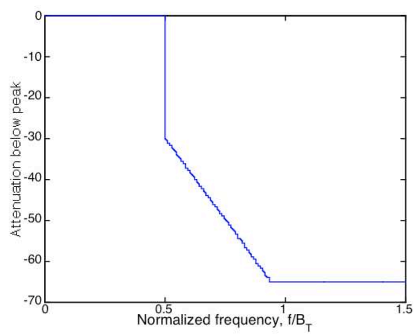
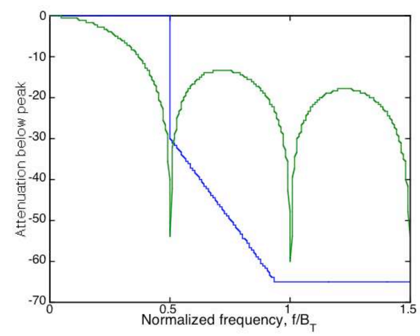
Spectrally Efficient Modulation [Ch. 10]:

- Out-of-band spectral content should be minimized.
- Often spectrum must fit into a "mask."
- The rectangular pulses we have assumed up until now have a sinc^2 spectrum: lots of spectral leakage!





Squared-cosine pulse family:

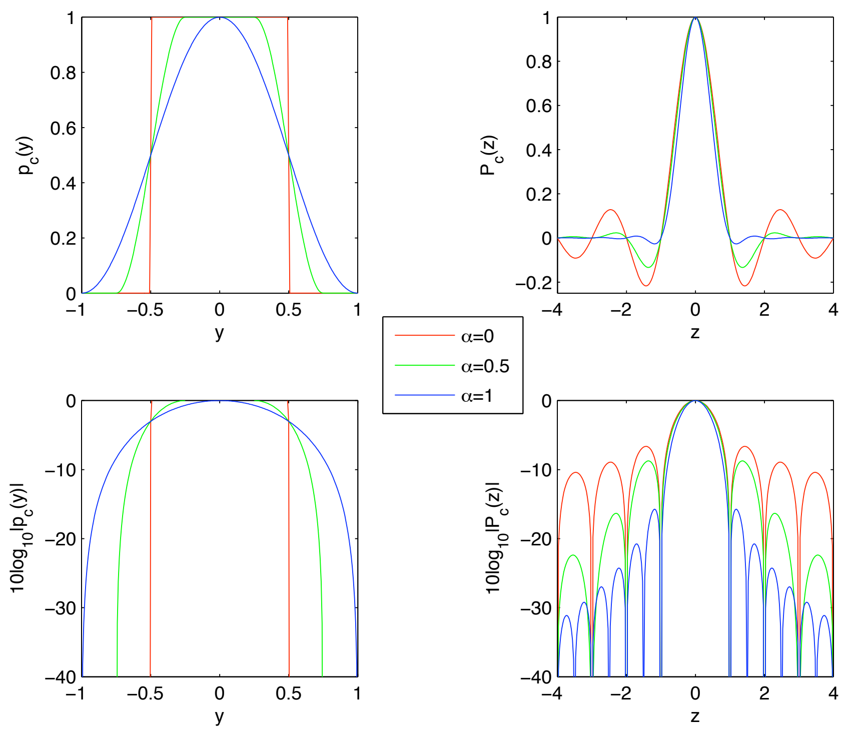
For parameter $\alpha \in [0,1]$:

$$p_c(y) = \begin{cases} 1 & |y| \le \frac{(1-\alpha)}{2} \\ \cos^2\left(\frac{\pi}{2\alpha}\left(|y| - \frac{(1-\alpha)}{2}\right)\right) & \frac{(1-\alpha)}{2} \le |y| \le \frac{(1+\alpha)}{2} \\ 0 & \text{else} \end{cases}$$

$$P_c(z) = \mathcal{F}\{p_c(y)\} = \frac{\cos(\pi\alpha z)}{1-(2\alpha z)^2}\,\mathrm{sinc}(z),$$

where $\mathrm{sinc}(y) \triangleq \frac{\sin(\pi y)}{\pi y}$.

Note: y and z could be either time or frequency domain!



Comments:

- $p_c(y)\big|_{\alpha=0}$ = rectangular pulse of width 1.
- $p_c(y)\big|_{\alpha=1} = \begin{cases} \frac{1}{2} + \frac{1}{2}\cos(\pi y) & |y| \leq 1 \\ 0 & \text{else} \end{cases}$
- Often called "raised cosine" family.
- Fitz's notes present $p_c(y)$ in time domain: $y = t/T_z$.
- $P_c(z) = 0$ for $z \in \mathbb{Z} \setminus 0$
- $P_c(z)$ decays faster as α gets larger.

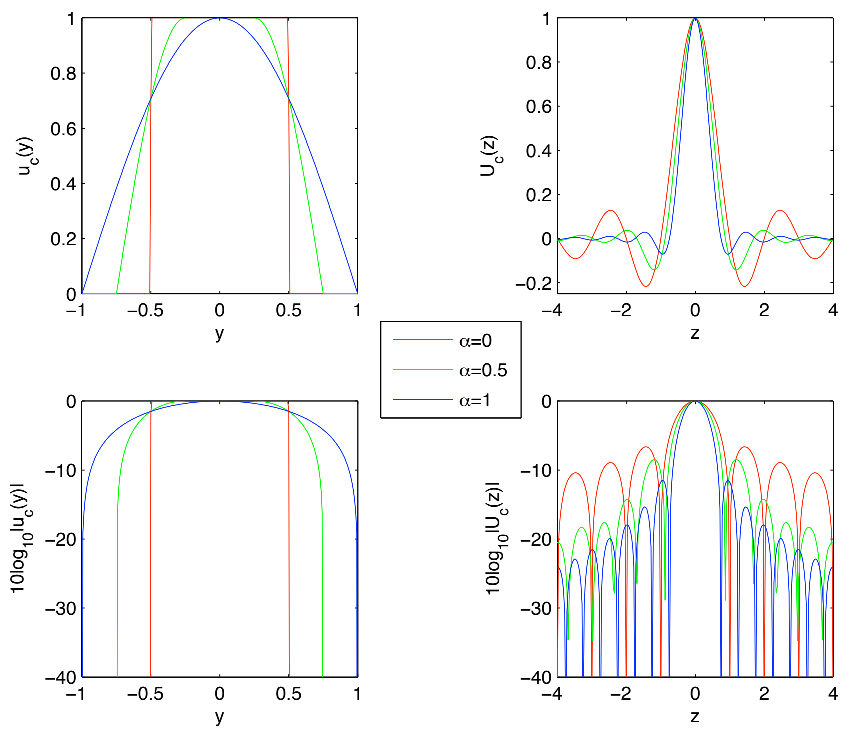
Cosine pulse family: For $\alpha \in [0,1]$:

$$u_c(y) = \begin{cases} 1 & |y| \le \frac{(1-\alpha)}{2} \\ \cos\left(\frac{\pi}{2\alpha}\left(|y| - \frac{(1-\alpha)}{2}\right)\right) & \frac{(1-\alpha)}{2} \le |y| \le \frac{(1+\alpha)}{2} \\ 0 & \text{else} \end{cases}$$

$$U_c(z) = \frac{(1-\alpha)\,\mathrm{sinc}(z(1-\alpha))}{1-(4\alpha z)^2} + \frac{4\alpha\cos(\pi z(1+\alpha))}{\pi(1-(4\alpha z)^2)}$$

Main points:

- $u_c(y) = \sqrt{p_c(y)}$
- Often called "square root raised cosine" family.
- $\int_{-\infty}^{\infty} U_c(z)U_c(z-n)dz = 0$ for $n \in \mathbb{Z}\setminus 0$



Spectral shaping for OFDM: Recall

$$X_z(t) = \sum_{l=1}^{K_b} D_z^{(l)} \sqrt{\frac{E_b}{T_p}} u_s(t) e^{j2\pi \frac{2l-K_b-1}{2T_p} t}$$

where previously $u_s(t) = 1_{[0,T_p]}(t)$ but now we keep $u_s(t)$ general. Orthogonality condition:

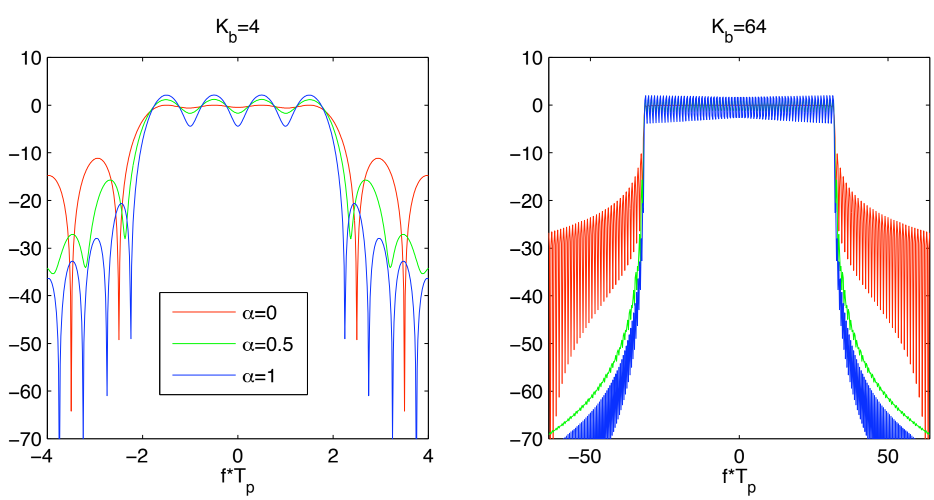
$$\begin{aligned} 0 &= \mathrm{Re}\left[D_z^{(l)} D_z^{(k)*} \int_{-\infty}^{\infty} |u_s(t)|^2 e^{-j2\pi \frac{k-l}{T_p} t} dt \right] \quad \forall k \neq l \\ &= \mathrm{Re}\left[D_z^{(l)} D_z^{(k)*} P_s(\tfrac{k-l}{T_p}) \right] \text{ where } p_s(t) \triangleq |u_s(t)|^2 \end{aligned}$$

The SRRC pulse guarantees orthogonality:

$$u_s(t) = u_c(\tfrac{t}{T_p}) \;\Rightarrow\; P_s(\tfrac{k-l}{T_p}) = P_c(k-l) = 0 \;\; \forall k \neq l.$$

Observations:

- Better out-of-band attenuation for larger α.
- Somewhat more effective for larger K_b.



Spectral shaping for linear stream modulation:

$$X_z(t) = \sum_{l=1}^{K_b} D_z^{(l)} \sqrt{E_b}\, u\big(t-(l-1)T\big)$$

Orthogonality condition:

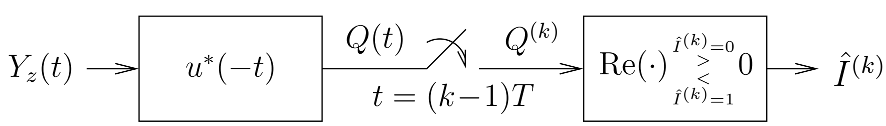
$$0 \;=\; \mathrm{Re}\Bigg[D_z^{(l)} D_z^{(k)*} \underbrace{\int_{-\infty}^{\infty} u(t-lT)u^*(t-kT)dt}_{\triangleq\, V_u\left((k-l)T\right)}\Bigg] \quad \forall k \neq l$$

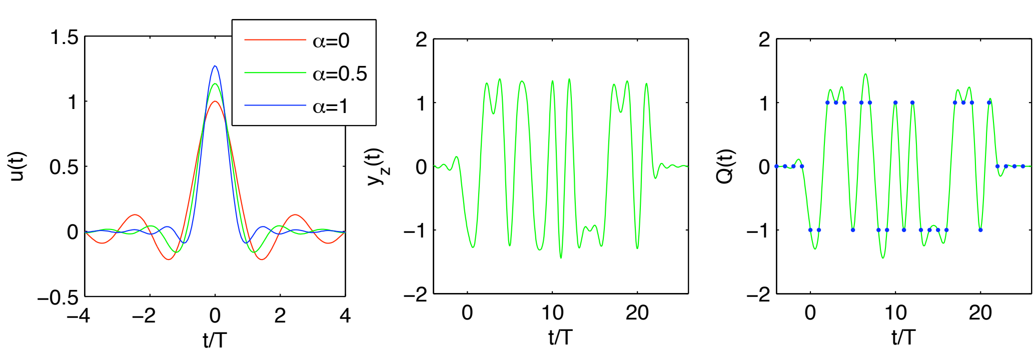
To guarantee orthogonality, use other form of SRRC pulse:

$$u(t) = U_c(\tfrac{t}{T}) \;\Rightarrow\; V_u\big((k-l)T\big) = 0 \;\; \forall k \neq l$$

Often called the "Nyquist criterion for zero ISI."

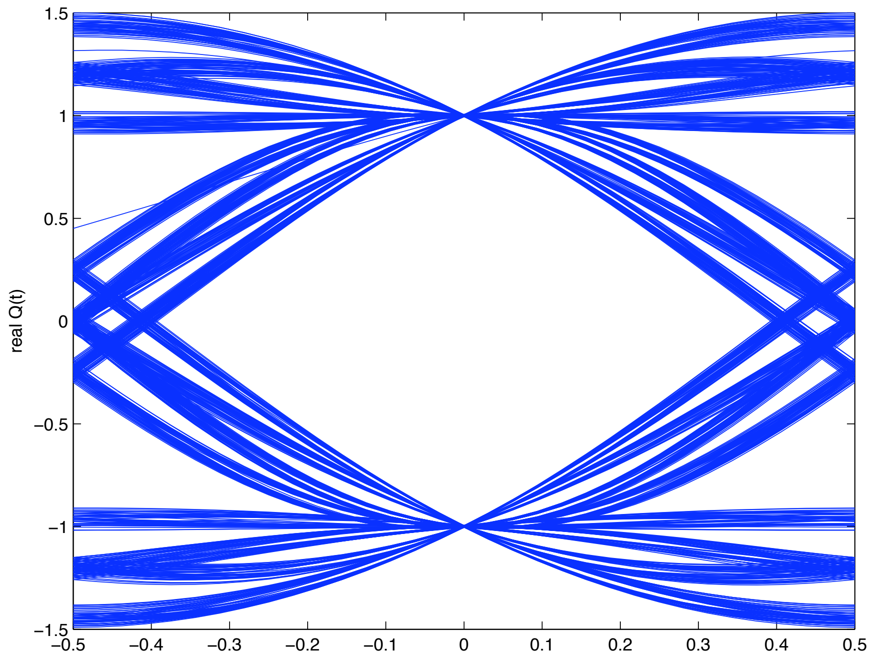
Binary stream demodulation:



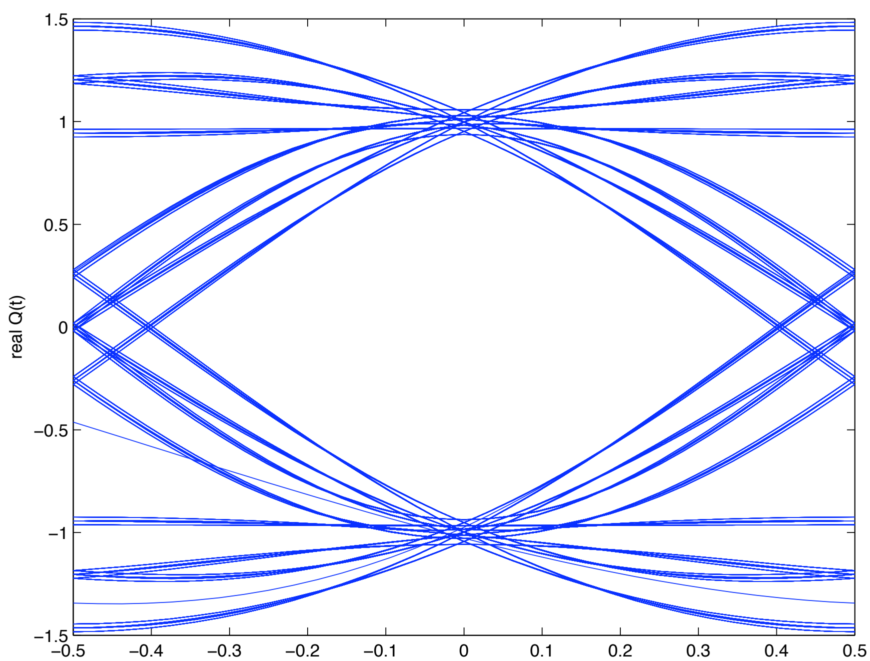


In practice, the pulse would be shifted forward $T_u/2$ seconds and then restricted to the interval $t \in [0, T_u]$.

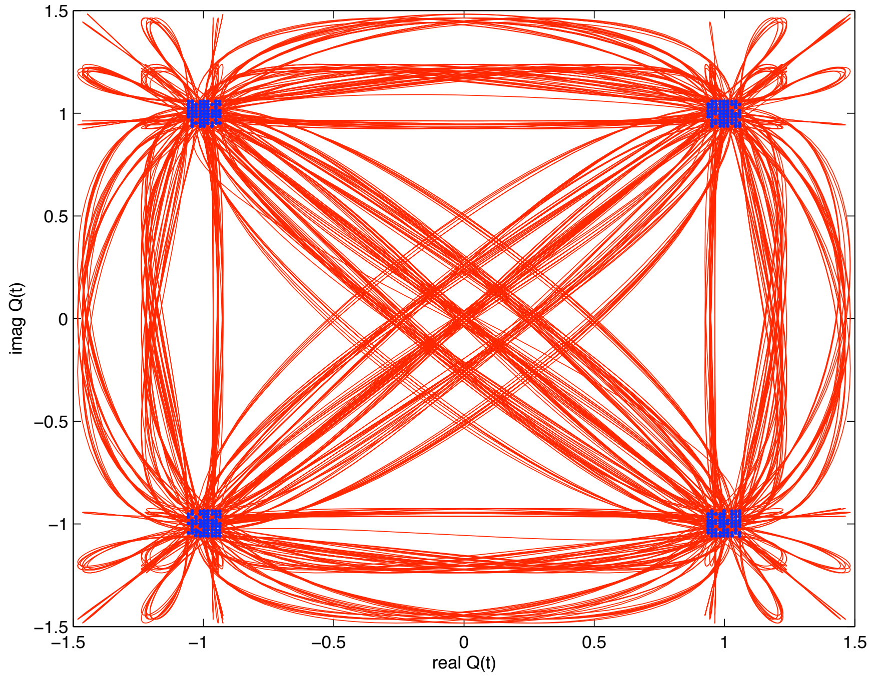
Eye diagram (BPSK, $\alpha = 0.5$, $T_u = 8T$):



Eye diagram (BPSK, $\alpha = 0.5$, $T_u = 3T$):



Vector diagram (QPSK, $\alpha = 0.5$, $T_u = 3T$):



Scatter plot (QPSK, $\alpha = 0.5$, $T_u = 3T$):