b) This time $\mathcal{F}$ consists of

$\{1\}, \{2\}, \{3,4\}, \{1,2\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}, \emptyset$

Again, note closure under unions, intersections, complements

As before, use $P[A \cup B] = P[A] + P[B]$ for $A \cap B = \emptyset$ to find probabilities...

SW 1.16)

$$P[X=1 \mid Y=1] = \frac{P[Y=1 \mid X=1]\, P[X=1]}{\sum_n P[Y=1 \mid X=n]\, P[X=n]} = \frac{P[Y=1 \mid X=1]\, P[X=1]}{\sum_n P[Y=1 \mid X=n]\,(n P[X=1])}$$

since $P[X=n] = n\,P[X=1]$

$$= \frac{P[Y=1 \mid X=1]}{\sum_n n\, P[Y=1 \mid X=n]}$$

$$= \frac{1-\alpha}{(1-\alpha) + (\beta/2)2 + (\gamma/2)3} = \frac{1-\alpha}{1-\alpha+\beta+3\gamma/2}$$

SW 1.24) For each BM, probability of a kill $= \underbrace{0.8 \cdot 0.2}_{\text{first AMM}} + \underbrace{0.8 \cdot 0.2}_{\text{2nd AMM}} + \underbrace{0.8 \cdot 0.8}_{\text{both AMMs}} = 0.96$

a) For all BMs to be destroyed, probability $= (0.96)^6 = 0.7828$

b) One getting through $\Leftrightarrow$ not all destroyed, so probability $= 1 - 0.78 = 0.2172$

c) Probability of n^{th} getting through with others destroyed $= (0.96)^5 (1-0.96)$

But there are $\binom{6}{1} = 6$ ways this could happen, so that

$$P = \binom{6}{1} (0.96)^5 (1-0.96) = 0.1957$$

SW 1.25) $\Pr[\text{exactly one BM lived} \mid \text{at least one BM lived}]$

$$= \frac{\Pr[\text{exactly one \underline{and} at least one}]}{\Pr[\text{at least one}]} = \frac{\Pr[\text{exactly one}]}{\Pr[\text{at least one}]} = \frac{0.1957}{0.2172} = 0.9008$$

SW 1.33)

$$\Pr[n_1 \text{ in } (0,t_1) \mid n_1+n_2 \text{ in } (0,T)] = \frac{P[n_1 \text{ in } (0,t_1) \text{ \& } n_1+n_2 \text{ in } (0,T)]}{P[n_1+n_2 \text{ in } (0,T)]}$$

$$= \frac{P[n_1 \text{ in } (0,t_1) \text{ \& } n_2 \text{ in } [t_1,T)]}{P[n_1+n_2 \text{ in } (0,T)]} = \frac{P[n_1 \text{ in } (0,t_1)]\, P[n_2 \text{ in } [t_1,T)]}{P[n_1+n_2 \text{ in } (0,T)]}$$

$$= \frac{e^{-\lambda t_1} \lambda^{n_1} t_1^{n_1}}{n_1!} \; \frac{e^{-\lambda(T-t_1)} \lambda^{n_2} (T-t_1)^{n_2}}{n_2!} \cdot \frac{(n_1+n_2)!}{e^{-\lambda T} \lambda^{n_1} \lambda^{n_2} T^{n_1+n_2}} = \frac{t_1^{n_1} (T-t_1)^{n_2}}{T^{n_1+n_2}} \frac{(n_1+n_2)!}{n_1!\, n_2!}$$