(3) We want that $\forall \varepsilon > 0$, $P_r(|X_n - \mu| > \varepsilon) \to 0$.
From Chebyshev inequality, $P(|X_n - \mu| > \varepsilon) \leq \frac{var(X_n)}{\varepsilon^2} = \frac{\sigma_n^2}{\varepsilon^2}$

Since $\sigma_n^2 \to 0$, for any $\varepsilon > 0$: $\frac{\sigma_n^2}{\varepsilon^2} \to 0$. Thus $P(|X_n - \mu| > \varepsilon) \to 0$. $\forall \varepsilon > 0$

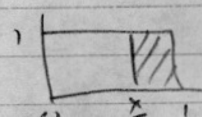
(4) a) When $\omega = 1 \in \Omega$, can see that $X_n(\omega) = a_n$ $\forall n$.
Since $a_n \to \infty$, $X_n(1) \to \infty$ and we claim $X_n \not\xrightarrow{s} 0$

b) When $\omega \in A \triangleq \{\omega \in \Omega : \omega \neq 1\}$, we claim
$\forall \varepsilon > 0$ $\exists N$ s.t. $\forall n > N$, $|X_n(\omega)| < \varepsilon$
since choosing $N = \lceil \frac{1}{1-\omega} \rceil$ makes $X_n(\omega) = 0$ $\forall n > N$.
Noting $P_r(A) = 1$, we find that $X_n(\omega) \to 0$ for a probability-one set of $\omega \in \Omega$. Thus $X_n \xrightarrow{as} 0$

c) Here, since $\Omega = [0, 1)$, we can find finite $N = \lceil \frac{1}{1-\omega} \rceil$ for any $\omega \in \Omega$ for which $X_n(\omega) = 0$ $\forall n > N$.
Thus $X_n \xrightarrow{s} 0$.

(5) $F_{nY_n}(x) = P_r[nY_n \leq x] = 1 - P_r[nY_n > x] = 1 - P_r[Y_n > \frac{x}{n}]$
$= 1 - P_r[\min(X_1, \cdots X_n) > \frac{x}{n}] = 1 - P_r[X_1 > \frac{x}{n}, X_2 > \frac{x}{n}, \cdots, X_n > \frac{x}{n}]$
$= 1 - P_r[X_k > \frac{x}{n}]^n$ since independent

$$= \begin{cases} 1 - (1 - x/n)^n, & 0 \leq \frac{x}{n} < 1 \\ 1, & 1 \leq \frac{x}{n} \\ 0, & \frac{x}{n} \geq 0 \end{cases}$$



Since $\lim_{n\to\infty} 1 - (1 - \frac{x}{n})^n = 1 - e^{-x}$ and $\lim_{n\to\infty} \{x : 0 \leq \frac{x}{n} < 1\} = \mathbb{R}^+$
we find

$$F_{nY_n}(x) \to \begin{cases} 1 - e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases} \triangleq F_Y(x)$$

Thus $nY_n \xrightarrow{d} Y$, where Y is given by above F_Y