HOMEWORK ASSIGNMENT #2

Due Wed. Apr. 14, 2004 (in class)

1. **Computer Exercise**

   In this problem you will visually verify properties of the Bayes risk function $V(\pi_0)$. Once you have written your code, you should be able to investigate a variety of system parameters. You are encouraged to play with different system parameters.

   Consider the Location Testing problem with Gaussian error. Assume that $\mu_0 = -1, \mu_1 = 5, \sigma^2 = 4$. Furthermore, assume that you have the following cost matrix:

   $$C = \begin{bmatrix} 0.1 & 1.0 \\ 0.8 & 0.25 \end{bmatrix}$$

   (a) Give an expression (or collection of expressions) for the Bayes risk function $V(\pi_0)$.

   (b) Plot this function and verify that it is concave and that the end points ($V(\pi_0 = 0)$ and $V(\pi_0 = 1)$) of the function are the predicted values.

   (c) Determine (numerically if need be) the least favorable prior probability π_L. For π_L, evaluate $R_0(\delta_{\pi_L})$ and $R_1(\delta_{\pi_L})$ where δ_{π_L} is the Bayes rule for π_L. What is the difference between these two conditional risk values? Do you have an equalizer rule?

   (d) Plot $r(\pi_0, \delta_{\pi_0'})$ versus π_0 for $\pi_0' \in \{\pi_L, 0.5, 0.8\}$. Superimpose these plots over a plot of $V(\pi_0)$.

   (e) Discuss your findings.

   Note that what you will turn in will be a well-labeled plot, the values of π_L, $R_0(\delta_{\pi_L})$, and $R_1(\delta_{\pi_L})$, and a concise paragraph describing your findings.

2. [Poor: II.2(b,c)] Suppose Y is a random variable that, under hypothesis H_0, has pdf

   $$p_0(y) = \begin{cases} (2/3)(y+1) & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   and, under hypothesis H_1, has pdf

   $$p_1(y) = \begin{cases} 1 & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   (b) Find the minimax rule and minimax risk for uniform costs.

   (c) Find the NP rule and the corresponding detection probability for false-alarm probability $\alpha \in (0, 1)$.

3. [Poor: II.6(b,c)] Repeat the previous exercise for the hypothesis pair

   $$\begin{aligned} H_0 &: Y = N - s \\ H_1 &: Y = N + s \end{aligned}$$

where $s > 0$ is a fixed real number and N is a continuous random variable with density

$$p_N(n) = \frac{1}{\pi(1+n^2)}, \quad n \in \mathbb{R}$$

4. [Poor: II.16] Generalize the Bayesian hypothesis testing method to $M > 2$ hypotheses. Provide extended definitions for Γ_i, $C_{i,j}, \delta(y), R_j(\delta), \pi_j$, and $r(\delta)$, and formulate the optimal decision rule.

Note: Some of the problems above require solving a quadratic, thus giving two possible solutions. In such cases, only one of the solutions may be valid. Clearly explain your reasoning when discarding a solution.