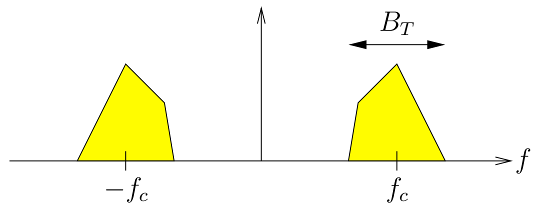
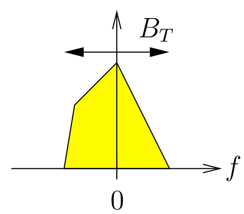
Complex Baseband Representations:

Many systems transmit (real) passband signals:



but modem processing is done at baseband. Hence, need a complex baseband signal representation [Ch. 2].

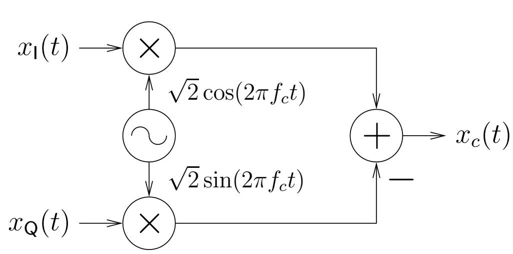


Baseband signal ("complex envelope"):

$$x_z(t) = x_{\mathsf{I}}(t) + j x_{\mathsf{Q}}(t) \qquad \begin{cases} x_{\mathsf{I}}(t) \in \mathbb{R} & \text{"in phase"} \\ x_{\mathsf{Q}}(t) \in \mathbb{R} & \text{"quadrature"} \end{cases}$$

Conversion to passband signal $x_c(t)$:

$$\begin{aligned} x_c(t) &= \sqrt{2}\,\mathbb{R}\big[x_z(t) e^{j2\pi f_c t}\big] \\ &= \sqrt{2}\Big[x_{\mathsf{I}}(t)\cos(2\pi f_c t) - x_{\mathsf{Q}}(t)\sin(2\pi f_c t)\Big] \end{aligned}$$



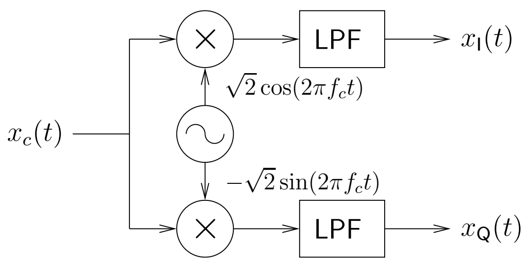
"quadrature modulator"

"I/Q upconverter"

Conversion from passband to baseband:

$$
\begin{aligned}
x_c(t)\sqrt{2}\cos(2\pi f_c t) &= x_{\mathsf{I}}(t) + x_{\mathsf{I}}(t)\cos(4\pi f_c t) \\
&\quad - x_{\mathsf{Q}}(t)\sin(4\pi f_c t) \\
-x_c(t)\sqrt{2}\sin(2\pi f_c t) &= x_{\mathsf{Q}}(t) - x_{\mathsf{Q}}(t)\cos(4\pi f_c t) \\
&\quad - x_{\mathsf{I}}(t)\sin(4\pi f_c t)
\end{aligned}
$$

LPF to remove double-frequency terms:



"quadrature demodulator"

"I/Q downconverter"

Signal spectra:

$$
\begin{aligned}
X_z(f) &:= \mathcal{F}\{x(t)\} && \text{Fourier transform} \\
G_{X_z}(f) &:= |X_z(f)|^2 && \text{“Energy spectrum”}
\end{aligned}
$$

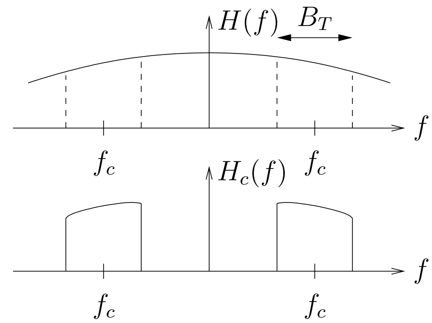
Note that

$$
G_{X_c}(f) = \frac{1}{2} G_{X_z}(f - f_c) + \frac{1}{2} G_{X_z}(-f - f_c)
$$

Filtering of bandpass $X_c(f)$:

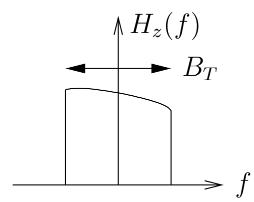
$$Y_c(f) = H(f)X_c(f),$$
$$\Rightarrow Y_c(f) = H_c(f)X_c(f)$$

via bandpass equivalent $H_c(f)$.



Translate filter to baseband:

$$\begin{aligned} h_c(t) &= 2\mathbb{R}\left[h_z(t)e^{j2\pi f_c t}\right] \\ h_z(t) &= h_\mathsf{I}(t) + jh_\mathsf{Q}(t) \end{aligned}$$



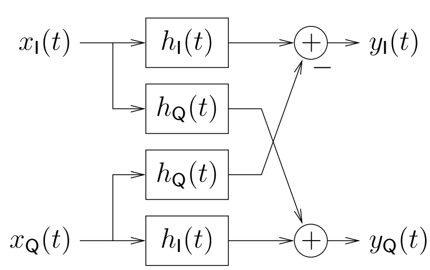
Baseband equivalent filtering:

$$\text{If} \left\{ \begin{array}{ccc} X_c(f) & \leftrightarrow & X_z(f) \\ Y_c(f) & \leftrightarrow & Y_z(f) \\ H_c(f) & \leftrightarrow & H_z(f) \end{array} \right\},$$

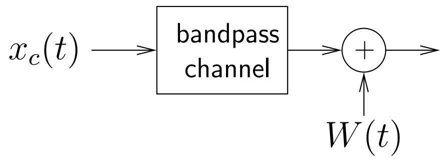
then

$$Y_c(f) = H_c(f)X_c(f) \;\leftrightarrow\; Y_z(f) = H_z(f)X_z(f).$$

Can show:



Passband additive white Gaussian noise model [Ch. 3]:



We assume $W(t)$ is zero-mean stationary Gaussian with

$$\begin{aligned} R_W(\tau) &= \mathrm{E}\{W(t)W(t-\tau)\} \quad \text{autocorrelation} \\ S_W(f) &= \mathcal{F}\{R_W(\tau)\} \quad \text{power spectrum} \end{aligned}$$

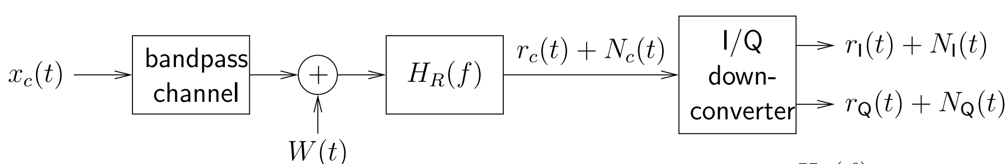
We also assume a constant PSD (i.e., "white noise"):

$$S_W(f) = \frac{N_0}{2}$$

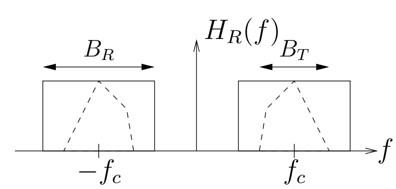
Thus

$$R_W(\tau) = \frac{N_o}{2}\delta(\tau), \qquad \sigma_W^2 = R_W(0) = \infty$$

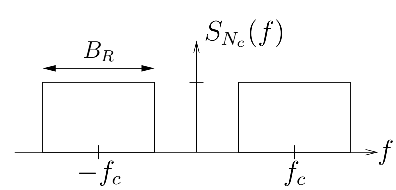
Received noise model [Ch. 4]:



Here $H_R(f)$ is the receive filter:



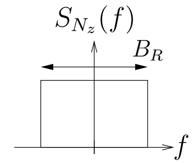
The passband noise spectrum is



$$S_{N_c}(f) = \frac{N_o}{2}|H_R(f)|^2$$

Baseband equivalent noise model:

- Say $\begin{cases} N_c(t) &= \sqrt{2}\mathbb{R}\Big[N_z(t)e^{j2\pi f_c t}\Big] \\ N_z(t) &= N_\mathsf{I}(t) + jN_\mathsf{Q}(t) \end{cases}$



- Fitz shows that $N_\mathsf{I}(t)$ and $N_\mathsf{Q}(t)$ are zero-mean, jointly stationary and jointly Gaussian with

$$R_{N_\mathsf{I}}(\tau) = R_{N_\mathsf{Q}}(\tau) \quad \text{and} \quad R_{N_\mathsf{I}N_\mathsf{Q}}(\tau) = -R_{N_\mathsf{I}N_\mathsf{Q}}(-\tau)$$

- Thus

$$R_{N_\mathsf{I}N_\mathsf{Q}}(0) = 0 \quad \Rightarrow \quad N_\mathsf{I}(t_o) \perp\!\!\!\perp N_\mathsf{Q}(t_o) \text{ for any } t_o$$

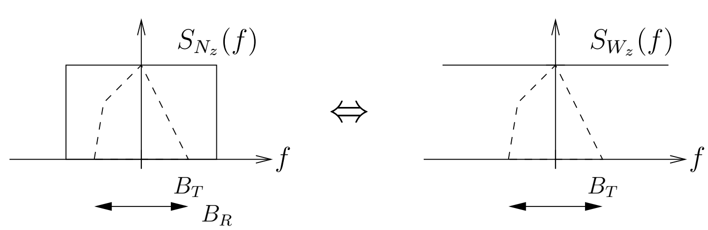
and

$$S_{N_z}(f) = 2\underbrace{S_{N_\mathsf{I}}(f)}_{\text{even}} - j2\underbrace{S_{N_\mathsf{I}N_\mathsf{Q}}(f)}_{\text{odd}}$$

Complex white noise model [Ch. 5]:

- With flat, unity-gain receive filter and $B_R > B_T$, can approximate $N_z(t)$ by <u>complex white Gaussian noise</u> $W_z(t)$ with statistics given by

$$S_{W_z}(f) = N_o \quad \Leftrightarrow \quad R_{W_z}(\tau) = N_o\delta(\tau)$$



- Note that:

$$S_{N_c}(f) = \frac{1}{2}S_{N_z}(f - f_c) + \frac{1}{2}S_{N_z}(-f - f_c)$$