EE-806 **Detection and Estimation Theory** **Spring 2004**

Homework #1 Apr. 2, 2004

HOMEWORK ASSIGNMENT #1

Due Wed. Apr. 7, 2004 (in class)

This problem set has two parts. The first part is designed to give you a feel for the hypothesis testing problem through simulations. The second set of problems are more classical, mathematical type problems. They will enable you to go through the details of designing optimal Bayesian decision rules for some interesting probability density functions. Peculiar probability density functions are considered so that you will have exposure to some of the tricky aspects of decision rule design (non-contiguous decision regions for example).

1. Computer Exercise 1

   Your task is to generate a binary decision rule for some sonar data that has been collected. You are trying to classify underwater objects from the Titanic. You are going to design the initial system which must simply distinguish between metal objects (H_1) and wooden objects (H_0). At your disposal are several data files: observations for H_1 only, observations for H_0 only and a test file containing observations from both H_1 and H_0 as well as the true source information.

   On the course Homework web page, you'll find a pointer to an ascii file called `hw1dat`. This file has four columns: data from H_0, data from H_1, source information for test data, and test data.

   (a) Design **two** possible decision rules. Play with the data to determine what might be good decision rules. (I found it instructive to plot approximate pdfs using the `hist` command.)

   (b) For each decision rule, use the test data to determine the four possible $P_i(\Gamma_j)$.

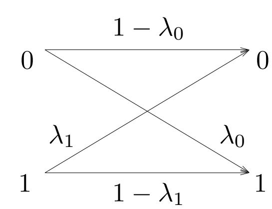
   (c) Estimate the prior probabilities and compute the Bayes risk for your two decision rules. Assume uniform costs.

   (d) Which rule is better and why?

   (e) Comment on your findings.

2. Computer Exercise 2

   The binary communications channel is a common toy model used to study simple communications systems. The system works as follows:



   Let π_1 be the prior probability that a "1" is transmitted, so that $\pi_0 = 1-\pi_1$ is the prior probability that a "0" is transmitted. Let λ_1 be the probability that a transmitted "1" is flipped by the channel, and λ_0 the probability that a transmitted "0" is flipped by the channel.

On the course Homework web page, you'll find the Matlab routine `binchan.m`. This Matlab file simulates the binary channel. The input parameters are π_0, λ_1 and λ_0. The outputs are transmitted bits and observed bits, respectively.

(a) Consider the following system parameters:

i. $\pi_0 = 0.3, \lambda_1 = 0.1, \lambda_0 = 0.2$

ii. $\pi_0 = 0.3, \lambda_1 = 0.7, \lambda_0 = 0.2$

iii. $\pi_0 = 0.8, \lambda_1 = 0.5, \lambda_0 = 0.6$

iv. $\pi_0 = 0.5, \lambda_1 = 0.5, \lambda_0 = 0.5$

Compute the probability of error of the **four** possible deterministic decision rules (i.e., always choose 0, always choose 1, keep observation, reverse observation). Also determine the probability of false alarm for these four rules.

(b) In many applications, you want to be able to achieve a particular value of false alarm $P_F \in (0,1)$. For the binary channel scenario, this implies that you would have to consider a *randomized* rule. The idea is that for the randomized rule, you flip an unfair coin to determine what your decision will be. Compute the probability of error and the false alarm rates for the following rules as a function of q. Consider all four sets of input parameters.

A. $\delta(1) = 1, \quad \delta(0) = \begin{cases} 1 & \text{with probability } \; q \\ 0 & \text{with probability } \; 1-q \end{cases}.$

B. $\delta(0) = 0, \quad \delta(1) = \begin{cases} 1 & \text{with probability } \; q \\ 0 & \text{with probability } \; 1-q \end{cases}.$

Comment on your findings.

NOTE: You will soon be able to easily derive all of these quantities requested above, but for now, we'll just focus on computing the values from experimental data.

3. (Poor: Chapter 2, Problem 1.) Find the minimum Bayes risk for the binary channel example discussed in the lecture. Assume uniform costs but general π_0, λ_0, λ_1.

4. (Poor: Chapter 2, Problem 2(a).) Suppose Y is a random variable that, under hypothesis H_0, has pdf

$$p_0(y) = \begin{cases} (2/3)(y+1) & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

and, under hypothesis H_1, has pdf

$$p_1(y) = \begin{cases} 1 & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the Bayes rule and minimum Bayes risk for testing H_0 versus H_1 with uniform costs and equal priors.

5. (Poor: Chapter 2, Problem 6(a).) Repeat the previous exercise for the hypothesis pair

$$\begin{aligned} H_0 &: \; Y = N - s \\ H_1 &: \; Y = N + s \end{aligned}$$

where $s > 0$ is a fixed real number and N is a continuous random variable with density

$$p_N(n) = \frac{1}{\pi(1+n^2)}, \quad n \in \mathbb{R}$$

6. Choose a broad research topic area. This might be your current or proposed area of research or simply an area that fascinates you. Describe briefly this area of research (3-4 sentences). List **two** hypothesis testing problems of interest to your specific area of research. What are the usual assumptions? Are there any difficulties associated with testing for these particular hypotheses? Be creative.