HOMEWORK ASSIGNMENT #6

Due Wed. Nov. 22, 2000 (in class)

Note: When in doubt over how to "prove" convergence, I suggest resorting to the definitions of the form "$\forall \epsilon > 0,\ \exists N$ s.t. $\forall n \geq N,\ |?| < \epsilon$" which were presented in class. To disprove something, it is sufficient to create a counterexample. You may find the five inequalities presented in class useful.

1. Say that R is a Rayleigh r.v. with parameter σ and Θ is a uniform r.v. on $[0, 2\pi)$. Show that $Z = Re^{j\Theta}$ (for $j = \sqrt{-1}$) is complex circular Gaussian when R and Θ are independent.

2. Consider two random sequences $\{X_n\}$ and $\{Y_n\}$. Suppose that $X_n \overset{ip}{\to} X$ and $Y_n \overset{ip}{\to} Y$. Prove that $X_n + Y_n \overset{ip}{\to} (X+Y)$.

3. Say $X_n \sim \mathcal{N}(\mu, \sigma_n^2)$ where $\sigma_n^2 \to 0$. Prove that $X_n \overset{ip}{\to} \mu$.

4. Consider $\omega \in \Omega = [0,1]$ with uniform probability measure, deterministic sequence $a_n \to \infty$, and random variables

$$X_n(\omega) = \begin{cases} 0 & \omega \in [0, 1-1/n), \\ a_n & \omega \in [1-1/n, 1]. \end{cases}$$

   (a) Disprove the claim $X_n \overset{s}{\to} 0$.

   (b) Prove $X_n \overset{as}{\to} 0$.

   Now make the change $\Omega = [0,1)$ and

$$X_n(\omega) = \begin{cases} 0 & \omega \in [0, 1-1/n), \\ a_n & \omega \in [1-1/n, 1). \end{cases}$$

   (c) Prove $X_n \overset{s}{\to} 0$.

5. Say X_n is a sequence of independent uniform r.v.s on $[0,1]$, and say that $Y_n = \min(X_1, X_2, \ldots, X_n)$. Show that $nY_n \overset{d}{\to} Y$ where Y is an exponential r.v. with parameter 1. [Hint: $(1 - y/n)^n \to e^{-y}$.]

6. Consider a sequence $\{X_n\}$ such that $X_n \overset{ms}{\to} X$.

   (a) Prove $E[X_n] \to E[X]$.

   (b) Prove $E[X_n^2] \to E[X^2]$.

   You can assume finite means and variances. [Hint: Be clever with the Schwarz inequality!]