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Problem fs. 10

a) Make

$$X_z(t) = \sum_{\ell=1}^{3} D_z(\ell) \exp[j 2\pi f_\ell t] \quad -10^{-4} \le t \le 10^{-3}$$

$$= 0 \text{ elsewhere}$$

$$\mathrm{Re}\left[Q(K) H_z^*(f_K) \right] \underset{\hat{I}(K)=1}{\overset{\hat{I}(K)=0}{\gtrless}} 0 \qquad K = 1, 3$$

b) $$\mathrm{Re}[Q(1)] = Q_I(1) \underset{\hat{I}(1)=1}{\overset{\hat{I}(1)=0}{\gtrless}} 0$$

$$\mathrm{Re}[Q(2)(0.1 - j0.1)] = Q_I(2) + Q_Q(2) \underset{\hat{I}(2)=1}{\overset{\hat{I}(2)=0}{\gtrless}} 0$$

$$\mathrm{Re}[Q(3) - j0.5] = Q_Q(3) \underset{\hat{I}(3)=1}{\overset{\hat{I}(3)=0}{\gtrless}} 0$$

c) $$\tilde{E}_b = \frac{T_d}{3} \sum_{\ell=1}^{3} |H_z(f_\ell)|^2 = \frac{1.27\, T_d}{3} = 0.4233\, T_d$$

$$P_B(E, K) = \frac{1}{2} \mathrm{erfc}\left[\sqrt{\frac{\tilde{E}_b}{N_0} g(K)} \right]$$

where $$g(K) = \frac{|H_z(f_K)|^2 T_d}{\tilde{E}_b}$$

$$P_w(E) = 1 - \frac{1}{8} \mathrm{erf}\left[\sqrt{\frac{\tilde{E}_b}{N_0} g(1)} \right] \mathrm{erf}\left[\sqrt{\frac{\tilde{E}_b}{N_0} g(2)} \right] \mathrm{erf}\left[\sqrt{\frac{\tilde{E}_b}{N_0} g(3)} \right]$$

Problem fs.10 (cont.)

d) Minimum distance error event is due to smallest channel

$$P_w(E) \approx \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{\tilde{E}_b}{N_0}\,\frac{0.02}{0.4233}}\right]$$

First attempt - All bits on strongest channel - 8PSK

$$P_w(E) \approx \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{\tilde{E}_b}{2N_0}\,\frac{3}{0.4233}\left(1-\cos\left(\frac{\pi}{4}\right)\right)}\right]$$

1.03 versus 0.09 $\Rightarrow$ 14.1 dB better

Second attempt - 2 bits on strongest channel
1 bit on 2nd strongest channel

$$P_{w1}(E) \approx \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{\tilde{E}_b}{N_0}\,\frac{1}{0.4233}}\right] \qquad \text{QPSK}$$

17.7 dB better

$$P_{w2}(E) \approx \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{\tilde{E}_b}{N_0}\,\frac{0.5}{0.4233}}\right] \qquad \text{BPSK}$$

14.7 dB better

The BPSK error will dominate the word error performance and it will be slightly better than the first case (8PSK on strongest)