HOMEWORK ASSIGNMENT #2

Due Fri. Oct. 13, 2000 (in class)

1. Let $\Omega = [0,1] \times [0,1]$, the unit square. Assume that for any event A, $P[A]$ is equal to the area of A. Can you specify A, B, C, three subsets of Ω, that are pairwise independent but for which $P[ABC] \neq P[A]P[B]P[C]$?

2. Consider $\Omega = \{1, 2, \ldots, n\}$. The set of events of interest is given by $\mathcal{F} = \big\{\emptyset, \Omega, \{1,2\}, \{3, 4, \ldots, n\}\big\}$. Construct a nonmeasurable function $f : \Omega \to \mathbb{R}$.

3. Suppose you flip a coin four times, and each time you observe the face of the coin.
   (a) Define the sample space Ω and a random variable X that counts the number of heads.
   (b) Calculate the CDF $F_X(x)$ for an unfair coin with $P[\text{heads}] = 0.8$.
   (c) Generalize (b) for n flips.

4. Consider the sample space $\Omega = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \leq 1\}$. For any subset $A \in \mathcal{F}$, we define its probability as the normalized area of A, or

$$P[A] = \frac{\text{area of } A}{\pi}.$$

   Consider the random variable X defined by $X(a, b) = \sqrt{a^2 + b^2}$.
   (a) Illustrate graphically the subset of Ω that corresponds to the event $\{X \leq x\}$.
   (b) Calculate and plot the CDF of X.

5. The arrival time of a professor to his office is a continuous r.v. uniformly distributed over the hour between 8am and 9am. Define the events:

$$\begin{aligned} A &= \{ \text{ the prof has not arrived by 8:30am}\}, \\ B &= \{ \text{ the prof will arrive by 8:31am}\}, \end{aligned}$$

   Find
   (a) $P[B|A]$.
   (b) $P[A|B]$.

6. Let $\Omega = [-10, 10]$.
   (a) Specify a continuous r.v. X on this sample space.
   (b) Specify a discrete r.v. Y.
   (c) Specify a mixed r.v. Z.

   Don't forget that you need to specify $\mathcal{F}$ in each case.

7. Define the subset A as the region of the unit square included between the curves $y = x^2$ and $y = \sqrt{x}$. Consider the random variables X, Y with joint pdf

$$f_{XY}(x, y) = \begin{cases} 3, & (x, y) \in A \\ 0, & \text{else.} \end{cases}$$

   (a) Plot the joint pdf.

   (b) Find and plot the marginal densities of X and Y.

8. Consider a Gaussian r.v. X with parameters $\mu = -1, \sigma^2 = 2$. Calculate the following probabilities

   (a) $P[X \geq 0]$.

   (b) $P[X < -0.3]$.

   (c) $P[|X| \leq 0.5]$.

   (d) $P[|X| > 10]$.

   Hint: use either `erf` or `erfc` in MATLAB, but first use the `help` command to see how they are specified.

9. Show that for all $x > 0$,

$$\frac{x}{1+x^2} e^{-\frac{x^2}{2}} \; < \; \int_x^\infty e^{-\frac{y^2}{2}} dy \; < \; \frac{1}{x} e^{-\frac{x^2}{2}}.$$