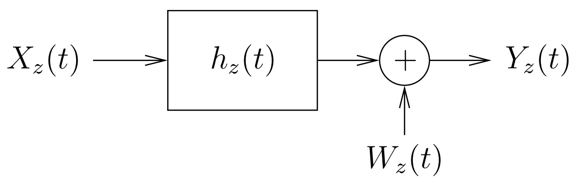
Frequency Selective Channels [Ch. 11]:

- Caused by multipath propagation and lossy media.
- Time dispersive; impulse response $h_z(t)$ on $[0, T_h]$.
- Assume $h_z(t)$ known at receiver but not transmitter.
- Goal: Communicate K_b bits with complexity $\mathcal{O}(K_b)$.
- Remember E_b denotes average *received* energy per bit.



General M-ary MLWD:

- Optimal demodulation performed as before, but with signal $x_i(t)$ replaced by $\tilde{x}_i(t) \triangleq x_i(t) * h_z(t)$

$$\underline{\hat{I}} = \arg\max_i \left[\mathrm{Re} \int_{-\infty}^{\infty} Y_z(t)\tilde{x}_i^*(t)dt - \frac{\tilde{E}_i}{2} \right]$$

$$\tilde{E}_i \triangleq \int_0^{T_p+T_h} |\tilde{x}_i(t)|^2 dt$$

  Note: orthogonality may be hard to preserve!

- Performance now determined by

$$\Delta_E(i,j) = \int_0^{T_p+T_h} |\tilde{x}_i(t) - \tilde{x}_j(t)|^2 dt$$

Binary OCDM in Frequency Selective Channels:

$$x_i(t) = \sum_{l=1}^{K_b} d_{m_l}^{(l)} s^{(l)}(t) \qquad \text{where } \underline{I} = i = [m_1, m_2, \ldots, m_{K_b}]$$

$$\tilde{x}_i(t) = \sum_{l=1}^{K_b} d_{m_l}^{(l)} \underbrace{\left(s^{(l)}(t) * h_z(t)\right)}_{\tilde{s}^{(l)}(t)}$$

$$\begin{aligned}\hat{\underline{I}} &= \arg\max_{i\in\{0,\ldots,M-1\}} \sum_{l=1}^{K_b} \mathrm{Re}\Big[d_{m_l}^{(l)*} \underbrace{\int_0^{T_p+T_h} Y_z(t)\tilde{s}^{(l)*}(t)dt\Big]}_{Q^{(l)}} \\ &\quad - \frac{1}{2}\sum_{k=1}^{K_b}\sum_{l=1}^{K_b} d_{m_k}^{(k)} d_{m_l}^{(l)*} \underbrace{\int_0^{T_p+T_h} \tilde{s}^{(k)}(t)\tilde{s}^{(l)*}(t)dt}_{V_{\tilde{s}}^{(k,l)}}\end{aligned}$$

Matrix Formulation:

$$\underline{\tilde{Q}} = \begin{pmatrix} \tilde{Q}^{(1)} \\ \vdots \\ \tilde{Q}^{(K_b)} \end{pmatrix}, \quad \underline{d}_i = \begin{pmatrix} d_{m_1}^{(1)} \\ \vdots \\ d_{m_{K_b}}^{(K_b)} \end{pmatrix}, \quad \underline{\tilde{N}} = \begin{pmatrix} \tilde{N}^{(1)} \\ \vdots \\ \tilde{N}^{(K_b)} \end{pmatrix}$$

where $\tilde{N}^{(k)} = \int_0^{T_p+T_h} W_z(t)\tilde{s}^{(k)*}(t)dt$.

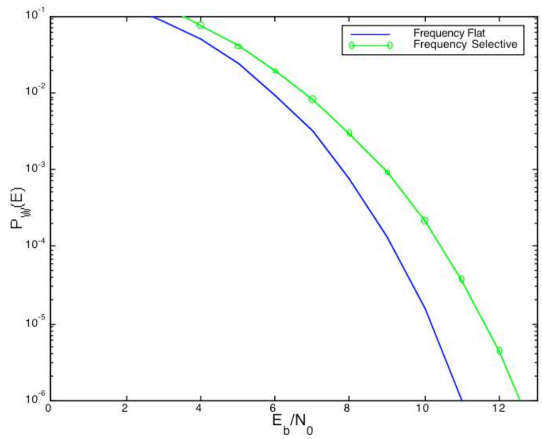
$$\begin{aligned}\underline{\tilde{Q}} &= E_b \boldsymbol{G}\underline{D} + \underline{\tilde{N}} \quad \text{where } [\boldsymbol{G}]_{k,l} = E_b^{-1} V_{\tilde{s}}^{(k,l)} \\ \boldsymbol{R}_{\underline{\tilde{N}}} &= \mathrm{E}\{\underline{\tilde{N}}\underline{\tilde{N}}^H\} = E_b N_o \boldsymbol{G} \\ \hat{\underline{I}} &= \arg\max_{i\in\{0,\ldots,M-1\}} \left\{\mathrm{Re}\left[\underline{d}_i^H \underline{\tilde{Q}}\right] - \frac{E_b}{2}\underline{d}_i^H \boldsymbol{G}\underline{d}_i\right\}\end{aligned}$$

Note: K_b matched filters but $\mathcal{O}(2^{K_b})$ processing.

Can approximate performance via union bound:

$$\text{PWE} = \frac{1}{M}\sum_{j=0}^{M-1}\sum_{i=0,i\neq j}^{M-1}\frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{\Delta_E(i,j)}{4N_o}}\right)$$

$$\Delta_E(i,j) = E_b\big(\underline{d}_i-\underline{d}_j\big)^H\boldsymbol{G}\big(\underline{d}_i-\underline{d}_j\big)$$



Sub-optimum Binary-OCDM Demodulators:

1. Decorrelating (or Zero-Forcing) Detector:

$$\underline{\hat{D}} = (E_b\boldsymbol{G})^{-1}\underline{\tilde{Q}} = \underline{D} + \underbrace{(E_b\boldsymbol{G})^{-1}\underline{\tilde{N}}}_{\underline{\tilde{N}}_d}$$

$$\text{Re}\left\{\hat{D}^{(k)}\right\} \underset{\hat{\imath}^{(k)}=1}{\overset{\hat{\imath}^{(k)}=0}{\gtrless}} 0 \quad \forall k$$

Performance:

$$\boldsymbol{R}_{\underline{\tilde{N}}_d} = \frac{N_o}{E_b}\boldsymbol{G}^{-1}$$

$$\text{PBE}^{(k)} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_o[\boldsymbol{G}^{-1}]_{k,k}}}\right)$$

$\rightsquigarrow$ *Suppress interference perfectly at cost of noise gain.*

2. <u>Linear MMSE Detector</u>:

$$\underline{\hat{D}} = \boldsymbol{W}^H_{\text{MMSE}} \underline{\tilde{Q}}$$

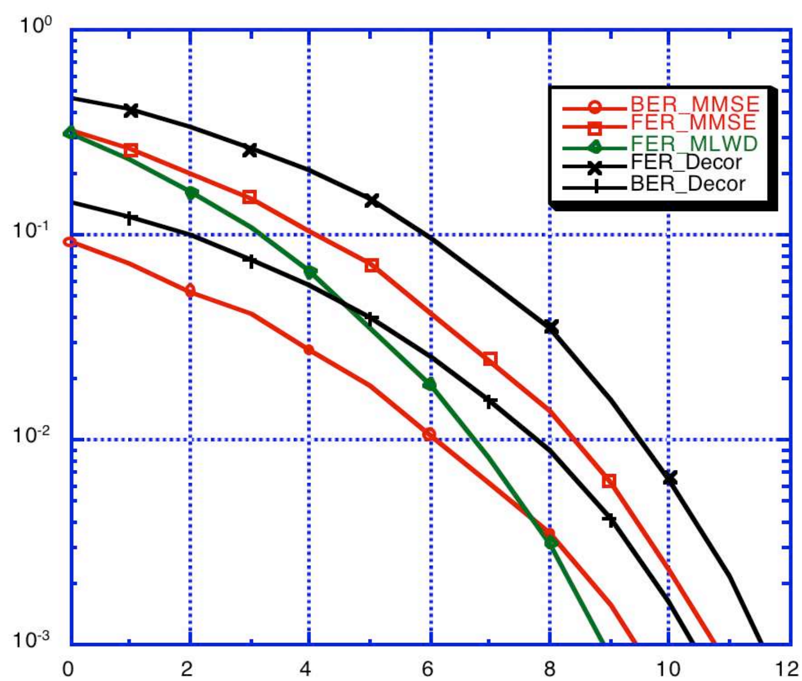
$$\text{Re}\left\{\hat{D}^{(k)}\right\} \underset{\hat{\imath}^{(k)}=1}{\overset{\hat{\imath}^{(k)}=0}{\gtrless}} 0, \quad \forall k$$

Noise+interference minimized in mean-square sense:

$$\boldsymbol{W}^H_{\text{MMSE}} = \arg\min_{\boldsymbol{W}^H} \text{E}\left[\left\|\boldsymbol{W}^H \underline{\tilde{Q}} - \underline{D}\right\|^2\right] = (E_b \boldsymbol{G} + N_o \boldsymbol{I}_{K_b})^{-1}$$

- Same $\mathcal{O}(K_b^2)$ complexity as ZF detector but better performance, since balances noise & interference.
- Problems: still some noise gain, and needs E_b/N_o.

Comparison of linear OCDM detectors:



3. <u>Successive Interference Cancellation (SIC)</u>:

$$\underline{\tilde{Q}}_1 = \underline{\tilde{Q}}$$

for $l = 1, \ldots, K_b$

1) find index k_l of highest-SINR remaining bit.

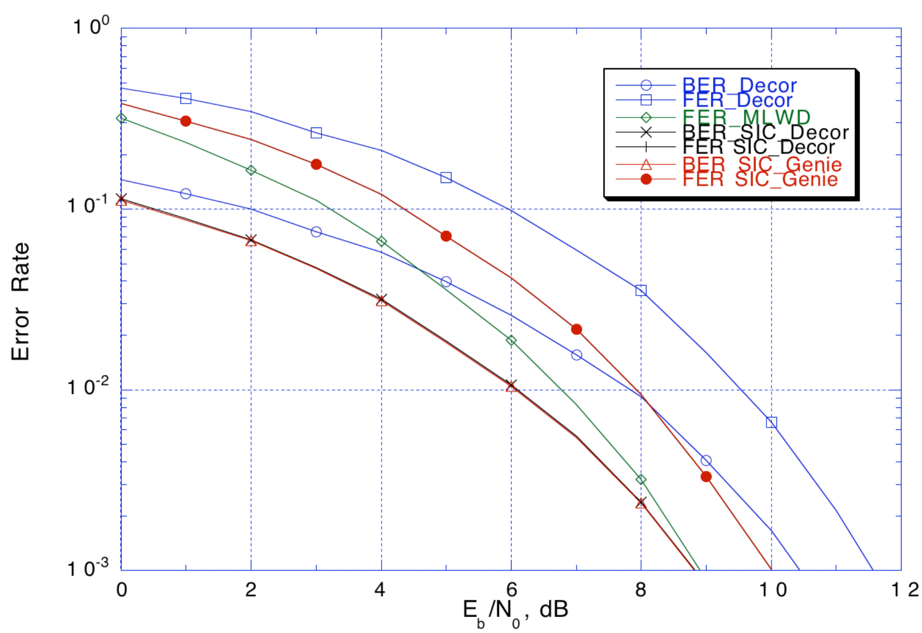
2) detect $I^{(k_l)}$ from $\underline{\tilde{Q}}_l$ (e.g., MMSE, ZF).

3) cancel interference: $\underline{\tilde{Q}}_{l+1} = \underline{\tilde{Q}}_l - E_b[\boldsymbol{G}]_{:,k_l} a(\hat{I}^{(k_l)})$.

end

- No noise gain, but possible error propagation.
- Same $\mathcal{O}(K_b^2)$ complexity but better performance than linear detectors (e.g., ZF, MMSE) when error propagation is not a problem.

Comparison of OCDM detectors:



Other sub-optimal OCDM detectors:

- Parallel Interference Cancellation (often multi-stage)
  - trys to cancel all interference from each bit estimate.
- Iterative Soft Interference Cancellation
  - soft cancellation avoids error propagation.
  - APPs for one iteration used as priors in next.
- Sequential Decoding (i.e., Tree Search)
  - evaluate a small fraction of the 2^{K_b} possibilities.
  - near-MLWD performance, polynomial complexity.
  - examples: sphere decoder, Fano alg, M-alg, T-alg.

Binary OFDM in Frequency Selective Channels:

Recall OFDM:

$$\begin{aligned} X_z(t) &= \sum_{l=1}^{K_b} D_z^{(l)} u(t) e^{j2\pi f_l t} \quad \text{for appropriate } u(t) \text{ and } \{f_l\} \\ Y_z(t) &= W_z(t) + X_z(t) * h_z(t) \\ &= W_z(t) + \sum_{l=1}^{K_b} D_z^{(l)} \int_0^{T_h} h_z(\tau) u(t-\tau) e^{j2\pi f_l (t-\tau)} d\tau \\ &= W_z(t) + \sum_{l=1}^{K_b} D_z^{(l)} e^{j2\pi f_l t} \underbrace{\int_0^{T_h} h_z(\tau) u(t-\tau) e^{-j2\pi f_l \tau} d\tau}_{\triangleq\, \tilde{u}^{(l)}(t)} \end{aligned}$$

Non-orthogonal $\{\tilde{u}^{(l)}(t)\}$ imply $\mathcal{O}(2^{K_b})$ complexity!

Suboptimal Binary-OFDM Demodulation:

If $u(t)$ is constant over the range $t \in [-T_h, T_d]$, then

$$\begin{aligned}\tilde{u}^{(l)}(t) &= \int_0^{T_h} h_z(\tau)u(t-\tau)e^{-j2\pi f_l \tau}d\tau \\ &= u(t)\int_0^{T_h} h_z(\tau)e^{-j2\pi f_l \tau}d\tau \\ &= u(t)H_z(f_l)\end{aligned}$$

for $t \in [0, T_d]$. Over the same range $t \in [0, T_d]$,

$$Y_z(t) = W_z(t) + \sum_{l=1}^{K_b} D_z^{(l)}\, u(t)\, e^{j2\pi f_l t}\, H_z(f_l),$$

which looks like frequency-flat OFDM except for $H_z(f_l)$!

Rather than using the optimal (but non-orthogonal) MFs

$$\left\{\tilde{u}^{(k)*}(t)e^{-j2\pi f_k t}, \quad t \in (-\infty, \infty)\right\}_{k=1}^{K_b},$$

we use the suboptimal (but orthogonal) receiver pulses

$$\left\{H_z^*(f_k)u(t)e^{-j2\pi f_k t}, \quad t \in [0, T_d]\right\}_{k=1}^{K_b},$$

yielding processing complexity $\mathcal{O}(K_b)$ instead of $\mathcal{O}(2^{K_b})$.

In this case, the k^{th} bit decision is made as follows:

$$\mathrm{Re}\left[H_z^*(f_k)\int_0^{T_d} Y_z(t)e^{-j2\pi f_k t}dt\right] \underset{\hat{\imath}^{(k)}=1}{\overset{\hat{\imath}^{(k)}=0}{\gtrless}} 0$$

Using $\overline{E}_b$ to denote avg bit energy received in $[0, T_d]$,

$$\begin{aligned}\text{PBE}^{(k)} &= \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{|H_z(f_k)|^2}{\frac{1}{K_b}\sum_{k=1}^{K_b}|H_z(f_k)|^2}\frac{\overline{E}_b}{N_o}}\right)\\ \text{PWE} &= 1 - \prod_{k=1}^{K_b}\big(1 - \text{PBE}^{(k)}\big)\end{aligned}$$

Notes:

- This scheme is called <u>cyclic prefix OFDM</u>.
- Weakest subchannel PBE dominates average PBE.
- Not all of received signal used: $\overline{E}_b < E_b$.
- Time spent on prefix $\rightsquigarrow$ reduced rate, spectral efficiency.