HOMEWORK ASSIGNMENT #6

Due Wed. May 26, 2004 (in class)

1. Poor, 4.12
2. Poor, 4.21 (a)-(c)
3. Assume that $X_1, \cdots, X_n$ are i.i.d. random variables that are uniformly distributed on $[0, \theta]$. Assume that θ is unknown, but deterministic.
   (a) Show that $T(\underline{x}) = \max(x_i)$ is a sufficient statistic for θ.
   Hint: when you write out $p_\theta(\underline{x})$ be sure to include the valid regions of support using indicator functions.
   (b) Find $p_\theta(T)$.
   (c) Is T a complete sufficient statistic?
   (d) What is the mean of T?
   (e) Determine the MVUE of θ; justify why it is the MVUE.
   (f) What is the variance of the MVUE?
4. **Computer Exercise**
   (a) Simulate the performance of the MVUE above for $\theta = 4.3$. Consider $n_{\max} = 500$ and average 50 runs for each realization of n samples. Plot the empirical mean and the variance (using `semilogy`) of the estimator as a function of n. Superimpose the theoretically calculated variance for comparison.
   (b) Simulate the performance of another unbiased estimator for θ by noticing that $\mathrm{E}\{\sum_{i=1}^{n} X_i\} = \frac{n}{2}\theta$. Calculate and plot the mean and variance of this estimator as in part (a). Derive and plot the theoretically calculated variance for comparison.
   (c) Comment on the results and explain any differences in performance you observe.

   Please include your annotated code.