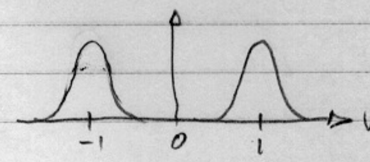
b) $f_R(r) = \int_{-\infty}^{\infty} f_{R|S}(r|s)\, f_S(s)\, ds = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(r-s)^2\right)\left[\frac{1}{2}\delta(s-1) + \frac{1}{2}\delta(s+1)\right] ds$

$$= \frac{1}{2\sqrt{2\pi\sigma^2}}\left[\underbrace{\exp\left(-\frac{1}{2\sigma^2}(r-1)^2\right)}_{N(1,\sigma^2)} + \underbrace{\exp\left(-\frac{1}{2\sigma^2}(r+1)^2\right)}_{N(-1,\sigma^2)}\right]$$



c) $f_{S|R}(s|r) = \dfrac{f_{R|S}(r|s)\, f_S(s)}{f_R(r)}$

where

$$f_{R|S}(r|s)\, f_S(s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(r-s)^2\right)\left[\frac{1}{2}\delta(s-1) + \frac{1}{2}\delta(s+1)\right]$$

$$= \frac{1}{2\sqrt{2\pi\sigma^2}}\left[\exp\left(-\frac{1}{2\sigma^2}(r-1)^2\right)\delta(s-1) + \exp\left(-\frac{1}{2\sigma^2}(r+1)^2\right)\delta(s+1)\right]$$

thus

$$f_{S|R}(s|r) = \frac{\exp\left(-\frac{1}{2\sigma^2}(r-1)^2\right)\delta(s-1) + \exp\left(-\frac{1}{2\sigma^2}(r+1)^2\right)\delta(s+1)}{\exp\left(-\frac{1}{2\sigma^2}(r-1)^2\right) + \exp\left(-\frac{1}{2\sigma^2}(r+1)^2\right)}$$

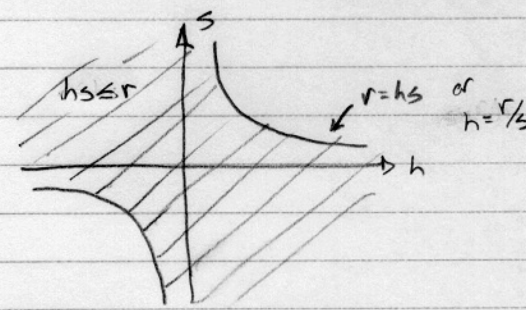
$$= \frac{\delta(s-1)}{1+\exp\left(\frac{(r-1)^2}{2\sigma^2} - \frac{(r+1)^2}{2\sigma^2}\right)} + \frac{\delta(s+1)}{1+\exp\left(\frac{(r+1)^2}{2\sigma^2} - \frac{(r-1)^2}{2\sigma^2}\right)}$$

$$= \frac{\delta(s-1)}{1+\exp\left(-\frac{2r}{\sigma^2}\right)} + \frac{\delta(s+1)}{1+\exp\left(+\frac{2r}{\sigma^2}\right)}$$

d) when $\sigma = 1$ $\quad P[S=1 \mid R=1] = \dfrac{1}{1+\exp(-2)} = 0.8808$

$$P[S=-1 \mid R=1] = \frac{1}{1+\exp(2)} = 0.1192$$

4) $F_R(r) = \displaystyle\int_0^{\infty}\int_{-\infty}^{r/s} f_H(h)\, f_S(s)\, dh\, ds + \int_{-\infty}^{0}\int_{r/s}^{\infty} f_H(h)\, f_S(s)\, dh\, ds$ $\quad$ since



where $f_H(h) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{h^2}{2\sigma^2}\right)$, $f_S(s) = \frac{1}{2}\delta(s-1) + \frac{1}{2}\delta(s+1)$

$$F_R(r) = \int_0^{\infty}\int_{-\infty}^{r/s} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{h^2}{2\sigma^2}\right)\frac{\delta(s-1)}{2}\, dh\, ds + \int_{-\infty}^{0}\int_{r/s}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{h^2}{2\sigma^2}\right)\frac{\delta(s+1)}{2}\, dh\, ds$$

Next we use Leibnitz Rule to find $f_R(r) = \dfrac{dF_R(r)}{dr}$

(over)