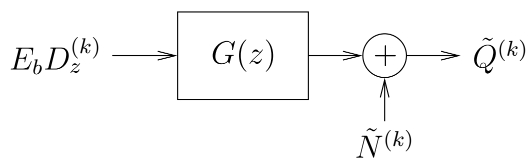
Decision Feedback Equalization:

- A technique with performance close to MLWD but complexity close to linear equalization.
- The DFEs derived here are in "standard" form, and so my development differs from that of Fitz!
- Assume front-end matched-filtering that yields $\{\tilde{Q}^{(k)}\}$, giving the effective discrete-time channel $G(z)$:



where we recall that $S_{\tilde{N}}(e^{j2\pi f}) = E_b N_o G(e^{j2\pi f})$.

Minimum-Phase Spectral Factorization:

Any rational power spectrum S_x can be factored as follows:

$$\begin{aligned} S_x(z) &= \gamma_x F_x^+(z) F_x^-(z) \\ F_x^+(z) &= 1 + \sum_{l=1}^{\infty} f_x[l] z^{-l} \quad \text{(monic, causal, min-phase)} \\ F_x^-(z) &= 1 + \sum_{l=1}^{\infty} f_x^*[l] z^{l} \quad \text{(monic, anti-causal, max-phase)} \\ \gamma_x &= \exp\left[\int_{-0.5}^{0.5} \ln S_x(e^{j2\pi f}) df\right] = \langle S_x \rangle_{\mathsf{G}} \end{aligned}$$

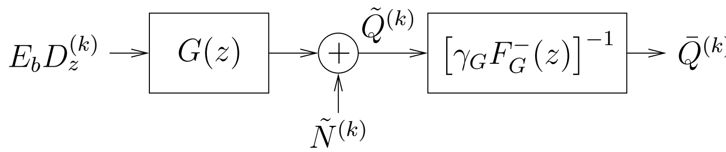
Notes:

- $F_x^-(z) = [F_x^+(1/z^*)]^*$, $F_x^-(e^{j2\pi f}) = [F_x^+(e^{j2\pi f})]^*$.
- $[F_x^+(z)]^{-1}$ is causal, $[F_x^-(z)]^{-1}$ is anti-causal.

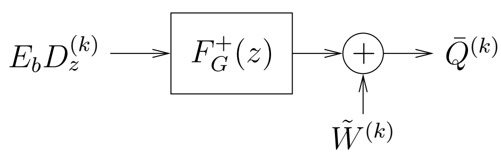
The Whitened Matched Filter Front-end:

Idea: Whiten the noise $\{\tilde{N}^{(k)}\}$ in the discrete time model.

$$\begin{aligned} S_{\tilde{N}}(e^{j2\pi f}) &= E_b N_o G(e^{j2\pi f}) \\ &= E_b N_o \gamma_G F_G^+(e^{j2\pi f}) F_G^-(e^{j2\pi f}) \end{aligned}$$



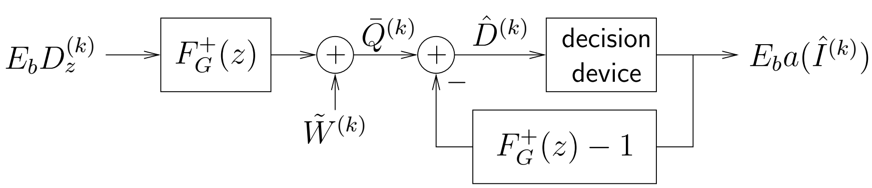
Note the causal monic channel and uncorrelated noise:



$$S_{\tilde{W}}(e^{j2\pi f}) = E_b N_o \gamma_G^{-1}$$

The ZF Decision Feedback Equalizer:

Cancel the *post-cursor* ISI after the WMF front-end:



Notice that the feedback filter is *strictly causal*.
Without error propagation, $\hat{D}^{(k)} = E_b D_z^{(k)} + \tilde{W}^{(k)}$, so that

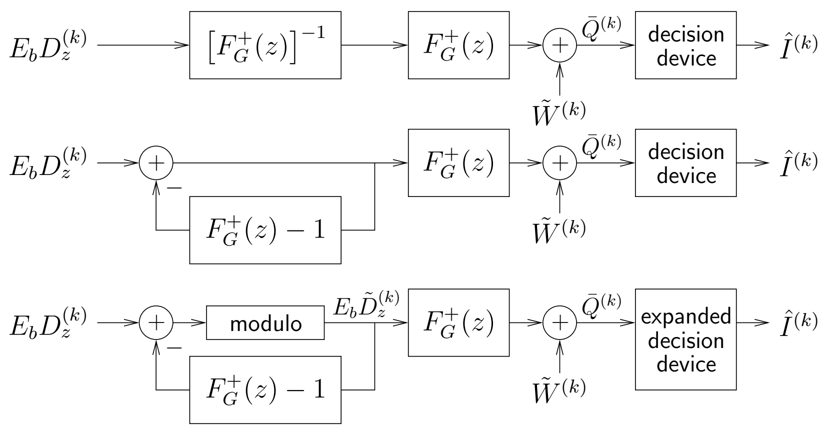
$$\text{PBE} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b \gamma_G}{N_o}}\right)$$

Compare to ZF-LE:

$$\frac{\gamma_G}{\langle G^{-1}\rangle_{\mathsf{A}}^{-1}} = \frac{\langle G\rangle_{\mathsf{G}}}{\langle G^{-1}\rangle_{\mathsf{A}}^{-1}} = \frac{\langle G^{-1}\rangle_{\mathsf{G}}^{-1}}{\langle G^{-1}\rangle_{\mathsf{A}}^{-1}} = \frac{\langle G^{-1}\rangle_{\mathsf{A}}}{\langle G^{-1}\rangle_{\mathsf{G}}} \geq 1.$$

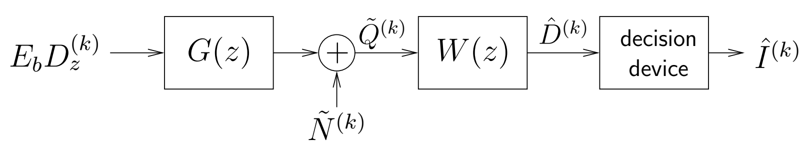
Tomlinson-Harashima Precoding:

- If channel known at transmitter, can pre-equalize (i.e., "pre-code") to eliminate ISI at receiver:
- To avoid increase in transmitted power, use modulo transmission and periodically expand the decision device:

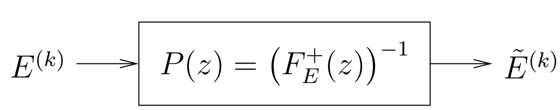


The MMSE Decision Feedback Equalizer:

Recall the linear MMSE equalizer: $W(z) = \left(G(z) + \frac{N_o}{E_b}\right)^{-1}$



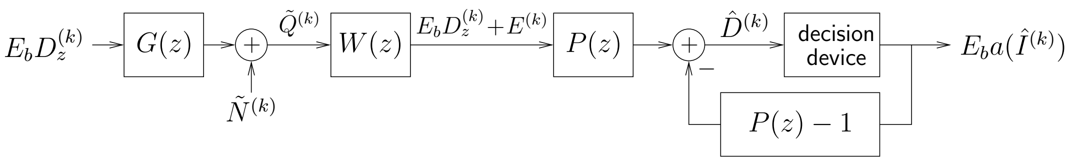
Say $\hat{D}^{(k)} = E_b D_z^{(k)} + E^{(k)}$. Detection performance will increase if we reduce σ_E^2. Can maximally reduce σ_E^2 by whitening $\{E^{(k)}\}$ using optimal linear prediction:



$$S_E(e^{j2\pi f}) = \gamma_E F_E^+(e^{j2\pi f}) F_E^-(e^{j2\pi f}), \quad \sigma_E^2 = \gamma_E \sum_{l=0}^{\infty} |f[l]|^2$$

$$S_{\tilde{E}}(e^{j2\pi f}) = \gamma_E, \quad \sigma_{\tilde{E}}^2 = \gamma_E$$

Cleverly, we can whiten $\{E^{(k)}\}$ without affecting $E_b D_z^{(k)}$!



Assuming perfect decisions,

$$\begin{aligned} \hat{D}^{(k)} &= P(z)\big[E_b D_z^{(k)} + E^{(k)}\big] - \big[P(z) - 1\big] E_b D_z^{(k)} \\ &= E_b D_z^{(k)} + P(z) E^{(k)} \end{aligned}$$

Note that the feedback filter is strictly causal:
$P(z)$ is causal and monic because $P(z) = \big(F_E^+(z)\big)^{-1}$.

So what is $F_E^+(z)$?

Phil Schniter OSU ECE-809

Recall from MMSE equalization that

$$W(z) = \Big(G(z) + \tfrac{N_o}{E_b}\Big)^{-1} \quad \Leftrightarrow \quad 1 = G(z)W(z) + \tfrac{N_o}{E_b} W(z)$$

which is useful in simplifying

$$\begin{aligned} E^{(k)} &= \big(G(z)W(z) - 1\big) E_b D_z^{(k)} + W(z)\tilde{N}^{(k)} \\ &= -W(z) N_o D_z^{(k)} + W(z)\tilde{N}^{(k)}. \end{aligned}$$

Assuming zero-mean white $\{D_z^{(k)}\}$, independent of $\tilde{N}^{(k)}$,

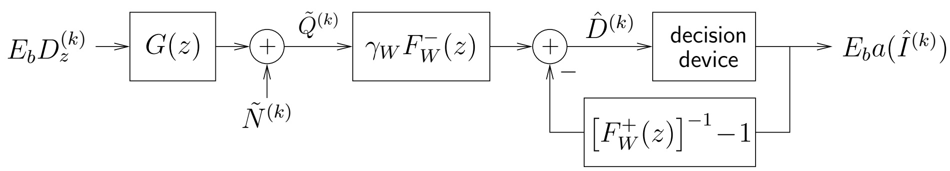
$$\begin{aligned} S_E(e^{j2\pi f}) &= W^2(e^{j2\pi f})\big(N_o^2 + E_b N_o G(e^{j2\pi f})\big) \\ &= E_b N_o W(e^{j2\pi f}) \\ &= E_b N_o \gamma_W F_W^+(e^{j2\pi f}) F_W^-(e^{j2\pi f}) \\ \Rightarrow F_E^+(z) &= F_W^+(z) \end{aligned}$$

The MMSE-DFE forward filter equals

$$W(z)P(z) \;=\; W(z)\big(F_W^+(z)\big)^{-1} \;=\; \gamma_W F_W^-(z)$$

and so is referred to as a "precursor equalizer."

Summary of MMSE-DFE:



It is interesting to note that the MMSE estimate is *biased*:

$$\begin{aligned}
\hat{D}^{(k)} &= E_b D_z^{(k)} + P(z)E^{(k)} \\
&= E_b D_z^{(k)} - P(z)W(z)N_o D_z^{(k)} + P(z)W(z)\tilde{N}^{(k)} \\
&= \big(1 - \tfrac{N_o}{E_b}\gamma_W F_W^-(z)\big) E_b D_z^{(k)} + \gamma_W F_W^-(z)\tilde{N}^{(k)} \\
&= \big(1 - \tfrac{N_o}{E_b}\gamma_W\big) E_b D_z^{(k)} - \tfrac{N_o}{E_b}\gamma_W \textstyle\sum_{l=1}^{\infty} f_W^*[l] D_z^{(k+l)} \\
&\quad + \gamma_W F_W^-(z)\tilde{N}^{(k)}
\end{aligned}$$

Bias removal will decrease PBE (and increase MSE).

$\rightsquigarrow$ *unbiased MMSE-DFE*

Unbiasing can be incorporated into the "decision device."