Quiz 1

Handed out Monday, March 31, 1997
Due in lecture, Monday, April 7, 1997

This is a take-home quiz covering review material from the topics studied in EE 804. This exam is **completely open book**. You may use any external materials you wish. Do pay attention to the *support* of the relevant probability density functions, this is very important for this course.

Please note how much time it took you to complete the exam.

After completing your test, copy the following pledge onto the front page of your exam and sign it *if it applies*: **No aid given, received, or observed**.

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1. Stark & Woods **Problem 2.15**:
   Consider a communication channel corrupted by noise. Let X be the value of the transmitted signal and Y be the value of the received signal. Assume that the conditional density of Y given $\{X = x\}$ is Gaussian, i.e.,

   $$f_{Y|X}(y|x) \;=\; \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(y-x)^2}{2\sigma^2}\right),$$

   and X is uniformly distributed on $[-1, 1]$. What is the conditional probability density of X given Y (i.e., $f_{X|Y}(x|y)$)?

2. Stark & Woods **Problem 2.17**:
   Let X be a random variable with pdf

   $$f_X(x) \;=\; \begin{cases} 0, & x < 0 \\ ce^{-2x}, & x \geq 0 \end{cases} \quad (c > 0)$$

   (a) Find c;
   (b) Let $a > 0, x > 0$, find $P[X \geq x + a]$;
   (c) Let $a > 0, x > 0$, find $P[X \geq x + a | X \geq a]$.

3. We'll be doing some simulations of several detection schemes in future homeworks. Matlab does not have the cumulative distribution function for a standard Gaussian random variable ($\mathcal{N}(0, 1)$) given by

   $$\Phi(x) \;=\; P[X < x] = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp -\frac{y^2}{2} dy$$

   But, it does have the *error function*, defined as,

   $$\text{erf}(x) \;=\; \int_{0}^{x} \frac{2}{\sqrt{\pi}} \exp -y^2 dy$$

   Write out $\Phi(x)$ in terms of erf(x).

4. **Multi-variate Gaussians**

(a) Let $\underline{X}$ be a random vector (of dimension $N \times 1$) with the Gaussian density, $\mathcal{N}(\underline{m}, C)$. What is the mean vector and covariance matrix of $A\underline{X} + \underline{b}$, where A is a constant matrix of dimension $N \times N$ and $\underline{b}$ is a constant vector of dimension $N \times 1$.

(b) Let $p_i(\underline{Y})$ be the Gaussian multivariate density $\mathcal{N}(\underline{s}_i, \Sigma)$ where $i = 1, 2$. Determine the ratio of the two densities : $\frac{p_1(\underline{Y})}{p_2(\underline{Y})}$. **Simplify the expression as much as possible.** There should be **no ratios** in the answer except for $\frac{1}{2}$. Assume that $\underline{s}_1 \neq \underline{s}_2$.

5. Assume you had a likelihood ratio test of the form:

$$(y-7)^2 \geq \tau \rightarrow \text{choose } H_1 \qquad -\infty \leq y \leq \infty$$

Describe the decision regions Γ_1 and Γ_0 as functions of subsets of the real line.

Now consider

$$\ln(y-7) \geq \tau \rightarrow \text{choose } H_1 \qquad -\infty \leq y \leq \infty$$

Describe the decision regions Γ_1 and Γ_0 as functions of subsets of the real line.

6. Based on this exam, what topics in EE 804 are important for understanding the concepts in EE 806?

7. Describe briefly your proposed (or current) area of research (3-4 sentences). List **two** hypothesis testing problems of interest to your specific area of research (or a topic you are interested in if you are not doing research right now). What are the usual assumptions? Are there any difficulties associated with testing for these particular hypotheses?

   I realize that most of you are not working directly on problems that involve hypothesis testing. Be creative! Within your area of research (or an area that you are interested in) make up two relevant problems.