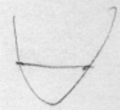

(6) Know $E(X_n-X)^2 \to 0$ or $E|X_n-X|^2 \to 0$

a) Want to show $E(X_n-X) \to 0$.

Since $E|X_n-X|^2 \to 0$, $\sqrt{E|X_n-X|^2} \to 0$.

Using Jensen's Ineq. with convex $g(y) = |y|^2$, $E|X_n-X|^2 \geq |E(X_n-X)|^2$

thus $\sqrt{E(X_n-X)^2} \geq |E(X_n-X)|$

Since $|E(X_n-X)|$ is bounded from above by a sequence $\to 0$,
and bounded from below by zero,

$$|E(X_n-X)| \to 0$$
$$\Rightarrow E(X_n-X) \to 0$$

Note: The absolute value signs are needed here. Why?

Say $E(Y_n) \geq E(Z_n)$ and $E(Y_n) \to 0$

Could still have $E(Z_n) \to -\infty$, or any other negative # !

b) Want to show $E(X_n)^2 \to E(X)^2$

Know $E(X_n)^2 - 2E(XX_n) + E(X)^2 = E(X_n-X)^2 \to 0$.

So if we can show that $E(XX_n) \to E(X)^2$, this would
imply that $E(X_n)^2 \to E(X)^2$

Showing $E(XX_n) \to E(X)^2$ equivalent to showing $E(X(X_n-X)) \to 0$

Use Schwarz Inequality : $E^2[XY] \leq E[X^2]E[Y^2]$

$$E^2[X(X_n-X)] \leq E[X^2]\,E[(X_n-X)^2]$$
$$|E[X(X_n-X)]| \leq \underbrace{\sqrt{E[X^2]}}_{\text{finite}} \cdot \underbrace{\sqrt{E[(X_n-X)^2]}}_{\to 0}$$

Thus since $|E(X(X_n-X))|$ bounded above by convergent sequence
and below by zero,

$$|E(X(X_n-X))| \to 0$$
$$\Rightarrow E(X(X_n-X)) \to 0$$

To review, the last statement implies $E(XX_n) \to E(X)^2$
and we know that $E(X_n)^2 + E(X)^2 \to 2E(XX_n)$
Thus we must have that $E(X_n)^2 \to E(X)^2$.