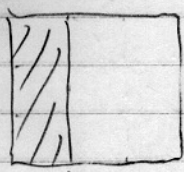
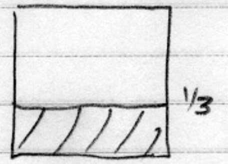
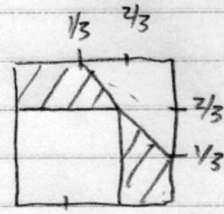
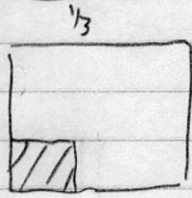
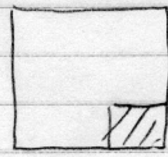
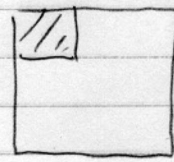
EE804 HW #2 SOLUTIONS P. SCHNITER

) Say A =  , B =  C = 

AB =  BC =  AC =  $ABC = \phi$

$P[AB] = P[BC] = P[CA] = 1/9$, $P[ABC] = 0$

) Let $f(\omega) = \omega$ for $\omega = 1, 2, 3, \ldots, n$, noting $\omega \in \Omega$ and $\omega \in \mathbb{R}$
Then $f^{-1}(1) = 1$, but $1 \notin \mathcal{F}$ so f non-measurable.

) $\Omega = \{$ HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT $\}$

$X(\omega) = 0$ for $\omega =$ HHHH
$= 1$ for $\omega =$ HHHT, HHTH, HTHH, THHH
$= 2$ for $\omega =$ HHTT, HTHT, THHT, HTTH, THTH, TTHH
$= 3$ for $\omega =$ HTTT, THTT, TTHT, TTTH
$= 4$ for $\omega =$ TTTT

b) $P[X=0] = (0.2)^4 = 0.0016$
$P[X=1] = (0.8)^1 (0.2)^3 \binom{4}{1} = 0.0256$
$P[X=2] = (0.8)^2 (0.2)^2 \binom{4}{2} = 0.1536$
$P[X=3] = (0.8)^3 (0.2)^1 \binom{4}{3} = 0.4096$
$P[X=4] = (0.8)^4 = 0.4096$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 0.0016 & 0 \le x < 1 \\ 0.0272 & 1 \le x < 2 \\ 0.1808 & 2 \le x < 3 \\ 0.5904 & 3 \le x < 4 \\ 1.0000 & 4 \le x \end{cases}$$

c) In general, $P[X=k] = (0.8)^k (0.2)^{n-k} \binom{n}{k}$ $F_X(x) =$ 0 $x < 0$
$k \le x < k+1$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \sum_{k=0}^{m} (0.8)^k (0.2)^{n-k} \binom{n}{k} & m \le x < m+1 \quad \text{for } m \in \mathbb{Z},\ 0 \le m < n \end{cases}$$