HOMEWORK ASSIGNMENT #4

Due Wed. Oct. 19, 2005 (in class)

1. Recall the equalization setup from the last homework, shown again in Fig. 1. Assume that $H(z) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}z^{-1}$, $\boldsymbol{R}_w = 0.01 \cdot \boldsymbol{I}$, $\boldsymbol{R}_s = \boldsymbol{I}$, $\Delta = 0$, and $F(z) = f_0 + f_1 z^{-1}$. Also assume that $\{w(n)\}$ and $\{s(n)\}$ are real-valued and uncorrelated.

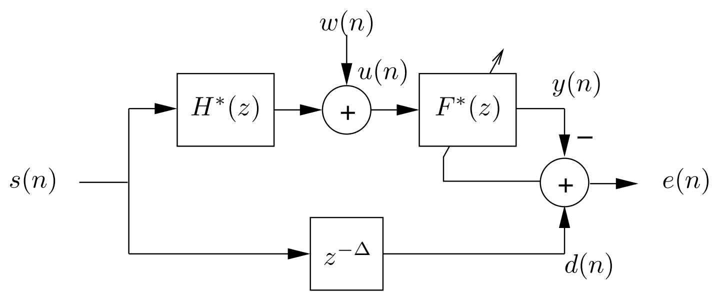

Figure 1: Linear Channel Equalization

   (a) What is $\mu_{\max}$, the maximum stepsize for MSE-GD stability?

   (b) Write Matlab code that calculates MSE-GD parameter trajectories (i.e., $\{\boldsymbol{f}(n),\ n \geq 0\}$) and learning curves (i.e., $\{J_{\text{mse}}(n),\ n \geq 0\}$) for the equalization application. Generate a single plot showing the parameter trajectories superimposed on the MSE cost contour plot for all combinations of $\mu \in \{\frac{3}{4}\mu_{\max}, \frac{1}{10}\mu_{\max}\}$ and $\boldsymbol{f}(0) \in \{[-1,1]^t, [-1,0]^t, [-1,-1]^t\}$. Also generate a single plot of the learning curves for these combinations of μ and $\boldsymbol{f}(0)$, plotting MSE on a logarithmic scale.

   (c) Comment on the important features of the plots.

2. Now we will examine GD-MSE for the system identification setup illustrated in Fig. 2.

   (a) Assuming $F^*(z) = \sum_{k=0}^{K-1} f_k^* z^{-k}$ and $H^*(z) = \sum_{k=0}^{K-1} h_k^* z^{-k}$, derive an expression for $J_{\text{mse}} = \text{E}\{|e(n)|^2\}$ in terms of $\boldsymbol{R}_s, \sigma_w^2, \boldsymbol{f} = [f_0, f_1, \ldots, f_{K-1}]^t$, and $\boldsymbol{h} = [h_0, h_1, \ldots, h_{K-1}]^t$, where $\boldsymbol{R}_s$ denotes the autocorrelation matrix of $\{s(n)\}$ and σ_w^2 denotes the power of white $\{w(n)\}$ (uncorrelated with $\{s(n)\}$). Also derive the MMSE parameters $\boldsymbol{f}_\star$. What is $\mu_{\max}$, the maximum stepsize for MSE-GD stability?

   (b) Write Matlab code that calculates MSE-GD parameter trajectories (i.e., $\{\boldsymbol{f}(n),\ n \geq 0\}$) and learning curves (i.e., $\{J_{\text{mse}}(n),\ n \geq 0\}$) for the system identification application and plots the parameter trajectories on top of the MSE cost contour plot. Generate a single plot that shows the trajectories-on-cost for all combinations of $\mu \in \{\frac{3}{4}\mu_{\max}, \frac{1}{10}\mu_{\max}\}$ and $\boldsymbol{f}(0) \in \{[-1,1]^t, [-1,0]^t, [-1,-1]^t\}$, assuming the $\{s(n)\}, \{w(n)\}, H^*(z)$, and $F^*(z)$ from Problem 1. Also generate a single plot of the learning curves for these combinations of μ and $\boldsymbol{f}(0)$, plotting MSE on a logarithmic scale.

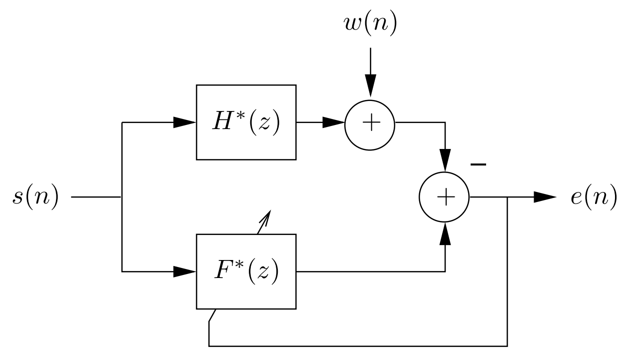

Figure 2: Linear System Identification

(c) Comment on the important features of the plot in 2(b) and compare to the plot in 1(b). Which application is more sensitive to initialization? Does one lead to faster convergence?