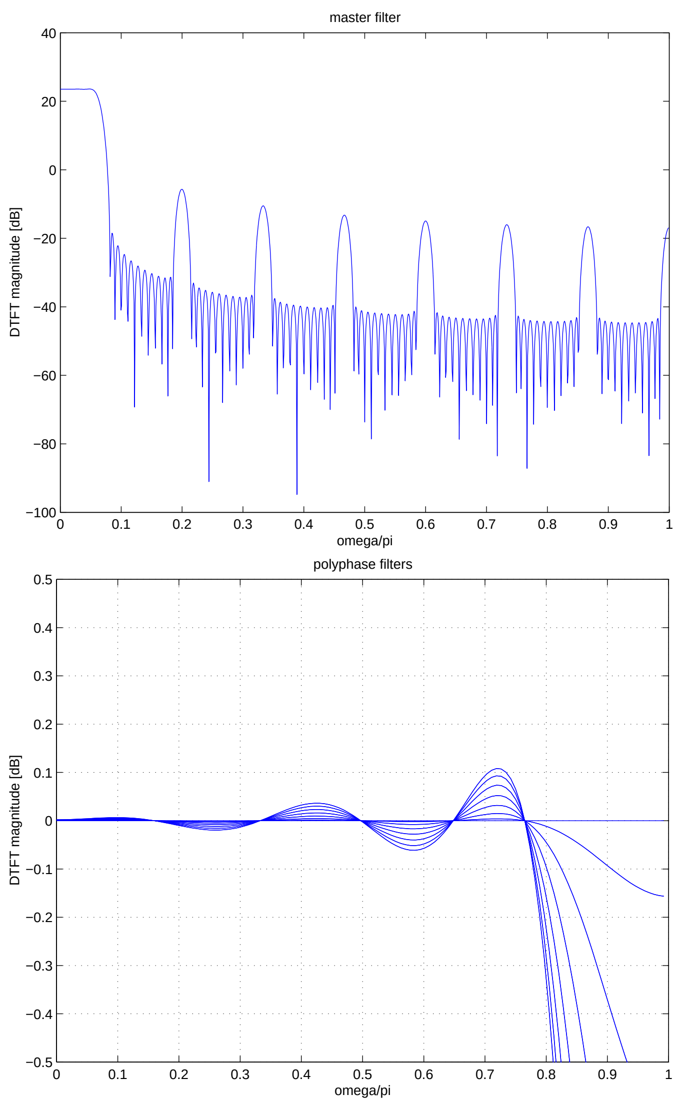
ECE-700 **Digital Signal Processing** **Winter 2007**

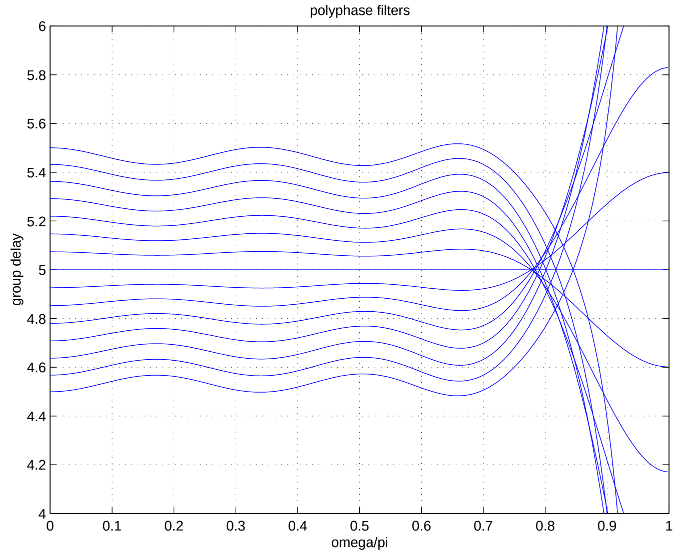
Homework #3 Jan. 24, 2007

HOMEWORK SOLUTIONS #3

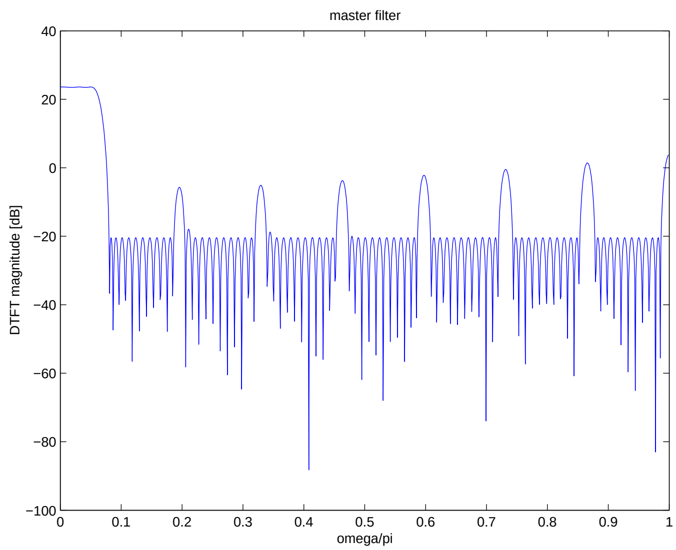
1. (a) Since we have an odd length ($NL = 165$) master filter and signal bandwidth of 0.8π, we have
   passband: $[0, 0.8)\frac{\pi}{L}$
   stopbands: $[2k - 0.8, 2k + 0.8)\frac{\pi}{L}$, for $k = 1 \ldots \frac{L-1}{2}$
   The remaining regions in $[0, \pi)$ are transition bands.

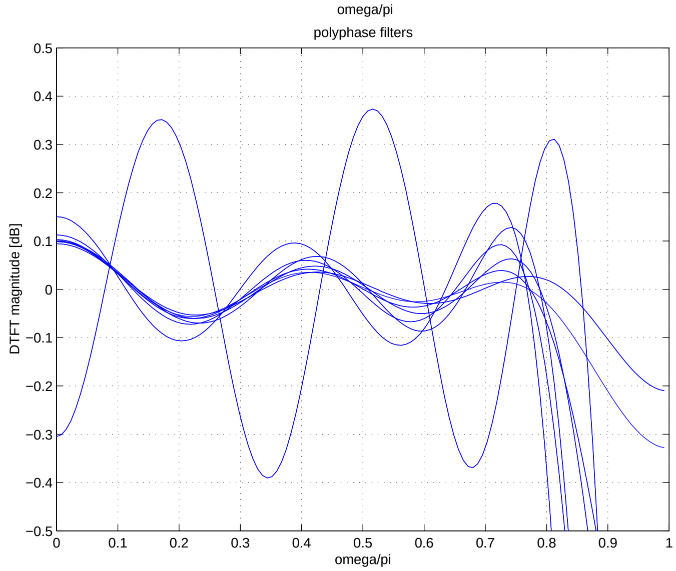
   (b) With the indirect least-squares (`firls`) polyphase design, we get

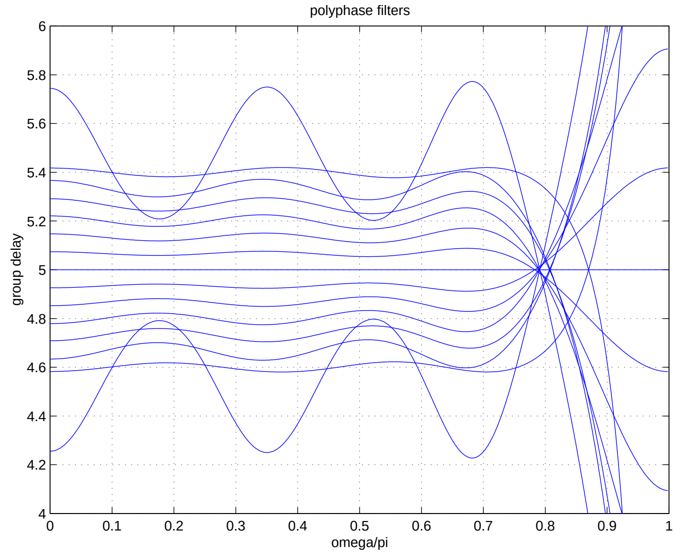




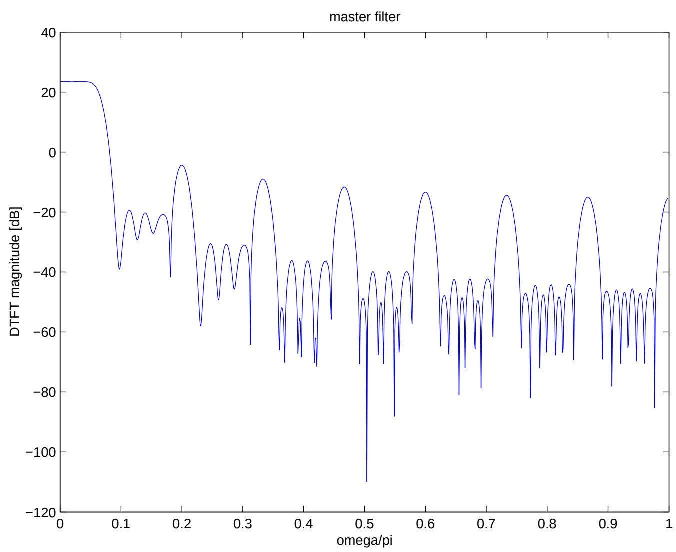
(c) With the indirect equi-ripple (`remez,firpm`) polyphase design, we get

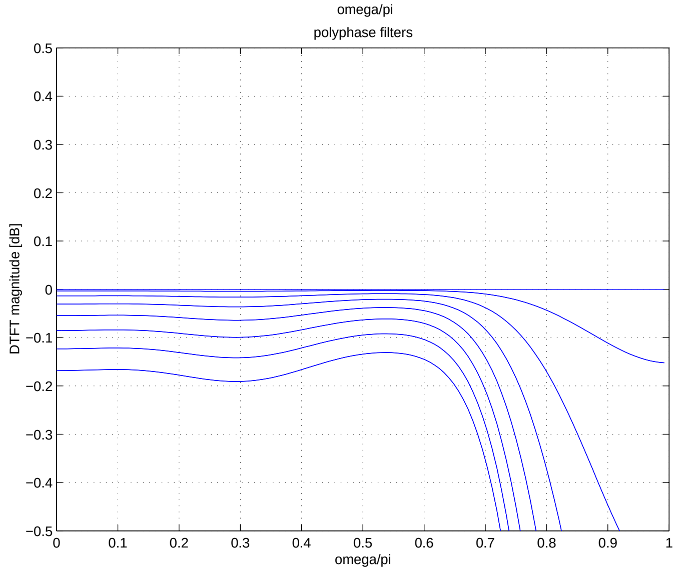


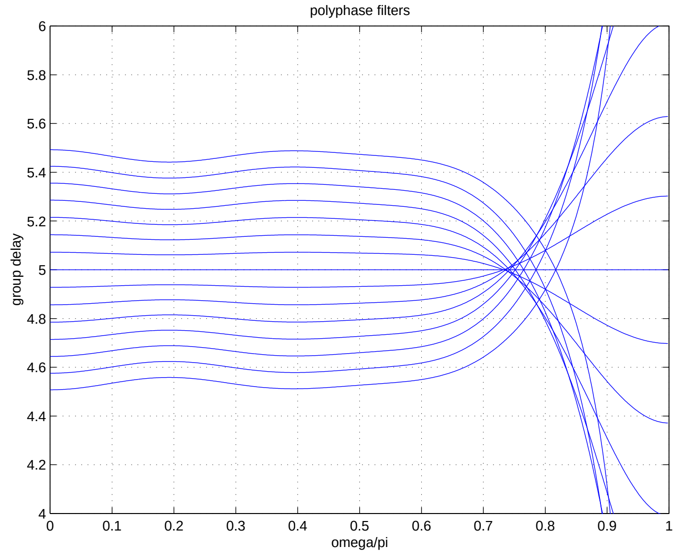




(d) With the direct Hamming-window polyphase design, we get

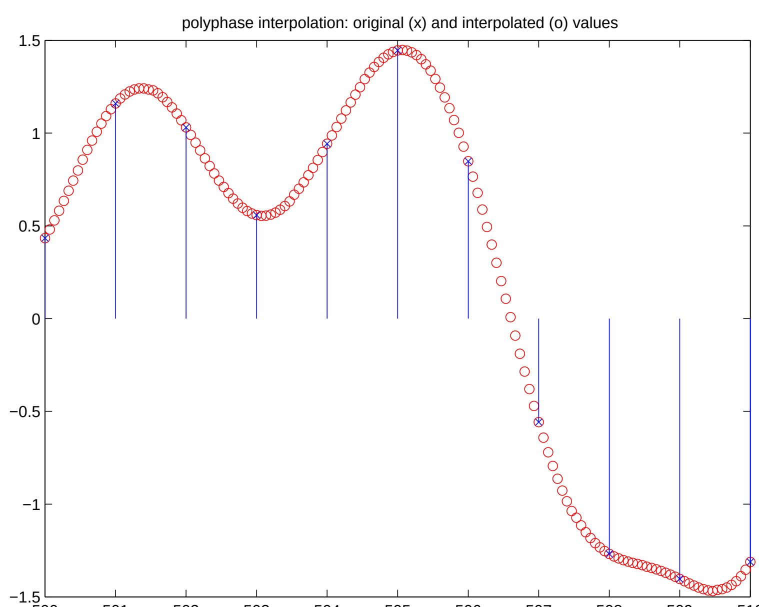




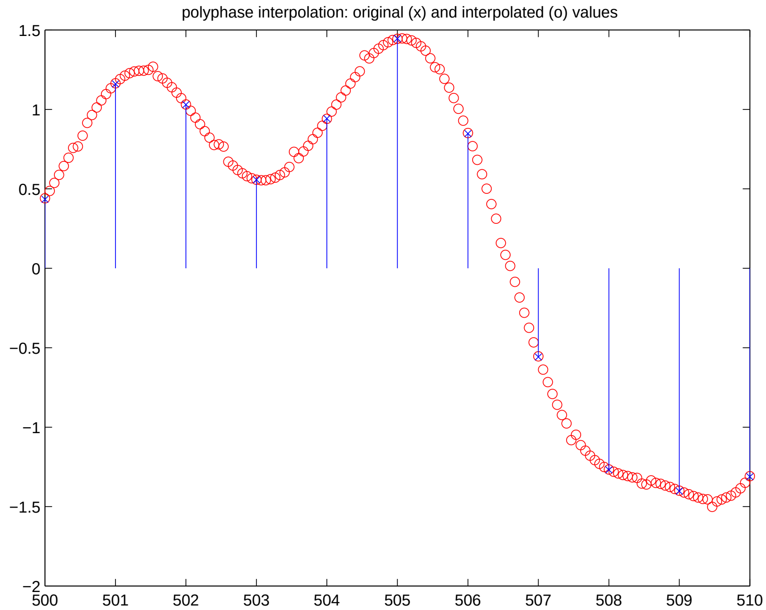


(e) The LS indirect design has the best looking plots. For example, the polyphase DTFT magnitude is far superior to either of the other two designs. The equi-ripple indirect design seems, at first glace, to have a decent master filter DTFT plot. When we look at the polyphase filters, however, we see poor performance. The direct Hamming-window design has, interestingly enough, a master filter DTFT response that is very similar to that of the LS *in*direct method: the transition bands are all in the right places! Its polyphase group delay plots look quite good, but the polyphase magnitude response starts to droop a bit for some filters. Thus, we expect that the indirect LS design will perform the best, the direct windowed design the second best, and the indirect equi-ripple design the worst. Note: one could work out a *direct* LS design that would perform as well as the indirect one, with the advantage of breaking up a potentially long filter design into a set of smaller ones.

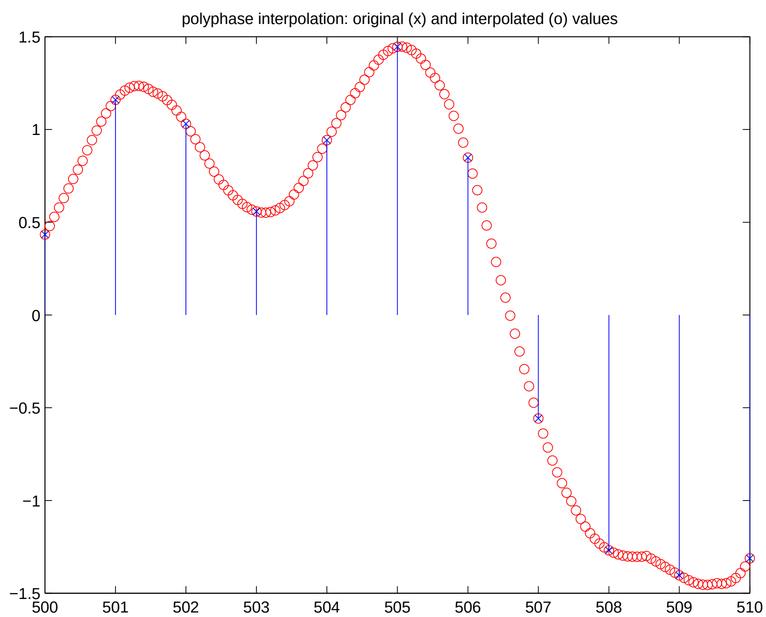
2. (b) The indirect LS design gives the following polyphase interpolation output



(c) The equi-ripple design gives the following polyphase interpolation output

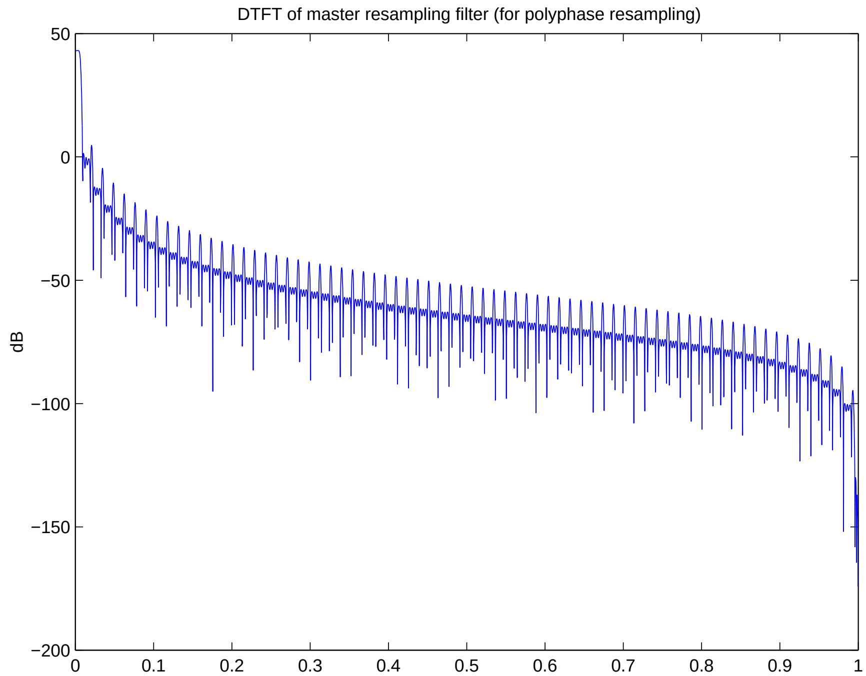


(d) The direct windowed design gives the following polyphase interpolation output

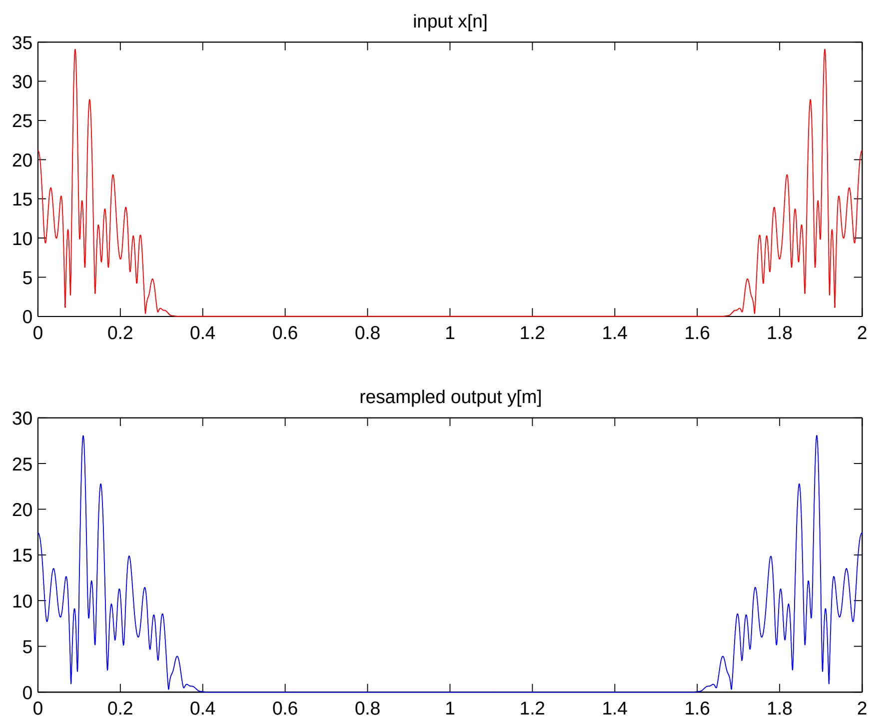


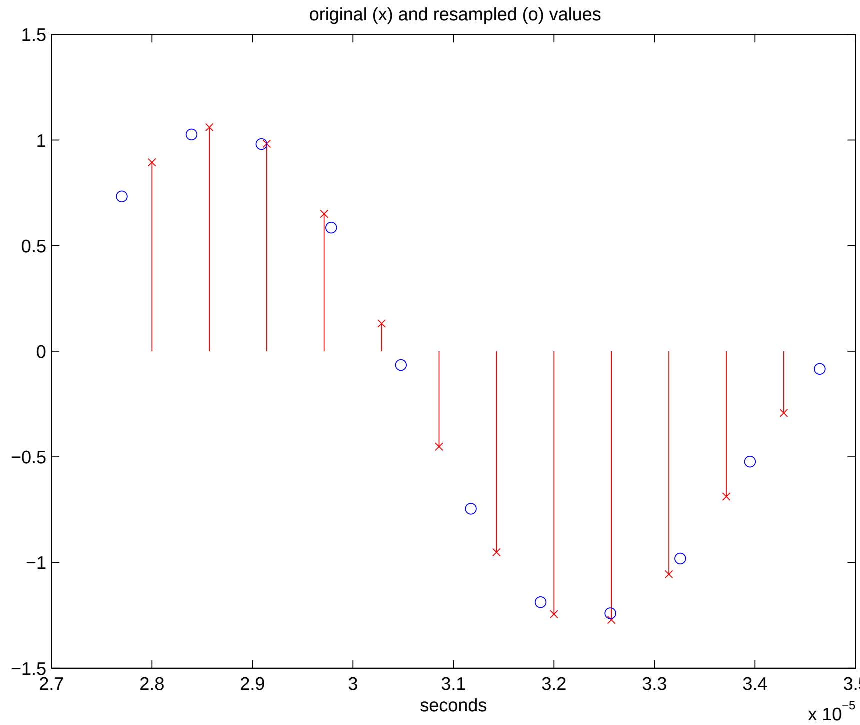
(e) The interpolation performance of the indirect LS design looks quite good, as we expect. Most of the equi-ripple interpolates look good, but every so often we get a terribly inaccurate value. Recalling that all but one pair of polyphase filters taken from the equi-ripple master filter had good group delay and magnitude responses, this makes sense. Finally, the windowed design looks good except for a couple places on the waveform, such as between 508 and 509. Here we see the effect of inaccurate passband gain in the polyphase components. (I used `randn('state',5)` to reset the random number generator so that I could compare each design on the same waveform.)

3. (b) Using the Hamming window direct design of 144 length-10 polyphase filters (the indirect designs were too computationally intensive for length 1440 master filter), we obtain the following DTFT magnitude:

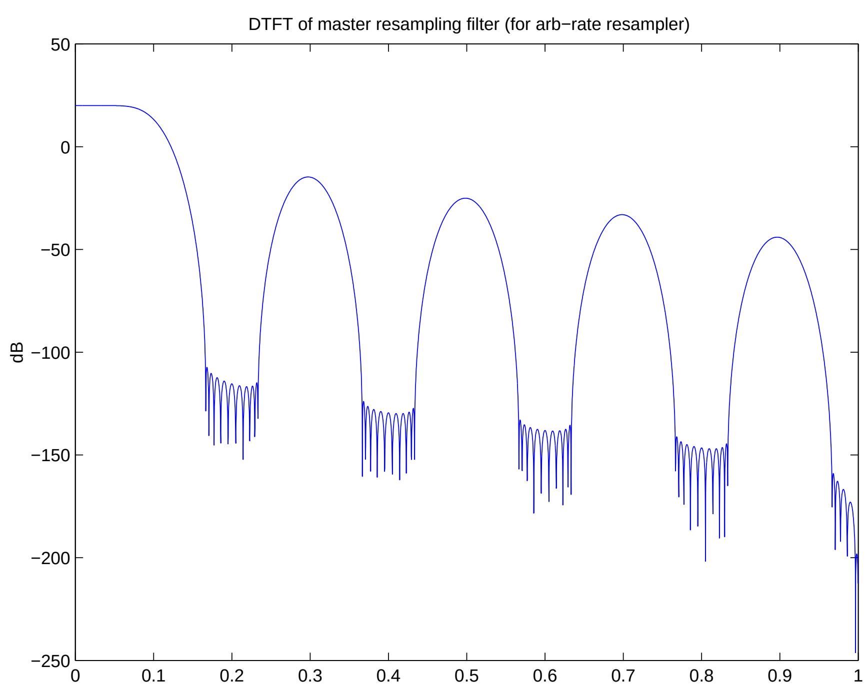


(c) The frequency-domain and time-domain effects of rational polyphase resampling by $\frac{144}{175}$ are illustrated below. Note the inaccuracy at time point 32.5 μs.





4. (b) Since the arbitrary-rate resampler uses only 10 length-10 polyphase filters, we can use the indirect LS filter design method to compute a length-100 master filter. The result is shown below. Note the excellent stopband attenuation that results from the generous transition bands.



(c) The frequency-domain and time-domain effects of arbitary-rate polyphase resampling by $\frac{144}{175}$ are illustrated below. Note the accuracy at time point 32.5 μs relative to the rational polyphase structure.