ECE-700 **Digital Signal Processing** **Winter 2007**

Homework #5 Jan. 31, 2007

HOMEWORK ASSIGNMENT #5

Due Wed. Feb. 7, 2007 (in class)

1. *Polyphase/DFT Filterbank:*
   In this problem, you will derive the equivalence between the uniformly modulated filterbank in Fig. 1 and its polyphase/DFT implementation in Fig. 2. Assume that the impulse response lengths of $\bar{H}(z)$ and $H(z)$ both equal N. The impulse responses of $\bar{P}_\ell(z)$ and $P_\ell(z)$ are related to those of $\bar{H}(z)$ and $H(z)$ via $\bar{p}_\ell[m] = \bar{h}[mM+\ell]$ and $p_\ell[m] = h[mM+\ell]$.

   (a) Show the equivalence between the synthesis banks in Fig. 1 and Fig. 2. (*Hint*: reverse the procedure used in the lecture to study the analysis bank.)

   (b) Implement the filterbank pairs of Fig. 1 and Fig. 2 in Matlab using $M = 8$ branches, master filter length $N = 160$, and input created via `x = randn(1,300)`. Using the following impulse response for both $\bar{H}(z)$ and $H(z)$.

   ```
   h = firls(N,[0,.8/M,1.2/M,1],[sqrt(M),sqrt(M),0,0]); h=h(1:end-1);
   ```

   Note that the group delay of each filter is exactly $N/2$ samples. Plot the output from both filters, the delayed input $\bar{x}[n-N]$, and the reconstruction error $\bar{x}[n-N] - x[n]$, as done in Fig. 3.

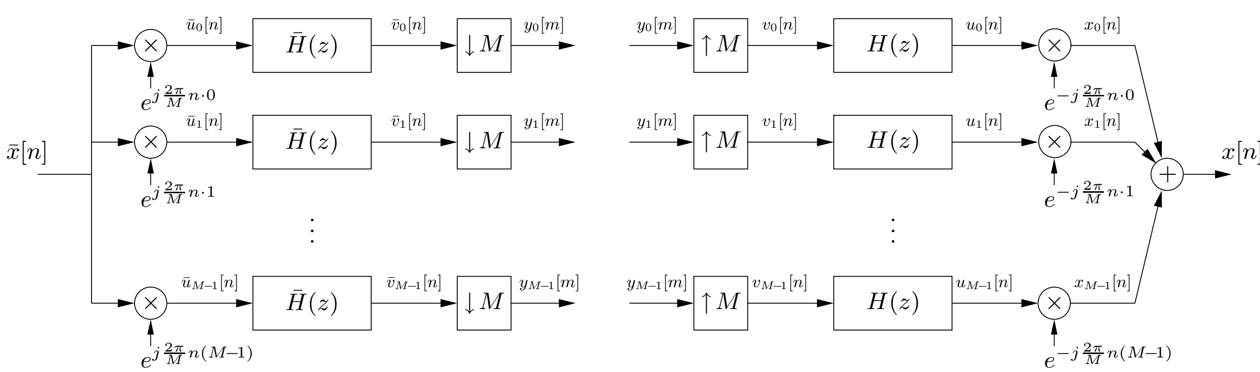


Figure 1: M-band uniformly-modulated analysis/synthesis filterbanks.

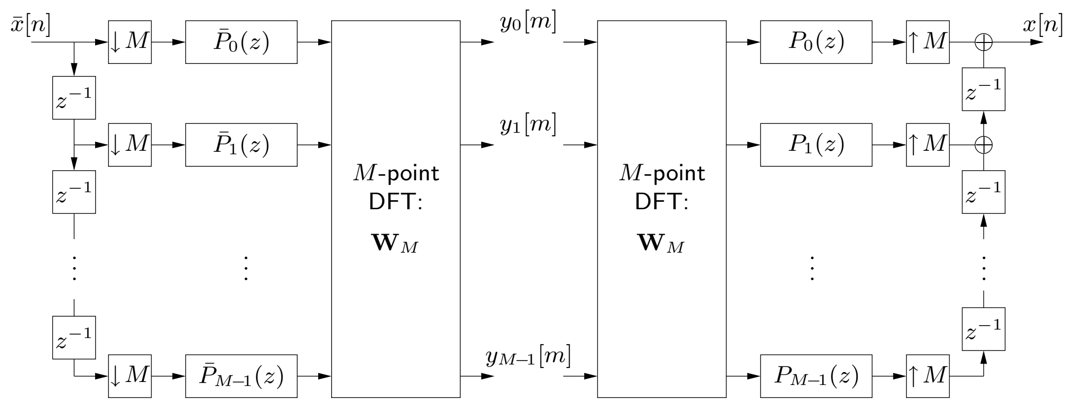


Figure 2: Polyphase/DFT implementation of M-band uniformly modulated analysis/synthesis filterbanks.

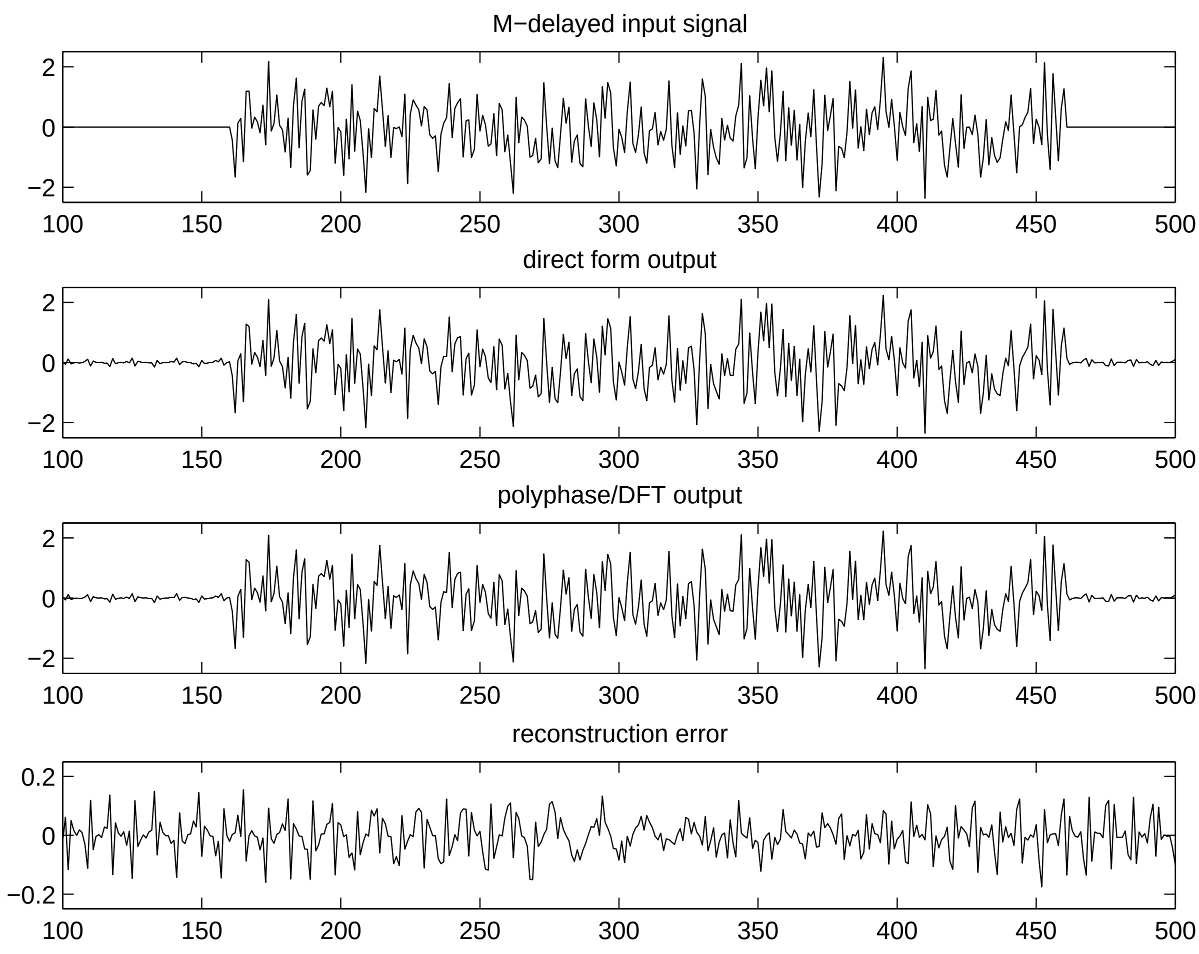


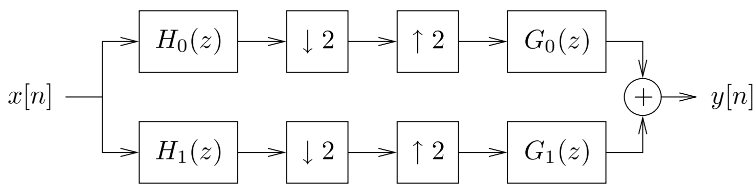
Figure 3: Matlab filterbank simulation outputs.

2. In this problem you will design a two-channel orthogonal perfect-reconstruction FIR filterbank. This involves the design of a prototype filter $H(z)$:

- Use a window (of your choice) to design the coefficients $q[n]$ of length-15 halfband filter:

$$Q(z) = \sum_{n=-7}^{7} q[n] z^{-n}.$$

- Find the minimum value attained by the (real-valued) DTFT $Q(e^{j\omega})$, denoted by $-\epsilon$.
- Using $q[n]$ and ϵ, create a raised halfband filter $f[n]$ with non-negative real DTFT.
- Collect the roots of $F(z)$ which have magnitude less than one, and form a new polynomial $\tilde{H}(z)$ using these roots. (There should be exactly 7 of them. Use `roots, poly` in Matlab.)
- Create $H(z)$, a scaled version of $\tilde{H}(z)$, so that $H(z)H(z^{-1}) + H(-z)H(-z^{-1}) = 1$. This is easily done by ensuring that $2\sum_n h^2[n] = 1$ (motivated by Parseval's theorem).



Using the prototype-filter design procedure above,

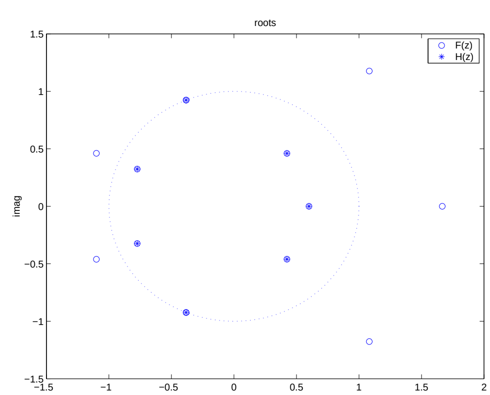
(a) Plot the roots of $F(z)$ and $H(z)$ superimposed on the unit circle in the complex plane.

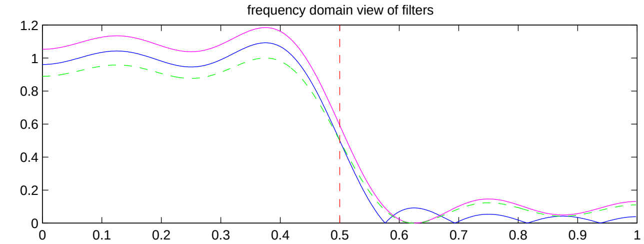
(b) Superimpose $Q(e^{j\omega})$, $F(e^{j\omega})$, and $H^2(e^{j\omega})$ on one plot. Also, show a zoomed view of the stopband to ensure that $F(e^{j\omega}) > 0$.

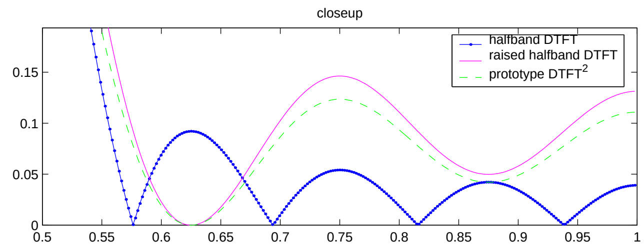
(c) Using $H(z)$, design the filters $H_0(z)$, $H_1(z)$, $G_0(z)$ and $G_1(z)$. Plot the DTFT magnitudes of the analysis filters, and show that they are "power complementary."

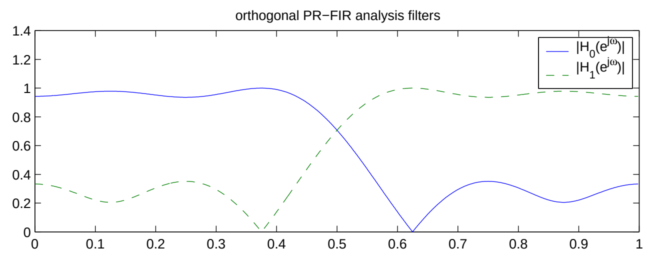
(d) Create $x[n]$ as a random sequence of length 200, and using the filters you designed generate output $y[n]$. Plot an appropriately delayed version of $x[n]$ on top of $y[n]$ to verify the perfect reconstruction property.

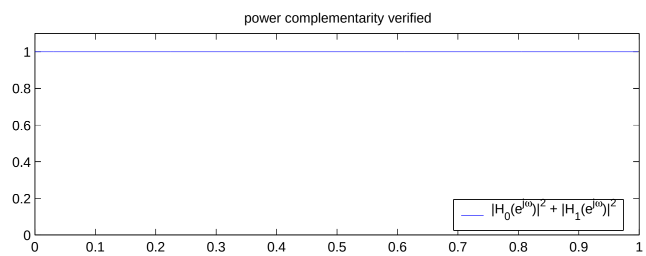
Examples below for a rectangular window: please use a different window for your homework!

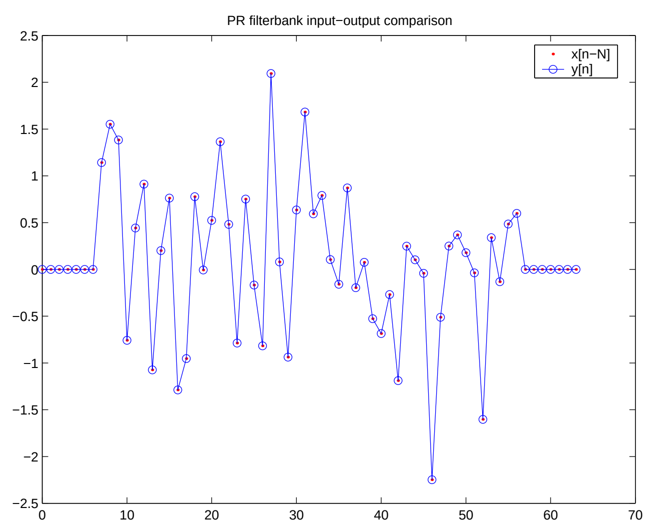



frequency domain view of filters


closeup
halfband DTFT
raised halfband DTFT
prototype DTFT^2


orthogonal PR−FIR analysis filters
|H_0(e^jω)|
|H_1(e^jω)|


power complementarity verified
|H_0(e^jω)|^2 + |H_1(e^jω)|^2


PR filterbank input−output comparison
x[n−N]
y[n]

3. Now we repeat the previous problem for bi-orthogonal perfect-reconstruction filterbank design.

- Use the same length-15 halfband filter design procedure as in the previous problem, but make sure to use a Hamming window.[1]
- From halfband $F(z)$, we choose real-valued linear-phase $H_0(z)$ and $H_1(z)$ such that

$$F(z) = H_0(z)H_1(-z).$$

  To do this, group the roots of $F(z)$ into complex-conjugate and mirror-image quadruplets (or real-valued mirror-image pairs). Regardless which groups you allocate to $H_0(z)$ versus $H_1(-z)$, you will get real-valued linear-phase filters. Different allocations will, however, lead to different filter responses. (Try different allocations to find the best one.)
- After root allocation, you will need to scale $H_0(z)$ and $H_1(z)$. I suggest to first pre-normalize $h_0[n]$ and $h_1[n]$ such that $\sum_n h_0^2[n] = \sum_n h_1^2[n] = 1$, then to scale them equally to achieve

$$H_0(z)H_1(-z) - H_0(-z)H_1(z) = z^{-N}.$$

- Generate the following plots:
  (a) Root plot of $F(z)$, $H_0(z)$, and $H_1(z)$, as in 2(a).
  (b) DTFT magnitude plots of analysis filters $h_0[n]$ and $h_1[n]$, as well as "cross-complementary" verification: $\left|H_0(e^{j\omega})H_1(-e^{j\omega}) - H_0(-e^{j\omega})H_1(e^{j\omega})\right| = 1$, similar to 2(c).
  (c) Time-domain perfect reconstruction plot, as in 2(d).

[1] When you are finished, it is interesting (and easy) to see what happens for other windows too.