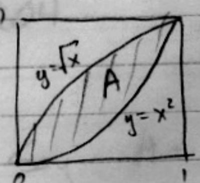
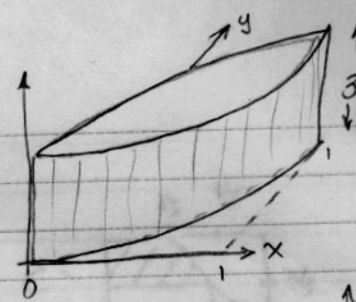
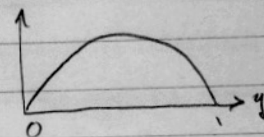
7) 

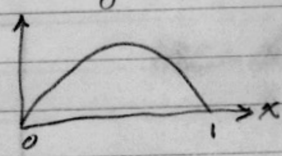
$$f_{XY}(x,y) = \begin{cases} 3 & (x,y)\in A \\ 0 & \text{else} \end{cases}$$



b) $f_Y(y) = \int_0^1 f_{XY}(x,y)\,dx = \int_{y^2}^{\sqrt{y}} 3\,dx = 3x\Big|_{y^2}^{\sqrt{y}} = 3\sqrt{y} - 3y^2$



$f_X(x) = \int_0^1 f_{XY}(x,y)\,dy = \int_{x^2}^{\sqrt{x}} 3\,dy = 3y\Big|_{x^2}^{\sqrt{x}} = 3\sqrt{x} - 3x^2$



8) $X \sim N(-1,2)$.

$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\,dt = \frac{2}{\sqrt{2\pi\sigma^2}} \int_{\sqrt{2}\sigma x+\mu}^\infty e^{-\frac{(y-\mu)^2}{2\sigma^2}}\,dy$ via $t = \frac{y-\mu}{\sqrt{2}\sigma}$, $dt = (\sqrt{2}\sigma)^{-1}dy$

$\Rightarrow \frac{1}{2}\operatorname{erfc}\left(\frac{z-\mu}{\sqrt{2}\sigma}\right) = \Pr[Z \geq z]$ for $Z \sim (\mu,\sigma^2)$

a) $\Pr[X \geq 0] = \frac{1}{2}\operatorname{erfc}\left(\frac{0-(-1)}{\sqrt{2}\cdot\sqrt{2}}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{1}{2}\right) = 0.2398$

b) $\Pr[X < -0.3] = 1 - \Pr[X \geq -0.3] = 1 - \frac{1}{2}\operatorname{erfc}\left(\frac{-0.3-(-1)}{\sqrt{2}\cdot\sqrt{2}}\right) = 0.6897$

c) $\Pr[|X| \leq 0.5] = \Pr[X \geq -0.5] - \Pr[X \geq 0.5] = 0.2174$

d) $\Pr[|X| > 10] = 1 - \Pr[|X| \leq 10] = 1 - (\Pr[X \geq -10] - \Pr[X \geq 10]) = 7.9312 \times 10^{-11}$

9) Using Integration by Parts, $\int_x^\infty y^{-2} e^{-y^2/2}\,dy = \frac{1}{x} e^{-x^2/2} - \int_x^\infty e^{-y^2/2}\,dy$

Because $\int_x^\infty y^{-2} e^{-y^2/2}\,dy > 0$ (since $x>0$), we have $\int_x^\infty e^{-y^2/2}\,dy < \frac{1}{x} e^{-x^2/2}$

Also, $\int_x^\infty y^{-2} e^{-y^2/2}\,dy < \int_x^\infty x^{-2} e^{-y^2/2}\,dy$, so that $(1+x^{-2})\int_x^\infty e^{-y^2/2}\,dy > \frac{1}{x} e^{-x^2/2}$

$\Leftrightarrow \int_x^\infty e^{-y^2/2}\,dy > \frac{x}{1+x^2} e^{-x^2/2}$