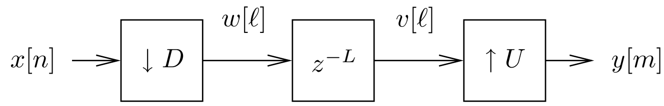
PRACTICE MIDTERM EXAMINATION

Name:

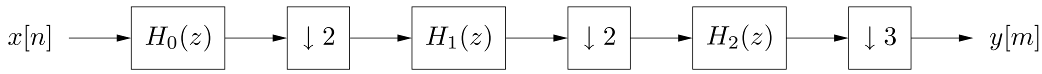
Instructions:

- Do not turn over this cover page until instructed to do so.
- You will have 120 minutes to complete this exam.
- You are allowed to consult the letter-sized piece of paper which you have prepared beforehand. You are not allowed to consult any other books, notes, or people.
- Please write clearly and include sufficient explanation with all of your answers.
- If you write on the backs of the pages, indicate this so the grader does not miss your work.
- Do not unstaple the test pages.

1. Given the following multirate system, derive an expression for the DTFT of $y[m]$ in terms of the DTFT of $x[n]$.



2. We are interested in decimating a real-valued input by factor 12 using the three-stage structure below. We want to prevent aliasing in the output spectrum up to bandwidth ω_o radians. Assume that the input is "fullband," i.e., has uniform spectral density.



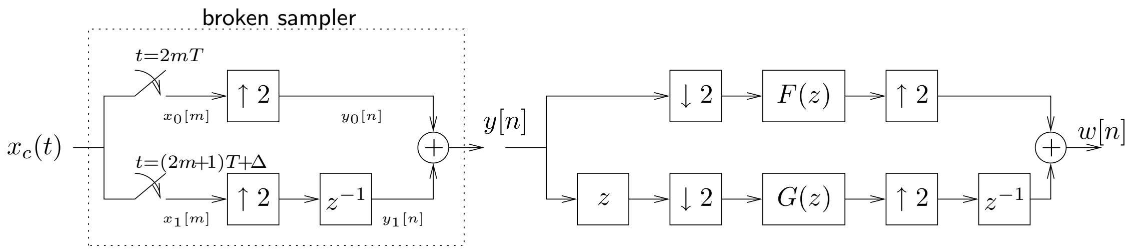
Specify the passband and stopband edges of $H_0(z)$, $H_1(z)$, and $H_2(z)$ which leave the widest possible transition band(s) to ensure the aliasing behavior described above. Summarize your answers in the table below. It is sufficient to specify the positive edge frequencies. (*Hint*: Consider the design of each stage separately.)

	passband edge	stopband edge(s)
$H_0(z)$		
$H_1(z)$		
$H_2(z)$		

3. Say that we are given a "broken" sampler that generates samples $y[n]$ of input signal $x_c(t)$ according to

$$y[n] \;=\; \begin{cases} x_c(nT) & \text{even integers } n, \\ x_c(nT+\Delta) & \text{odd integers } n, \end{cases}$$

where $\Delta \in (-T, T)$ is not necessarily an integer. Assume that $x_c(t)$ is bandlimited to $\frac{1}{2T}$ Hz. The structure in the figure below right is proposed to "fix" the sampler.



(a) Derive an expression for $Y(e^{j\omega})$, the DTFT of $y[n]$, in terms of $X_c(\Omega)$, the CTFT of $x(t)$.

(b) Evaluate your answer to (a) for the case $\Delta = 0$. Does it make sense?

(c) Show that the block diagram can be simplified such that the downsamplers (and several other elements) disappear.

(d) Suppose that $F(e^{j\omega}) = 1$ and $G(e^{j\omega}) = e^{-j\omega\frac{\Delta}{2T}}$ for $\omega \in [-\pi, \pi)$. Do these choices yield $w[n] = x_c(nT)$? In other words, have we fixed the broken sampler?

In answering the questions above, it may help to recall that $U(e^{j\omega})$, the discrete-time Fourier transform (DTFT) of $u[n]$, is related to $U_c(\Omega)$, the continuous-time Fourier transform (CTFT) of $u_c(t)$, as follows when $u[n] = u_c(nT)$:

$$U(e^{j\omega}) \;=\; \frac{1}{T}\sum_{k=-\infty}^{\infty} U_c\left(\frac{\omega - 2\pi k}{T}\right).$$