Professor Fitz
2/10/09

Problem mc.15

$$s_2(t) = \begin{cases} 1 & 0 \le t \le T_p \\ 0 & \text{elsewhere} \end{cases}$$

$$s_3(t) = \begin{cases} 1 & 0 \le t \le \frac{T_p}{3} \\ \exp\left[-j\frac{2\pi}{3}\right] & \frac{T_p}{3} \le t \le \frac{2T_p}{3} \\ \exp\left[-j\frac{4\pi}{3}\right] & \frac{2T_p}{3} \le t \le T_p \\ 0 & \text{elsewhere} \end{cases}$$

Professor Fitz
2/10/04

Problem mc. 18

a) $$R_u(\tau) = \int_{-\infty}^{\infty} u(t)\, u^*(t-\tau)\, dt$$

$$= \int_{\tau}^{T_u} \frac{2E_u}{T_u} \sin\left(\frac{\pi t}{T_u}\right) \sin\left(\frac{\pi(t-\tau)}{T_u}\right) dt \qquad 0 \le \tau < T_u$$

$$= \int_{\tau}^{T_u} \frac{E_u}{T_u} \cos\left(\frac{\pi \tau}{T_u}\right) dt - \int_{\tau}^{T_u} \frac{E_u}{T_u} \cos\left(\frac{2\pi t}{T_u} - \frac{\pi \tau}{T_u}\right) dt$$

$$= \frac{T_u - \tau}{T_u} E_u \cos\left(\frac{\pi \tau}{T_u}\right) - \frac{E_u}{2\pi} \sin\left(\frac{2\pi t}{T_u} - \frac{\pi \tau}{T_u}\right)\Bigg|_{\tau}^{T_u}$$

$$= E_u\left[\frac{T_u - \tau}{T_u} \cos\left(\frac{\pi \tau}{T_u}\right) - \frac{1}{2\pi}\left(\sin\left(\frac{-\pi \tau}{T_u}\right) - \sin\left(\frac{\pi \tau}{T_u}\right)\right)\right]$$

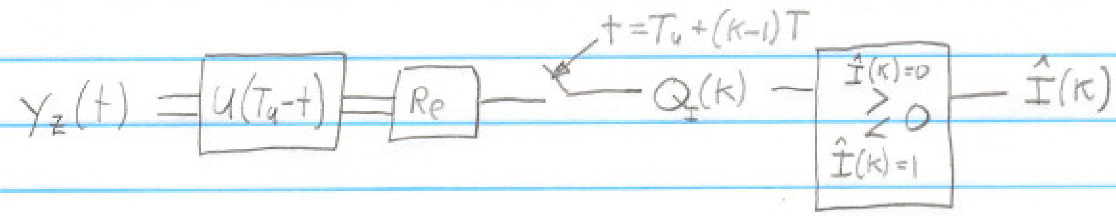
$$= E_u\left[\frac{T - \tau}{T_u} \cos\left(\frac{\pi \tau}{T_u}\right) + \frac{1}{\pi} \sin\left(\frac{\pi \tau}{T_u}\right)\right]$$

using the fact that $R_u(\tau)$ is real and hence even we have

$$R_u(\tau) = E_u\left[\cos\left(\frac{\pi \tau}{T_u}\right)\left[1 - \frac{|\tau|}{T_u}\right] + \frac{1}{\pi} \sin\left(\frac{\pi |\tau|}{T_u}\right)\right] \qquad |\tau| < T_u$$

b) Since $R_u(\tau) \neq 0$ $|\tau| < T_u$ implies that the fastest symbol rate is $W_0 = 1/T_u$

Problem mc. 18 (cont.)



c) $P_B(E) = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]$

d) $D_{x_z}(f) = G_u(f)$ when i.i.d BPSK modulation is used on a stream modulation

$$u(t) = \sqrt{2E_u}\, U_r(t) \sin\left(\frac{\pi t}{T_u}\right)$$

$$U(f) = \frac{\sqrt{2E_u}}{2j}\left[U_r\left(f - \frac{1}{2T_u}\right) - U_r\left(f + \frac{1}{2T_u}\right)\right]$$

$$= \frac{\sqrt{2E_u}}{2j}\left[\sqrt{T_u}\ \operatorname{sinc}\left(\pi\left(f - \frac{1}{2T_u}\right)T_u\right) \exp\left[-j\pi\left(f - \frac{1}{2T_u}\right)T_u\right] - \sqrt{T_u}\ \operatorname{sinc}\left(\pi\left(f + \frac{1}{2T_u}\right)T_u\right) \exp\left[-j\pi\left(f + \frac{1}{2T_u}\right)T_u\right]\right]$$

see Matlab file and plot

e) There is a one to one isometry between $\tilde{X}_z(t)$ and

$$\breve{X}_z(t) = \sum_{\ell=1}^{2K_b} D_z(\ell)\,(j)^{(\ell-1)}\, u\left(t - (\ell-1)\frac{T}{2}\right)$$

Problem mc. 18 (cont.)

It is clear that $\check{X}_z(t)$ is a form of stream modulation where

$$\check{X}_z(t) = \sum_{\ell=1}^{2K_b} D_z(\ell)\, u(\ell, t-(\ell-1)T_s) \qquad T_s = \frac{T}{2}$$

so the Nyquist criterion becomes

$$0 = \mathrm{Re}\left[D_z(\ell) D_z^*(k) \int_0^{T_u} u(\ell,t) u^*(k, t-(k-\ell)T_s)\, dt \right]$$

$$= \mathrm{Re}\left[D_z(\ell) D_z^*(k) (j)^{\ell-k} \int_0^{T_u} u(t) u^*(t-(k-\ell)T_s)\, dt \right]$$

$$= \mathrm{Re}\left[D_z(\ell) D_z^*(k) (j)^{\ell-k} R_u((k-\ell)T_s) \right]$$

Since $D_z(\ell)$ are BPSK (real) Nyquist criterion reduces to

$$R_u(2mT_s) = 0 = R_u(mT)$$

$\Rightarrow$ Nyquist criterion still satisfied when $T = T_u$

f) $$\tilde{X}_z(t) = \sqrt{\frac{2E_u}{T_u}} \left[D_z(1) \sin\left(\frac{\pi t}{T_u}\right) + j D_z(2) \sin\left(\frac{\pi(t - \frac{T_u}{2})}{T_u}\right) \right]$$

$$= \sqrt{\frac{2E_u}{T_u}} \left[D_z(1) \sin\left(\frac{\pi t}{T_u}\right) - j D_z(2) \cos\left(\frac{\pi t}{T_u}\right) \right]$$

$$\tilde{X}_A(t) = \sqrt{X_I^2(t) + X_Q^2(t)} = \sqrt{\frac{2E_u}{T_u}} \sqrt{D_z^2(1) \sin^2\left(\frac{\pi t}{T_u}\right) + D_z^2(2) \cos^2\left(\frac{\pi t}{T_u}\right)}$$

Problem mc.18 (cont.)

Since BPSK $D_z^2(\ell) = 1$

$$\tilde{X}_A(t) = \sqrt{\frac{2E_u}{T_u}} \sqrt{\sin^2\left(\frac{\pi t}{T_u}\right) + \cos^2\left(\frac{\pi t}{T_u}\right)} = \sqrt{\frac{2E_u}{T_u}}$$

$$\tilde{X}_z(t) = \sqrt{\frac{2E_u}{T_u}} \exp\left[j\left(D_z(2)\right)\frac{\pi t}{T_u} + D_z(1)\frac{\pi}{2}\right]$$

$$\tilde{X}_p(t) = D_z(2)\frac{\pi t}{T_u} + D_z(1)\frac{\pi}{2} \qquad \frac{T_u}{2} \leq t \leq T_u$$

g) FSK with $f_d = \dfrac{1}{2T_u}$ with and

initial phase shift of $D_z(1)\dfrac{\pi}{2}$