4) Pairwise independent $\{X_i\}_{i=1}^{n}$ imply $\mathrm{cov}(X_k X_\ell) = 0 \quad \forall\, k \neq \ell$ since $\{X_i\}$ are jointly Gaussian. But this implies that the $n \times n$ covariance matrix $K_{\underline{X}\underline{X}}$ is diagonal, since every off-diagonal term is of the form $\mathrm{cov}(X_k X_\ell)\big|_{k \neq \ell}$. Finally, for Gaussian random vector $\underline{X} = (X_1\ X_2 \cdots X_n)^t$ with diagonal covariance matrix, $\{X_i\}_{i=1}^{n}$ are independent.

5) $$E\left[\|\sum_i \underline{X}_i\|^2\right] = E\left[\left(\sum_i \underline{X}_i\right)^H \left(\sum_j \underline{X}_j\right)\right] = \sum_i \sum_j \underbrace{E\left[\underline{X}_i^H \underline{X}_j\right]}_{=0 \text{ when } i \neq j \text{ since orthogonal}}$$

$$= \sum_i E\left[\underline{X}_i^H \underline{X}_i\right] = \sum_i E\left[\|\underline{X}_i\|^2\right]$$

6) $$\hat{S}_{mmse} = E[S \mid R = r] = \int_{-\infty}^{\infty} s\, f_{S|R}(s|r)\, ds$$

From a previous homework, know

$$f_{S|R}(s|r) = \frac{\delta(s-1)}{1+\exp\left(-\frac{2r}{\sigma^2}\right)} + \frac{\delta(s+1)}{1+\exp\left(\frac{2r}{\sigma^2}\right)}$$

So

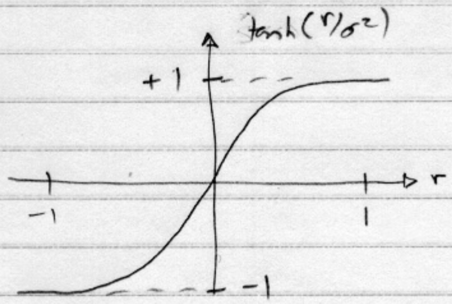
$$\hat{S}_{mmse}(r) = \frac{1}{1+\exp\left(-\frac{2r}{\sigma^2}\right)} - \frac{1}{1+\exp\left(\frac{2r}{\sigma^2}\right)}$$

using "sifting property": $\int_{-\infty}^{\infty} g(s)\,\delta(s-a)\, ds = g(a)$

Can rewrite

$$\hat{S}_{mmse}(r) = \frac{\exp\left(\frac{r}{\sigma^2}\right) - \exp\left(-\frac{r}{\sigma^2}\right)}{\exp\left(-\frac{r}{\sigma^2}\right) + \exp\left(\frac{r}{\sigma^2}\right)} = \tanh\left(r/\sigma^2\right)$$

Roughly...



(See plots on following pages...)