Problem 6.2 6/11/02

a) $C_z(f) = \int_{-\infty}^{\infty} C_z(t)\, e^{-j2\pi f t} dt$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \exp\left[-j\frac{\pi f T_p}{2} + j\theta\right]$$

See Matlab plot

b) $Y_z(f) = \mathcal{F}\{Y_z(t)\}$

$$Y_z(t) = \sqrt{\frac{E_b}{T_p}} \sum_{\ell=1}^{4} D_z(\ell)\, C_z(f_\ell) \exp\left[j2\pi f_\ell t\right] + W(t) \quad 0 \le t \le T_p$$

$$Q(\ell) = \frac{1}{\sqrt{T_p}} \int_0^{T_p} Y_z(t) \exp\left[-j2\pi f_\ell t\right] dt$$

$$f_\ell = \left[2(\ell-1) - 3\right] f_d = \frac{(\ell-1) - 1.5}{T_p}$$

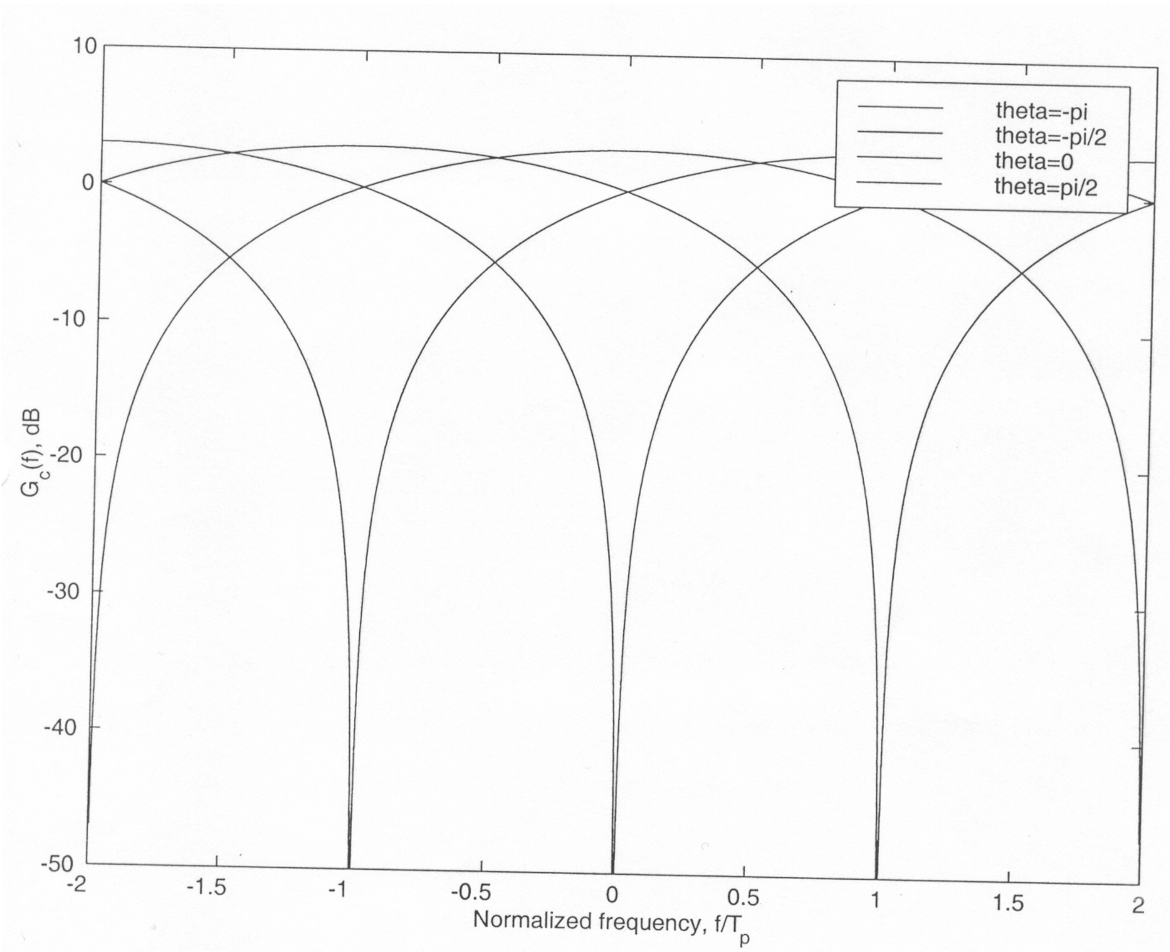
$$= \sqrt{E_b}\, D_z(\ell)\, C_z(f_\ell) + N(\ell)$$

due to orthogonality

c) Optimum bit decisions

$$\mathrm{Re}\left[Q(\ell)\, C_z^*(f_\ell)\right] \underset{\hat{I}(K)=1}{\overset{\hat{I}(K)=0}{\gtrless}} 0$$

d) $P_B(E) = \frac{1}{2} \operatorname{erfc}\left[\frac{m(f_\ell)}{\sqrt{2}\, \sigma_N(f_\ell)}\right]$



theta=-pi
theta=-pi/2
theta=0
theta=pi/2
10
0
-10
-20
-30
-40
-50
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
$G_c(f)$, dB
Normalized frequency, $f/T_p$

Problem 6.2 (cont.)

$$m(f_e) = 2|C_z(f_e)|^2 \sqrt{E_b}$$

$$\sigma_N^2(f_e) = \frac{N_0}{2} |C_z(f_e)|^2$$

$$P_B(E) = \frac{1}{2} \text{erfc}\left[\sqrt{|C_z(f_e)|^2 \frac{E_b}{N_0}}\right]$$

$$C_z(f_2) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \exp\left[ j\frac{\pi}{4} + j\theta \right]$$

setting $\theta = -\frac{\pi}{4}$ gives

$$C_z(f_2) = \frac{2}{\sqrt{2}}$$

$$P_B(E) = \frac{1}{2} \text{erfc}\left[\sqrt{\frac{2E_b}{N_0}}\right]$$

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% EE239
% Solution for Problem 6.2
% Author: M. Fitz
% Last modified: 6/11/02
%
clear all
close all
%
% Defining signal parameters
% T_p=1
numph=4;
theta=linspace(-pi,pi-2*pi/numph,numph);
numpts=1000;
freq=linspace(-2,2,numpts)';
czf=1/sqrt(2)+1/sqrt(2).*exp(-j*pi*freq/2*ones(1,numph)+j*ones(numpts,1)*theta);
figure(1)
plot(freq, 20*log10(abs(czf)));
axis([-2 2 -50 10])
legend('theta=-pi','theta=-pi/2','theta=0','theta=pi/2')
xlabel('Normalized frequency, f/T_p')
ylabel('G_c(f), dB')
```