with stepsize $\mu_{\text{lms}} = 1/\operatorname{tr}(\boldsymbol{R})$. To make a fair comparison, adjust $\mu_{\text{se-lms}}$ so that the steady-state MSE of both algorithms are equal. Superimpose on one plot the learning curves of both algorithms and the theoretical value of the steady-state MSE attained by LMS. (See Fig. 2 for an example.) Comment on the results.

(c) Repeat problem 2 with SR-LMS (adjusting $\mu_{\text{sr-lms}}$ to equate steady-state MSEs).

(d) Repeat problem 2 with DFT-based TDAF (adjusting μ_{tdaf} to equate steady-state MSEs). Also plot the averaged $\hat{\lambda}_k(n)$ trajectories and base your choice of forgetting factor γ on them. Initialize $\hat{\lambda}_k(0) = 1 \ \forall k$.

(e) In the previous problem, you should have noticed that the initial values of $\hat{\lambda}_k(n)$, those most important for fast convergence, do not well approximate the true $\boldsymbol{\lambda} = \operatorname{diag}(\boldsymbol{Q}^H\boldsymbol{R}\boldsymbol{Q})$; With incorrect values for $\hat{\lambda}_k(n)$, TDAF is not expected to perform much better than LMS. Repeat problem 4 using initialization $\hat{\lambda}_k(0) = \lambda_k$. Does this improve the performance of TDAF?

[1] The Rice University Signal Processing Information Base (SPIB) microwave channel database resides at `http://spib.rice.edu/spib/microwave.html`.

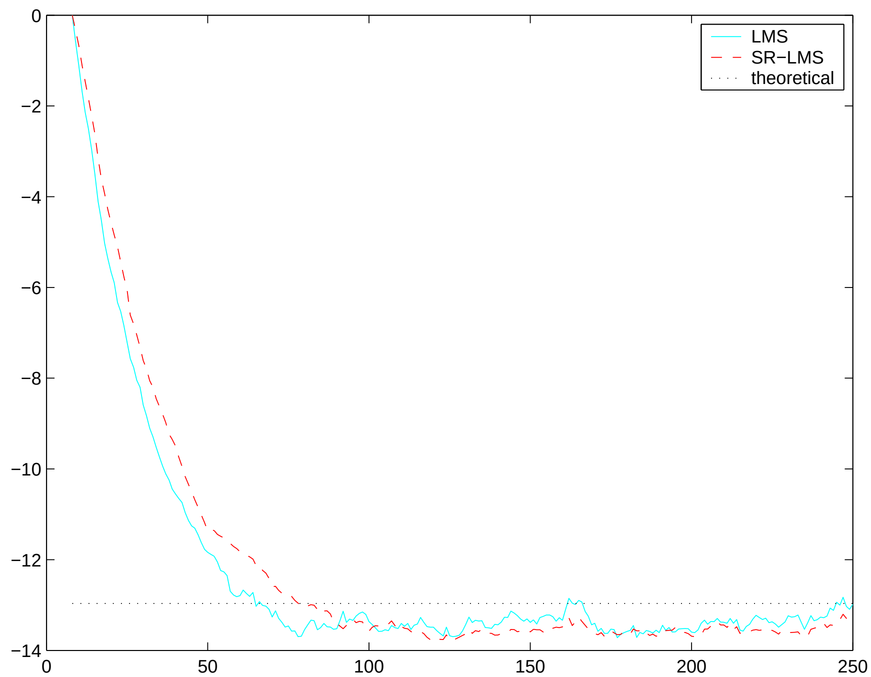

Figure 2: Example of (ensemble-averaged) learning curves with equal steady errors.