HOMEWORK ASSIGNMENT #6

Due Wed. Feb. 21, 2005 (in class)

1. Consider the following two definitions of a discrete-time STFT.

$$\begin{aligned} X(e^{j\omega}, n) &= \sum_{m=0}^{R-1} x[n-m]w[m]e^{j\omega m} \\ \bar{X}(e^{j\omega}, n) &= \sum_{m=-\infty}^{\infty} x[m]w[m-n+R-1]e^{-j\omega m}. \end{aligned}$$

Assume $w[m] \neq 0$ for $m = 0...R-1$ and $w[m] = 0$ otherwise, and assume $w[m] = w[R-1-m]$.

(a) Derive an expression for $\bar{X}(e^{j\omega}, n)$ in terms of $X(e^{j\omega}, n)$.

(b) Say that we are interested in evaluating the discrete-time STFT only at frequencies

$$\omega_k = \frac{2\pi k}{N}; \quad k = 0, \ldots, N-1, \quad \text{with} \quad N \geq R.$$

Which one of the STFTs above can be implemented as a bank of N LTI filters? Give expressions for the filter impulse responses.

(c) Define the discrete STFT as $X[k, n] = X(e^{j\frac{2\pi}{N}k}, n)$. Prove the inverse transform

$$x[n-p] = \frac{1}{Nw[p]} \sum_{k=0}^{N-1} X[k, n]e^{-j\frac{2\pi}{N}kp} \quad \text{for} \quad p = 0...R-1.$$

To reconstruct $x[n]$ for all n, do we really need access to $\{X[0, n], \ldots, X[N-1, n]\}$ for all n?

(d) Find the inverse transform for the discrete STFT $\bar{X}[k, n] := \bar{X}(e^{j\omega_k}, n)$.

2. Here we will experiment with a discrete STFT using an R-point window and computed at R frequency points:

$$X[k, n] = \sum_{m=0}^{R-1} x[n-m]w[m]e^{j\frac{2\pi k}{R}m}.$$

To save compute time, I suggest calculating $X[k, n]$ only at the times $n = 0, \frac{R}{2}, R, \frac{3R}{2}, 2R, \ldots$. (The previous problem should have convinced you that this sub-sampling is sufficient to capture all information in the input signal.)

(a) Assume $M = 1000$, $R = 32$, and a Hamming window. In Matlab, generate the linear chirp

$$x[n] = e^{j\omega_n n} \quad \text{for} \quad n = 0 \ldots M-1 \quad \text{with} \quad \omega_n = \frac{\pi}{M}n$$

and compute its discrete STFT. Plot the discrete STFT magnitude with time along the horizontal axis and frequency along the vertical axis using the `contour` command (with 20 contour lines). Is the slope of the line what you would expect?

(b) Repeat part (a) but with

$$\omega_n = \begin{cases} \frac{\pi}{M}n & n = 0 \ldots M/2-1 \\ \frac{\pi}{2} & n = M/2 \ldots M-1 \end{cases}$$

Can you explain the interesting results? (*Hint:* Perhaps "instantaneous frequency" = phase derivative.)

(c) Load the built-in Matlab speech signal by typing "`load mtlb`," and compare a time domain plot to its discrete STFT for $R = 32$ and $R = 256$. Comment on the tradeoff in time/frequency resolution.