Michael Fitz
2/8/03

Problem mc. 12

Approach 1

$$(1+x)^{K_b} = \sum_{n=0}^{K_b} \binom{K_b}{n} \cancel{x}^n$$

$$P_w(E) = 1 - \sum_{n=0}^{K_b} \binom{K_b}{n} \left(\frac{-1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]\right)^n$$

$$\approx \frac{K_b}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]$$

Approach 2

p = probability of bit error
$q = 1 - p$

$$P_w(E) = 1 - (p)^{K_b} \simeq (p+q)^{K_b} - (p)^{K_b}$$

$$= \sum_{n=0}^{K_b-1} \binom{K_b}{n} p^n q^{K_b-n}$$

$$P_w(E) \approx \frac{K_b}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right] \left(1 - \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]\right)^{K_b-1}$$

either way the most probable error codes have one bit in error

Example

$K_b = 4$ $\qquad \vec{I} = [0\ 0\ 1\ 0]$

Most Probable Errors

$$\hat{\vec{I}} = [1\ 0\ 1\ 0]$$
$$= [0\ 1\ 1\ 0]$$
$$= [0\ 0\ 0\ 0]$$
$$= [0\ 0\ 1\ 1]$$

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Problem mc.13

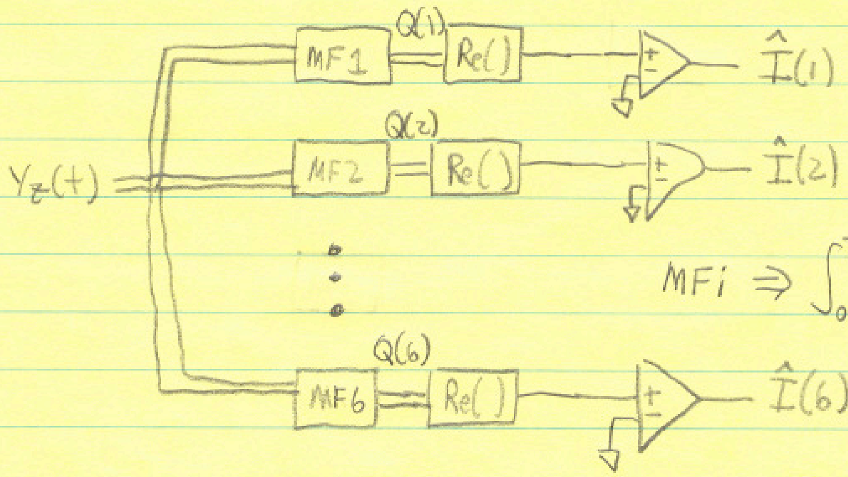
a) $$x_z(t) = \sum_{\ell=1}^{6} D_z(\ell)\exp\left[j2\pi f_d(2\ell - K_o - 1)\right] \quad 0 \le t \le T_p$$

$$= 0 \text{ elsewhere}$$

For BPSK the minimum spacing is $f_d = \frac{1}{4T_p}$

See spectrum plot

Optimum demodulator



$$MFi \Rightarrow \int_0^{T_p} Y_z(t)e^{-j2\pi f_i t}\,dt$$

$$P_W(E) = (1 - P_B(E))^6 = \left(1 - \frac{1}{2}\operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]\right)^6$$

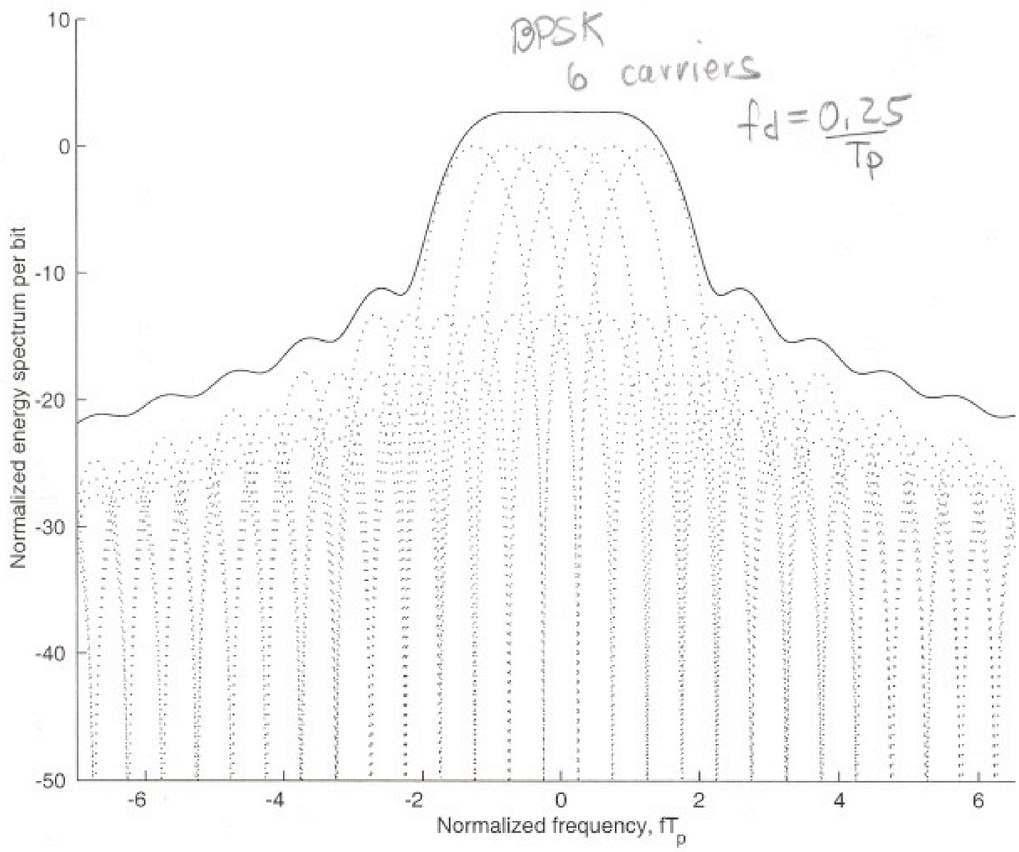
$$P_B(E) = \frac{1}{2}\operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]$$

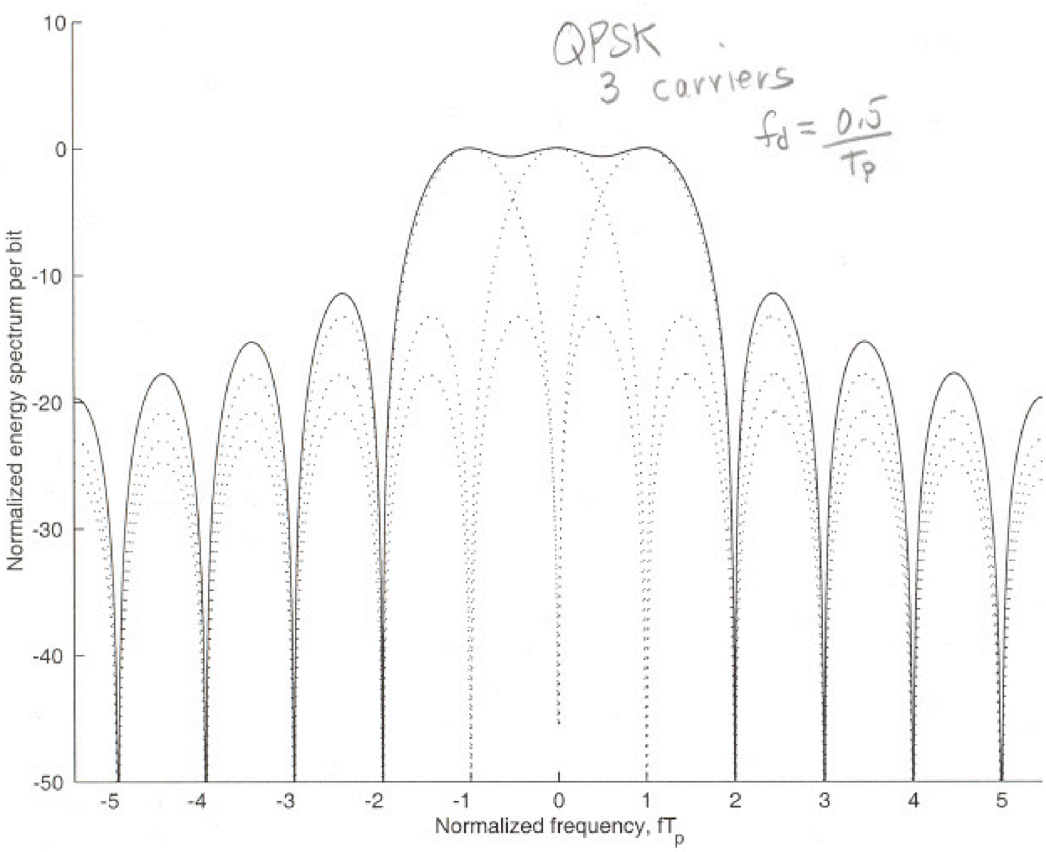
b) Same transmitted waveform roughly except

$\Omega_d = \{1+j, 1-j\ \ -1+j, -1-j\}$ and

$f_d = 0.5$

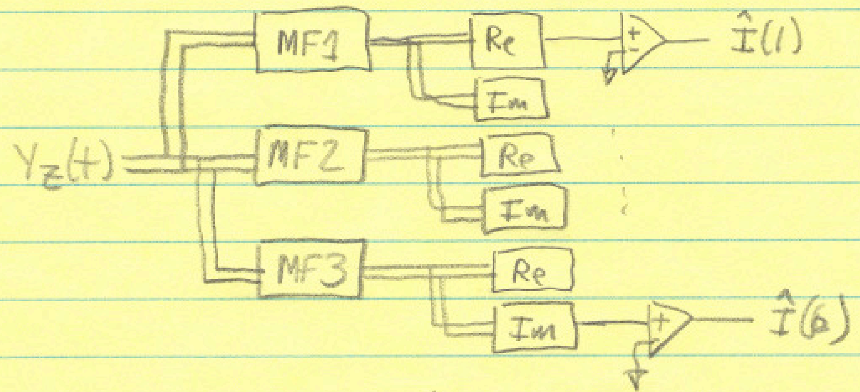
See spectrum plot


BPSK
6 carriers
fd = 0,25 / Tp
Normalized energy spectrum per bit
Normalized frequency, fT_p
10
0
-10
-20
-30
-40
-50
-6
-4
-2
0
2
4
6


QPSK
3 carriers
$f_d = \frac{0.5}{T_p}$
Normalized energy spectrum per bit
Normalized frequency, fT_p
10
0
-10
-20
-30
-40
-50
-5
-4
-3
-2
-1
0
1
2
3
4
5

Problem 7MC.13

Optimum demodulator



$$P_W(E) = (1 - P_w(E))^3 = \left(1 - \text{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right] - \frac{1}{2}\,\text{erfc}\left[\sqrt{\frac{2E_b}{N_0}}\right]\right)^3$$

$$P_B(E) = \frac{1}{2}\,\text{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]$$

Note same performance as a)

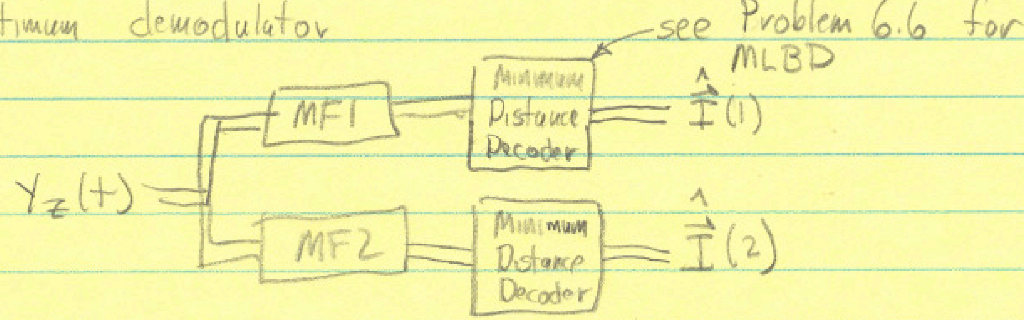
c) Same as before with

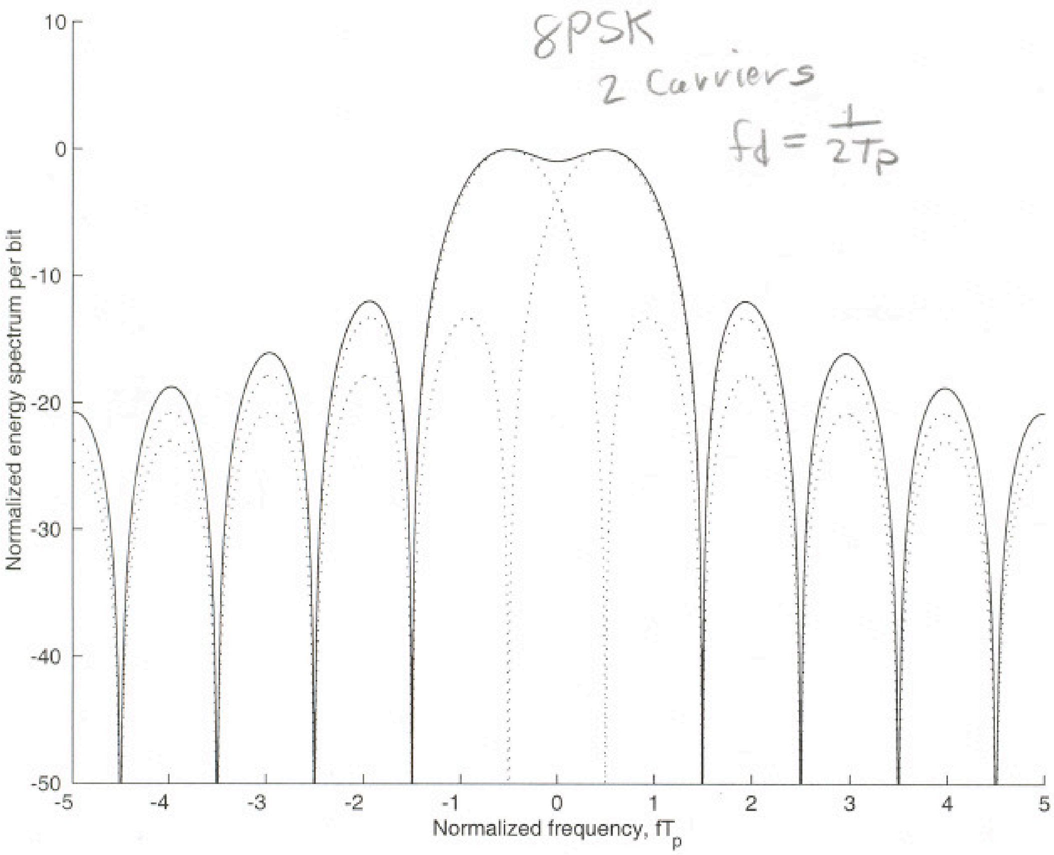
$$\Omega_d = \left\{ e^{j\frac{2\pi i}{8}} \quad i = 0, 7 \right\}$$

$$f_d = 0.5$$

see spectrum plot

Optimum demodulator




8PSK
2 Carriers
$f_d = \frac{1}{2T_p}$
Normalized energy spectrum per bit
Normalized frequency, fT_p
10
0
-10
-20
-30
-40
-50
-5
-4
-3
-2
-1
0
1
2
3
4
5

Problem mc.13

$P_W(E) = (1 - P_W(8PSK))^2$ Figure 6.7

$P_B(E) = P_B(8PSK)$ Problem 6.6