4) $E[X-Y^{10}|Y] = E[X|Y] - E[Y^{10}|Y] = E[X|Y] - Y^{10}$

But $E[X-Y^{10}|Y] = E[X-Y^{10}]$ due to independence

So $E[X|Y] = Y^{10} - E[X-Y^{10}|Y] = Y^{10} - \underbrace{E[X-Y^{10}]}_{c}$

Since $E[X-Y^{10}]$ is not a function of X nor Y.

5) $E[XZ|Y] = E[X|Y,Z]\,E[Z|Y] = E[X|Y]\,E[Z|Y]$

6) $\Phi_{X_i}(\omega) = \sum_i e^{j\omega i} p_X(i) = pe^{j\omega} + 1-p \qquad \forall i$

$\Phi_Y(\omega) = \Phi_{X_i}^n(\omega) = (pe^{j\omega}+1-p)^n = \sum_{k=0}^{n} \binom{n}{k} (pe^{j\omega})^k (1-p)^{n-k}$ via binomial series

$= \sum_{k=0}^{n} \underbrace{\binom{n}{k} p^k (1-p)^{n-k}}_{\text{pmf}} e^{j\omega k}$

Thus $p_Y(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & 0 \le k \le n \\ 0 & \text{else} \end{cases}$

7) $Y = \sum_{i=1}^{n} a_i Y_i \Rightarrow \Phi_Y = \prod_{i=1}^{n} \Phi_{a_i Y_i}(\omega) = \prod_{i=1}^{n} \Phi_{X_i}(a_i\omega)$

$\ln \Phi_Y = \sum_{i=1}^{n} \ln \Phi_{X_i}(a_i\omega) = \sum_{i=1}^{n} \sum_{n=0}^{\infty} K_{X,n} \frac{(ja_i\omega)^n}{n!}$

$= \sum_{n=0}^{\infty} \left(\sum_{i=1}^{n} a_i^n K_{X,n} \right) \frac{(j\omega)^n}{n!}$

Thus $K_{Y,n} = K_{X,n} \sum_{i=1}^{n} a_i^n$

8) $f_{X_1,X_2,\dots,X_n}(x_1,x_2,\dots,x_n) = f_{X_1|X_2\dots X_n}(x_1|x_2\dots x_n)\, \underbrace{f_{X_2\dots X_n}(x_2,\dots,x_n)}_{\text{examine}\dots}$

$f_{X_2,X_3\dots X_n}(x_2 x_3 \dots x_n) = f_{X_2|X_3\dots X_n}(x_2|x_3\dots x_n)\, \underbrace{f_{X_3\dots X_n}(x_3,\dots,x_n)}_{\text{examine}\dots}$

can see the pattern

$f_{X_k,X_{k+1}\dots X_n}(x_k x_{k+1} \dots x_n) = f_{X_k|X_{k+1}\dots X_n}(x_k|x_{k+1}\dots x_n)\, f_{X_{k+1}\dots X_n}(x_{k+1}\dots x_n)$

so by induction

$f_{X_1 X_2 \dots X_n}(x_1 x_2 \dots x_n) = f_{X_1|X_2\dots X_n}(x_1|x_2\dots x_n)\, f_{X_2|X_3\dots X_n}(x_2|x_3\dots x_n) \cdots$
$f_{X_k|X_{k+1}\dots X_n}(x_k|x_{k+1}\dots x_n) \cdots f_{X_n}(x_n)$