EE804 HW # SOLUTIONS P. SCHNITER

SW-1.9a) Ω and ϕ are disjoint, hence

$$P[\Omega \cup \phi] = P[\Omega] + P[\phi] \quad \text{by (1.4-3)}$$
$$= 1 + P[\phi] \quad \text{by (1.4-2)}$$

But $\Omega \cup \phi = \Omega$ thus $P[\Omega \cup \phi] = P[\Omega] = 1$, so

$$1 = 1 + P[\phi]$$
$$0 = P[\phi]$$

b) $E = E \cap \Omega = E \cap (F \cup F^c) = EF \cup EF^c$

But $EF \cap EF^c = EFF^c = \phi$ so EF & EF^c are disjoint.

Thus
$$P[E] = P[EF \cup EF^c]$$
$$= P[EF] + P[EF^c] \quad \text{by (1.4-3)}$$

So
$$P[EF^c] = P[E] - P[EF]$$

c) $\Omega = E \cup E^c$ where $E \cap E^c = \phi$

Thus
$$P[\Omega] = P[E \cup E^c]$$
$$= P[E] + P[E^c] \quad \text{by (1.4-3)}$$
$$1 = P[E] + P[E^c] \quad \text{by (1.4-2)}$$
$$P[E] = 1 - P[E^c]$$

SW-1.12a) Call 1=cat, 2=dog, 3=goat, 4=pig

$\mathcal{F}$ consists of 16 events:

$\{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}$
$\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}, \phi$

(Note closure under unions, intersections, complements)

- $P[\{1,2\}] = P[\{1\}] + P[\{2\}]$ since $\{1\} \cap \{2\} = \phi$
  Thus $P[\{1\}] = P[\{1,2\}] - P[\{2\}] = 0.9 - 0.5 = 0.4$
- $P[\{3,4\}] = P[\{3\}] + P[\{4\}]$ since $\{3\} \cap \{4\} = \phi$
  Thus $P[\{3\}] = P[\{3,4\}] - P[\{4\}] = 0.1 - 0.05 = 0.05$
- Remaining probabilities can be easily calculated from $P[A \cup B] = P[A] + P[B]$ for $A \cap B = \phi$ ...