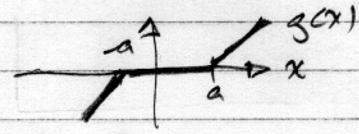
EE804 HW #3 SOLUTIONS P. SCHNITER

1) For this $g(x)$, we have



a) So for $y=0$, $P[Y=y] = P[-a \le X \le a]$

For $y<0$, there exists one root of $g(x)=y$, at $x=y-a$ and $g'(x)=x$, so that $f_Y(y) = f_X(y-a)$. Similar for $y>0$

Thus

$$f_Y(y) = \begin{cases} P[-a \le X \le a] & y=0 \\ f_X(y+a) & y>0 \\ f_X(y-a) & y<0 \end{cases}$$

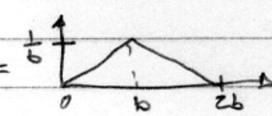
b) For $y<0$, $F_Y(y) = \int_{-\infty}^{y} f_X(\tilde{y}-a)\,d\tilde{y} = \int_{-\infty}^{y-a} f_X(z)\,dz = F_X(y-a)$

At $y=0$, $F_Y(y) = P[-a \le X \le a] + F_X(-a) = F_X(a)$

For $y>0$, $F_Y(y) = F_X(y+a)$

c) When $f_X(x) = \frac{1}{2}\delta(x-2a) + \frac{1}{2}\delta(x+2a)$, part (a) implies $f_Y(y) = \frac{1}{2}\delta(y-a) + \frac{1}{2}\delta(y+a)$

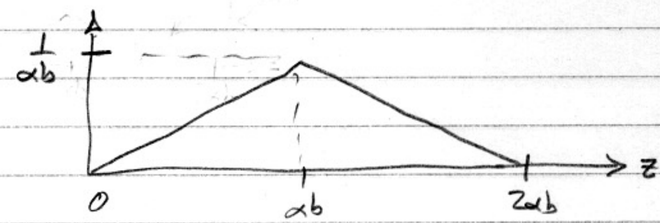
2) $Z = \alpha(X_1+X_2)$. If $Y = X_1+X_2$, we know $f_Y = f_{X_1} * f_{X_2} =$



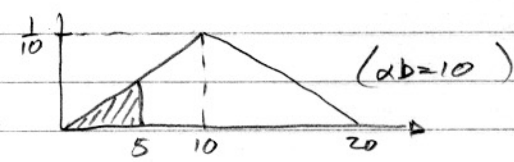
Then since $Z = \alpha Y$, we know $f_Z(z) = \frac{1}{|\alpha|} f_Y(z/\alpha)$.

Putting these together,

$$f_Z(z) = \begin{cases} \frac{z}{\alpha^2 b^2} & 0 \le z < \alpha b \\ \frac{2}{\alpha b} - \frac{z}{\alpha^2 b^2} & \alpha b \le z < 2\alpha b \\ 0 & \text{else} \end{cases}$$



b) $P[Z>5] = 1 - P[Z \le 5] = 1 - F_Z(5)$ where



$(\alpha b = 10)$

$$= 1 - \int_{-\infty}^{5} f_Z(z)\,dz = 1 - \int_{0}^{5} \frac{z}{10^2}\,dz = 1 - \frac{1}{2}\frac{z^2}{100}\Big|_{z=0}^{z=5}$$

$$= 1 - \tfrac{1}{8} = \boxed{7/8}$$

3) When $S=s$, $R = s+N$ and thus $R \sim N(s, \sigma^2)$

$$f_{R|S}(r|s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(r-s)^2\right)$$