$$f_R(r) = \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} s^{-1} \exp\left(-\frac{1}{2\sigma^2}\frac{r^2}{s^2}\right) \frac{\delta(s-1)}{2}\, ds + \int_{-\infty}^0 \frac{1}{\sqrt{2\pi\sigma^2}} s^{-1} \exp\left(-\frac{1}{2\sigma^2}\frac{r^2}{s^2}\right) \frac{\delta(s+1)}{2}\, ds$$

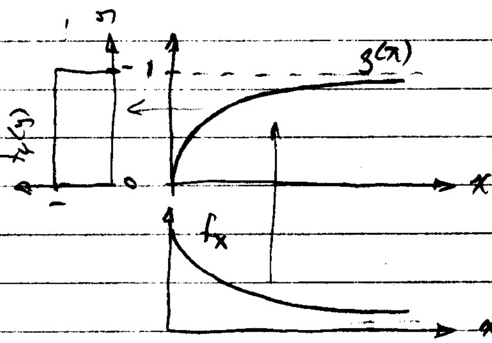
$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2} r^2\right) \cdot \frac{1}{2} + \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2} r^2\right) \cdot \frac{1}{2}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2} r^2\right) \quad \sim \mathcal{N}(0, \sigma^2)$$

b) R and S are independent, since $f_{R|S}(r|s) = f_R(r) \sim \mathcal{N}(0,\sigma^2)$

c) R and H are dependent, since $f_{R|H}(r|h) = \frac{1}{2}\delta(r-h) + \frac{1}{2}\delta(r+h) \neq f_R(r)$

5) $f_X(x) = \begin{cases} e^{-x} & \text{for } x \geq 0 \\ 0 & \text{else} \end{cases}$ $\quad f_Y(y) \sim \mathcal{U}[0,1)$



Guess something like

- let's try $g(x) = 1 - e^{-x}$
- Root of $0 = 1 - e^{-x} - y$ is $x_1 = -\ln(1-y)$ and is unique.

  Note that $x \geq 0 \Rightarrow 0 \leq y < 1$
- Then

$$f_Y(y) = f_X(x_1)\,|g'(x_1)|^{-1} \quad \text{for } x_1 = -\ln(1-y),\ 0 \leq y < 1$$

$$= e^{-x_1}\,|e^{-x_1}|^{-1}$$

$$= (1-y)\,|1-y|^{-1} \quad 0 \leq y < 1$$

$$= 1 \quad 0 \leq y < 1 \quad ✓$$