HOMEWORK ASSIGNMENT #7

Due Wed. Feb. 28, 2007 (in class)

1. Consider the wavelet series expansion of continuous-time signal $f(t) \in L_2$ using the Haar wavelet $\psi(t)$.

$$f(t) = \sum_{k,m} d_k[m]\, \psi_{k,m}(t)$$

   (a) Find the expansion coefficients when $f(t) = \phi(t)$, where $\phi(t)$ denotes the Haar scaling function.

   (b) For $f(t) = \phi(t)$, verify that $\sum_{k,m} |\ \langle \psi_{k,m}, f \rangle\ |^2 = 1$. (Parseval's identity for the wavelet series expansion.)

   (c) Consider $f(t) = \phi(t + 2^{-i})$ where i is a positive integer. Which coefficients are nonzero?

2. Consider a scaling function $\phi(t)$ such that $\{\phi(t-n), n \in \mathbb{Z}\}$ forms an orthonormal basis for V_0, and say that $\{h[n]\}$ are the coefficients satisfying the scaling equation $\phi(t) = \sqrt{2} \sum_n h[n]\phi(2t-n)$. Prove the following. (You can assume that all quantities are real-valued, but you cannot assume that $\phi(t)$ is the Haar scaling function.)

   (a) $\sum_n |h[n]|^2 = 1$.

   (b) $h[n] = \sqrt{2}\ \langle \phi(2t-n), \phi(t) \rangle$.

   (c) $\underbrace{\Phi(\Omega)}_{\text{CTFT}} = \frac{1}{\sqrt{2}} \underbrace{H(e^{j\Omega/2})}_{\text{DTFT}} \Phi(\Omega/2)$.

3. Using the same assumptions as the previous problem, as well as $0 < |\Phi(0)| < \infty$ and $\Phi(\Omega)$ continuous at $\Omega = 0$, prove the following.

   (a) $\sum_n h[n] = \sqrt{2}$

   (b) $\sum_n h[2n] = \sum_n h[2n+1]$