Managing Complexity [Ch. 9]:

Problem:

- Demodulating K_b bits generally requires $\mathcal{O}(2^{K_b})$ complexity.
- For a practical system, we want $\mathcal{O}(K_b)$ complexity.

Insights:

- MPSK only needed one filter for K_b bits.
  $\rightsquigarrow$ linear modulation.
- Gray-coded QPSK demodulated each bit in parallel.
  $\rightsquigarrow$ orthogonal modulation.

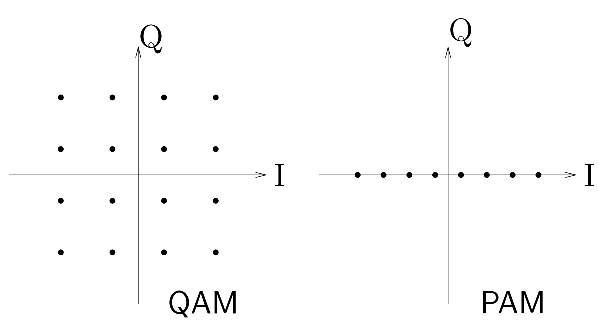
Linear Modulation:

$$\underline{I} = i \quad \Rightarrow \quad x_i(t) = d_i \sqrt{E_b} u(t)$$

Normalization so that E_b = average energy per bit.

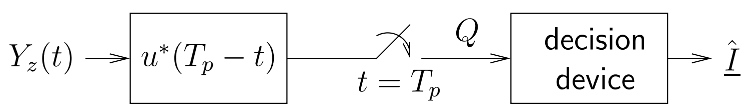
- $\int_{-\infty}^{\infty} |u(t)|^2 dt = 1$.
- $E_s = \sum_{i=0}^{M-1} |d_i|^2 \pi_i = K_b$.

Example constellations:



MLWD for linear modulation:

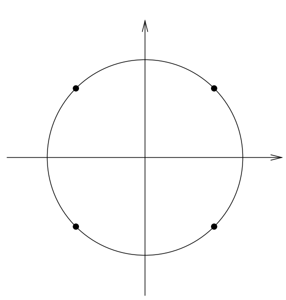
$$\begin{aligned}
E_i &= |d_i|^2 E_b \\
V_i(T_p) &= \int_0^{T_p} Y_z(t) x_i^*(t)dt \;=\; d_i^* \sqrt{E_b} \underbrace{\int_0^{T_p} Y_z(t) u^*(t) dt}_{Q} \\
\hat{\underline{I}} &= \arg\max_i \operatorname{Re}\{V_i(T_p)\} - E_i/2 \\
&= \arg\max_i \sqrt{E_b} \operatorname{Re}\{d_i^* Q\} - |d_i|^2 E_b/2 \\
&= \arg\max_i -\left|Q - d_i\sqrt{E_b}\right|^2 \\
&= \arg\min_i \left|Q - d_i\sqrt{E_b}\right|^2 \quad (\text{“minimum distance decoder”})
\end{aligned}$$

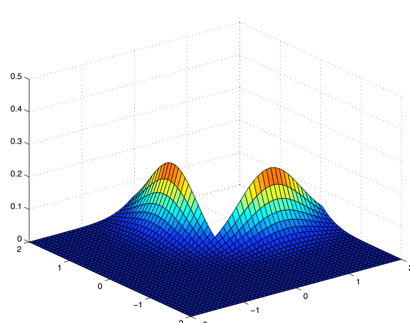


Observations on MLWD for linear modulation:

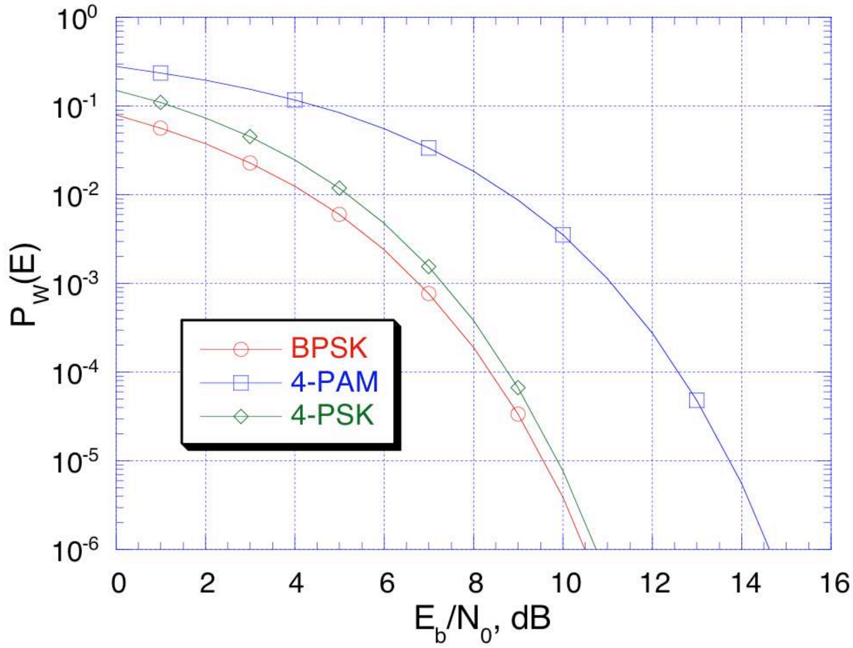
- Only a single filter required.
- Decision $\hat{\underline{I}} = i$ inferred when Q in decision region A_i.
- WEP $= \Pr\big(\hat{\underline{I}} \neq i | \underline{I} = i\big) = \Pr\big(Q \notin A_i | \underline{I} = i\big)$

Note conditional Gaussianity: $Q\big|_{\underline{I}=i} = \sqrt{E_b} d_i + N_z(T_p)$:





Linear modulation examples:



Decoupled Optimal Bit Decisions:

- This happened with Gray-coded QPSK.
- Motivation: gives complexity $\mathcal{O}(K_b)$.

Recall MAPBD for k^{th} bit:

$$\Pr\big(I^{(k)} = 1 | Y_z(t) = y_z(t)\big) \overset{\hat{I}=1}{\underset{\hat{I}=0}{\gtrless}} \Pr\big(I^{(k)} = 0 | Y_z(t) = y_z(t)\big)$$

$$\Leftrightarrow \quad \sum_{n=0}^{M/2-1} \exp\left[\frac{2T_{\{1,n\}}}{N_o}\right] \pi_{\{1,n\}} \overset{\hat{I}=1}{\underset{\hat{I}=0}{\gtrless}} \sum_{n=0}^{M/2-1} \exp\left[\frac{2T_{\{0,n\}}}{N_o}\right] \pi_{\{0,n\}}$$

For decoupled MAPBD decisions need

1. separable priors (i.e., independently generated bits)
2. separable ML metrics

1. Independently generated bits:

$$\forall k: \quad \pi_{\{m,n\}} = \Pr\big(I^{(k)} = m, \underline{I}^{(-)} = n\big) = \underbrace{\Pr\big(I^{(k)} = m\big)}_{\pi_m^{(k)}} \underbrace{\Pr\big(\underline{I}^{(-)} = n\big)}_{\pi_n^{(-)}}$$

Can be restated as follows. Say

$$\underline{I} = i \quad \Leftrightarrow \quad \underline{I} = [m_1, m_2, \ldots, m_{K_b}]$$

Then need

$$\pi_i = \prod_{k=1}^{K_b} \pi_{m_k}^{(k)}$$

2. Separable ML metric:

$$\forall k: \quad T_{\{m,n\}} = \underbrace{T_m^{(k)}}_{f_2\left(I^{(k)}\right)} + \underbrace{T_n^{(k-)}}_{f_1\left(\underline{I}^{(k-)}\right)}$$

Can be restated as follows. Say

$$\underline{I} = i \quad \Leftrightarrow \quad \underline{I} = [m_1, m_2, \ldots, m_{K_b}]$$

Then need

$$T_i = \sum_{k=1}^{K_b} T_{m_k}^{(k)}$$

Under separability conditions, MAPBD becomes

$$\sum_{n=0}^{M/2-1} \exp\left[\frac{2T_{\{1,n\}}}{N_o}\right]\pi_{\{1,n\}} \underset{\hat{I}=0}{\overset{\hat{I}=1}{\gtrless}} \sum_{n=0}^{M/2-1} \exp\left[\frac{2T_{\{0,n\}}}{N_o}\right]\pi_{\{0,n\}}$$

$$\Leftrightarrow e^{\frac{2T_1^{(k)}}{N_o}}\pi_1^{(k)} \sum_{n=0}^{M/2-1} e^{\frac{2T_n^{(k-)}}{N_o}}\pi_n^{(k-)} \underset{\hat{I}=0}{\overset{\hat{I}=1}{\gtrless}} e^{\frac{2T_0^{(k)}}{N_o}}\pi_0^{(k)} \sum_{n=0}^{M/2-1} e^{\frac{2T_n^{(k-)}}{N_o}}\pi_n^{(k-)}$$

$$\Leftrightarrow \exp\left[\frac{2T_1^{(k)}}{N_o}\right]\pi_1^{(k)} \underset{\hat{I}=0}{\overset{\hat{I}=1}{\gtrless}} \exp\left[\frac{2T_0^{(k)}}{N_o}\right]\pi_0^{(k)}$$

Note the M-ary MAPBD becomes K_b binary MAPBDs.

Example – Gray Coded QPSK:

$$\underline{I} = i \in \{0,1,2,3\} \quad \Leftrightarrow \quad \underline{I} = [m_1, m_2]$$

Bit-to-symbol mapping

$$d_i \;=\; d_{m_1} + j d_{m_2}$$

implies that

$$\begin{aligned} T_i &= \sqrt{E_b}\,\mathrm{Re}\{d_i^* Q\} - E_b \\ &= \underbrace{\sqrt{E_b}\,d_{m_1}Q_{\mathrm{I}} - \frac{E_b}{2}}_{T_{m_1}^{(1)}} + \underbrace{\sqrt{E_b}\,d_{m_2}Q_{\mathrm{Q}} - \frac{E_b}{2}}_{T_{m_2}^{(2)}} \end{aligned}$$

Orthogonal Modulation:

Say $\underline{I} = i = [m_1, m_2, \ldots, m_{K_b}]$. MLBD metric separable if

$$\begin{aligned} x_i(t) &= \sum_{l=1}^{K_b} x_{m_l}^{(l)}(t) \quad \text{and} \\ 0 &= \operatorname{Re} \int_{-\infty}^{\infty} x_{m_l}^{(l)}(t) x_{m_k}^{(k)*}(t) dt \quad \forall m_k, m_l, k \neq l \end{aligned}$$

Since M-ary problem decouples into K_b binary problems,

$$\begin{aligned} \text{BEP} &\geq \frac{1}{2} \operatorname{erfc}\Big(\sqrt{\frac{E_b}{N_o}}\Big) \\ \text{WEP} &= 1 - \big(1 - \text{BEP}\big)^{K_b} \end{aligned}$$

Example 1: Orthogonal Frequency Division Multiplexing:

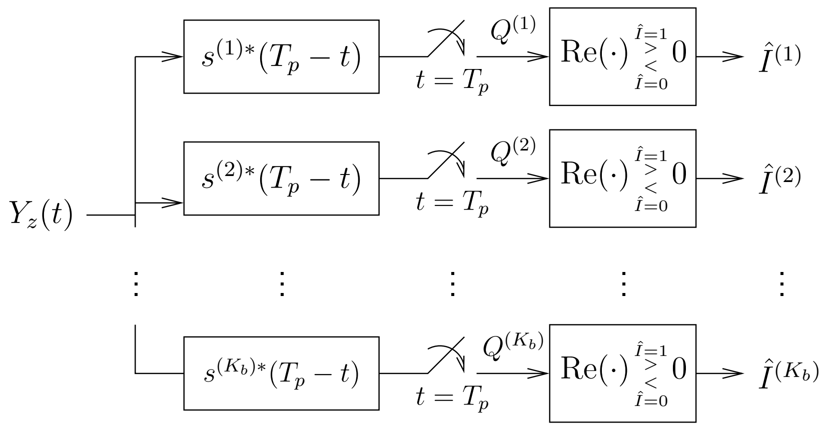
We focus on the case of *one bit per subcarrier*.

$$\begin{aligned} s^{(l)}(t) &= \begin{cases} \sqrt{\frac{1}{T_p}} \exp\big(j2\pi f_d(2l - K_b - 1)t\big) & t \in [0, T_p] \\ 0 & t \notin [0, T_p] \end{cases} \\ X_z(t) &= \sum_{l=1}^{K_b} \underbrace{D_z^{(l)} \sqrt{E_b} s^{(l)}(t)}_{x_{I^{(l)}}^{(l)}(t)} \end{aligned}$$

using BPSK: $D_z^{(l)} = a(I^{(l)})$, i.e., $a(0) = 1,\ a(1) = -1$.

For orthogonality (i.e., $\operatorname{Re} \int_{-\infty}^{\infty} x_{m_l}^{(l)}(t) x_{m_k}^{(k)*}(t) = 0$), generally need $f_d = \frac{1}{2T_p}$, though $f_d = \frac{1}{4T_p}$ suffices for real-valued constellations. Still, we focus on $f_d = \frac{1}{2T_p}$.

Binary OFDM demodulator:



BEP identical to that of BPSK. Spectral efficiency is

$$\left.\begin{array}{l} W_b = \frac{K_b}{T_p} \text{ bits/sec,} \\ B_T = 2 f_d K_b, \;\; f_d = \frac{1}{2T_p}\text{Hz} \end{array}\right\} \Rightarrow \eta_B = 1 \text{ bit/sec/Hz}$$

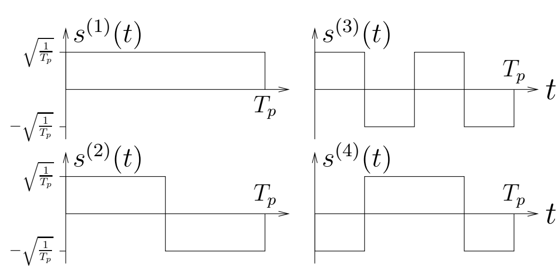
Example 2: Orthogonal Code Division Multiplexing:

We focus on the case of *one bit per spreading waveform*

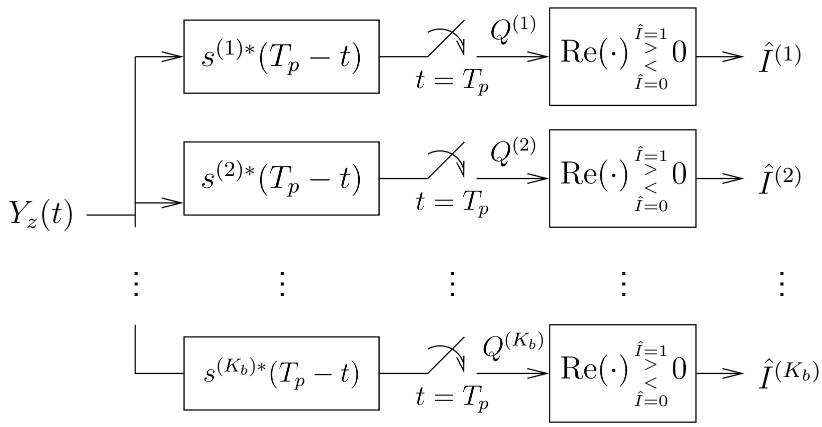
$$X_z(t) \;\; = \;\; \sum_{l=1}^{K_b} D_z^{(l)} \sqrt{E_b}\, s^{(l)}(t)$$

using BPSK $D_z^{(l)} = a(I^{(l)})$, i.e., $a(0) = 1, \;\; a(1) = -1.$ The spreading waveforms $\{s^{(l)}(t)\}_{l=1}^{K_b}$ are orthonormal on $[0, T_p]$:

Walsh codes:



Binary OCDM demodulator:



BEP identical to that of BPSK. Spectral efficiency is

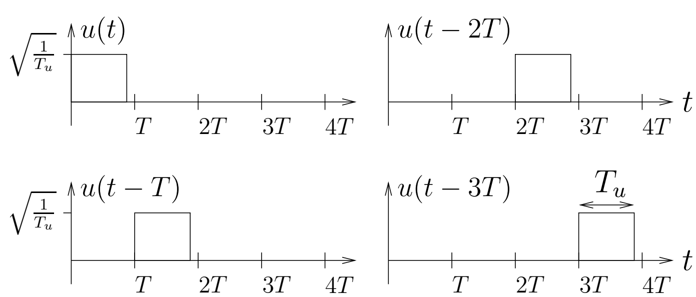
$$\left.\begin{aligned} W_b &= \tfrac{K_b}{T_p} \text{ bits/sec,} \\ B_T &\geq \tfrac{K_b}{T_p} \text{ Hz} \end{aligned}\right\} \Rightarrow \eta_B \leq 1 \text{ bit/sec/Hz}$$

Example 3: Binary Stream Modulation:

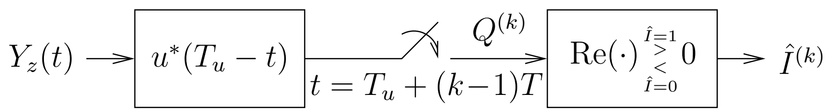
Could be called "orthogonal time-division multiplexing."

$$X_z(t) \;=\; \sum_{l=1}^{K_b} D_z^{(l)} \sqrt{E_b}\, u\big(t-(l-1)T\big)$$

using BPSK $D_z^{(l)}$ as before. The pulse waveform $u(t)$ is orthogonal to its T-shifts.



Binary stream demodulator:



for $k \in \{1, \ldots, K_b\}$.

BEP identical to that of BPSK. Spectral efficiency is

$$\left.\begin{array}{l} W_b = \frac{1}{T} \text{ bits/sec,} \\ B_T \geq \frac{1}{T} \text{ Hz} \end{array}\right\} \Rightarrow \eta_B = \frac{W_b}{B_T} \leq 1 \text{ bit/sec/Hz}$$

Note: In practice, Linear/OFDM/OCDM modulations are combined with stream modulation.