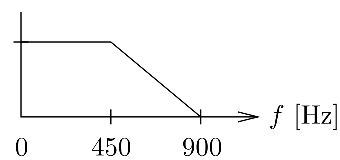
ECE-700 **Digital Signal Processing** **Winter 2007**

Homework #4 Jan. 24, 2007

HOMEWORK ASSIGNMENT #4

Due Wed. Jan. 31, 2007 (in class)

1. Multistage interpolation: A signal with spectrum



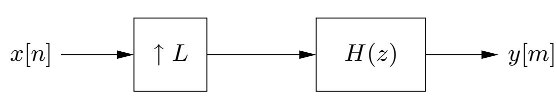
is sampled at a rate of 2 kHz. We would like to interpolate the signal to a rate of 20 kHz.

(a) Create length-200 signal $x[n]$ with spectrum above by filtering the output of `randn` with an appropriate length-200 Hamming-window designed filter. (*Hint:* Use `fir2`.)

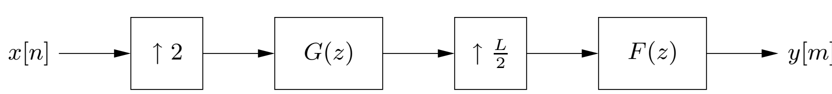
(b) Design a standard (one-stage) interpolation filter $H(z)$ with length determined by the Kaiser approximation, where the passband and stopband ripple are set at -40 dB.

$$N \approx \frac{-10\log_{10}(\delta_1\delta_2) - 13}{2.3\,\Delta\omega}$$

Clearly sketch your design procedure for the filters. Then produce a 4-part plot showing DTFT magnitudes of (i) the input signal, (ii) the upsampled signal and the filter, (iii) the interpolated signal over $[0, \pi)$, and (iv) the interpolated signal over $[0, \frac{\pi}{L})$.

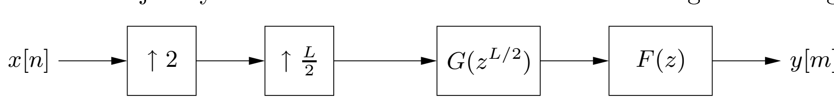


(c) Using the method outlined in class, design $F(z)$ and $G(z)$ for the multistage interpolator shown below. Clearly sketch your design procedure for the filters.



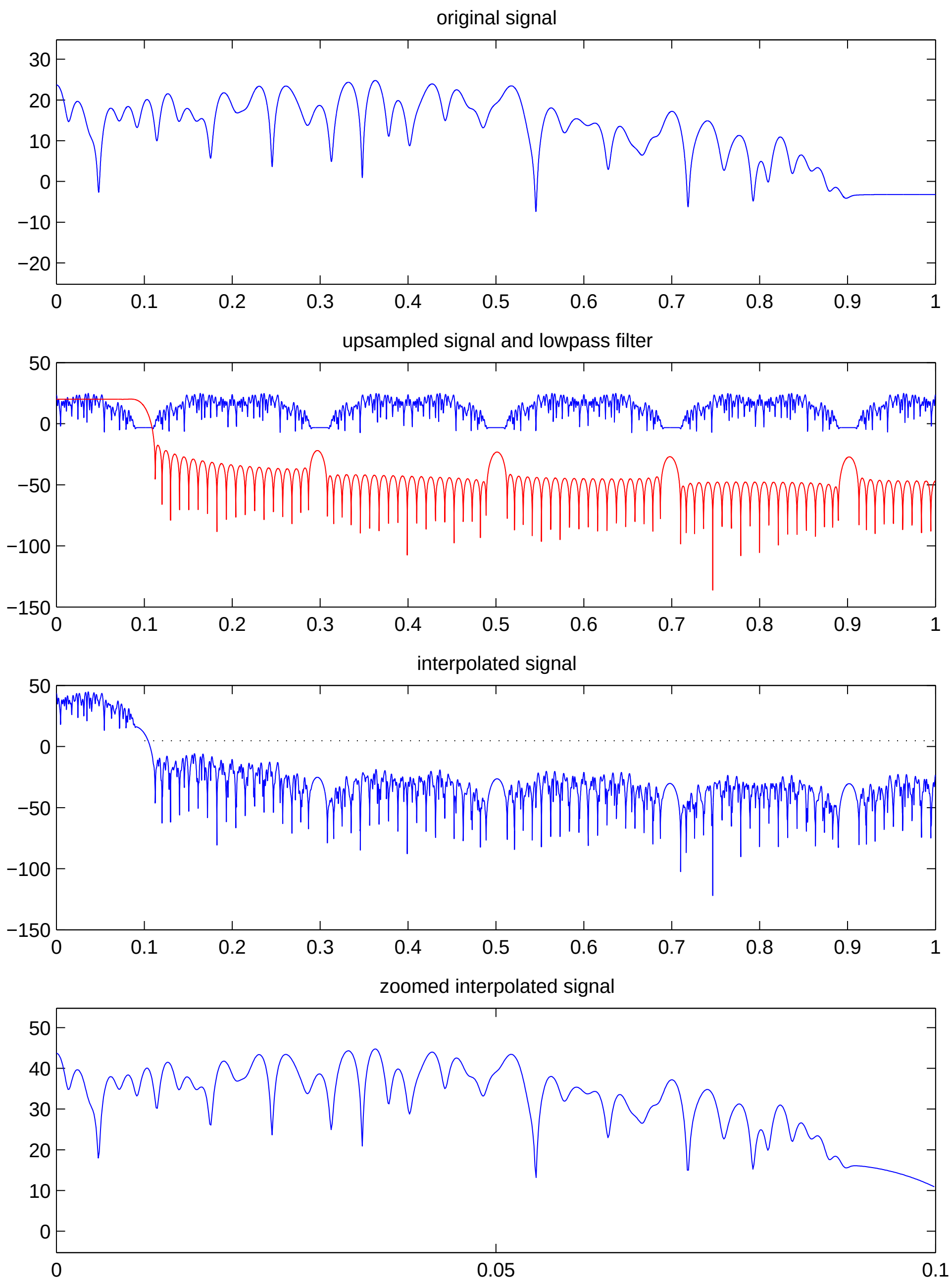
Implement the multistage interpolator and produce a 5-part plot showing DTFT magnitudes of (i) the input signal, (ii) the first upsampler output and first filter, (iii) the second upsampler output and second filter, (iv) the interpolated signal over $[0, \pi)$, and (v) the interpolated signal over $[0, \frac{\pi}{L})$.

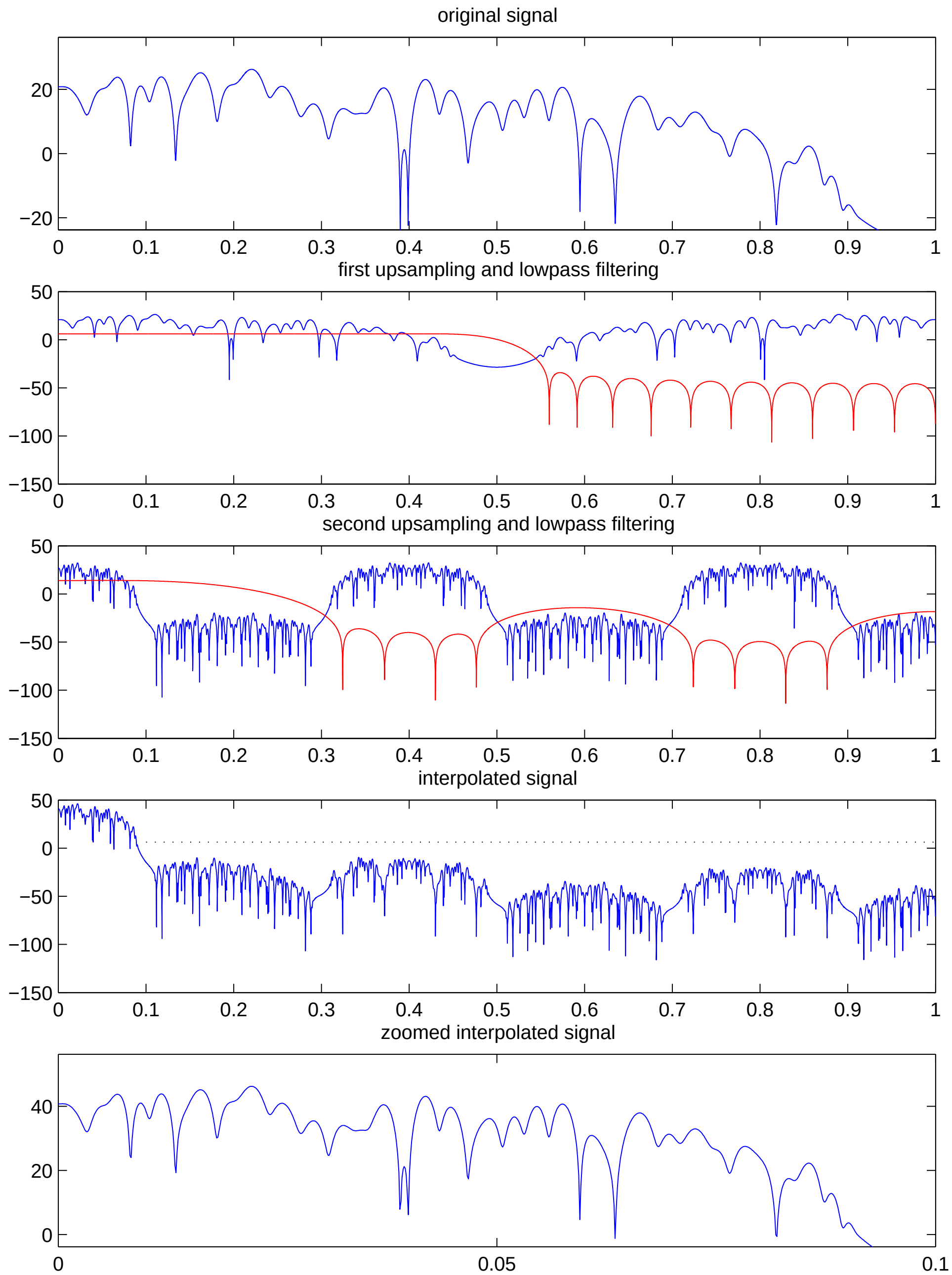
(d) The Noble identities justify the fact that we can rewrite the two-stage block diagram as:



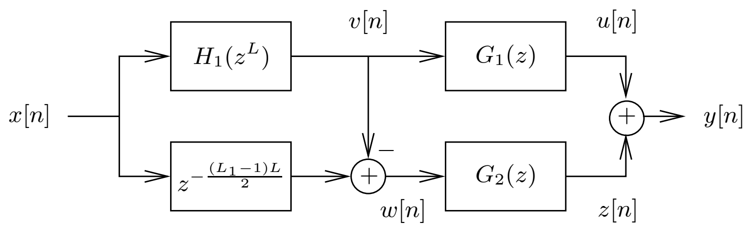
Thus we can do a direct comparison of the one- and two-stage methods by superimposing the DTFT magnitude of $H(z)$ on that of $G(z^{L/2})F(z)$. How many multiplications per input point is required by each method (assuming polyphase implementations)?

Your plots for (b) and (c) should resemble those on the next pages. Use 16384-pt FFTs to compute all DTFTs!! The commands `hold, subplot, axis` and `orient tall` should be useful. (*Note*: You do not need to use polyphase implementations in this problem.)




original signal
20
0
−20
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
first upsampling and lowpass filtering
50
0
−50
−100
−150
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
second upsampling and lowpass filtering
50
0
−50
−100
−150
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
interpolated signal
50
0
−50
−100
−150
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
zoomed interpolated signal
40
20
0
0 0.05 0.1

2. Here we study an efficient structure for lowpass filtering with a very narrow transition band. Assume, in the structure below, that $H_1(z)$ is a length-L_1 (L_1 odd) linear-phase lowpass filter with cutoff *centered* at $\omega = \pi/2$, transition bandwidth of $\Delta\omega$, and passband gain of 1. Assume that $G_1(z)$ and $G_2(z)$ are linear phase lowpass filters of equal length. The structure is capable of filtering with a effective transition bandwidth of $\Delta\omega/L$. The location of the effective cutoff results from the choice of $G_1(z)$ and $G_2(z)$.

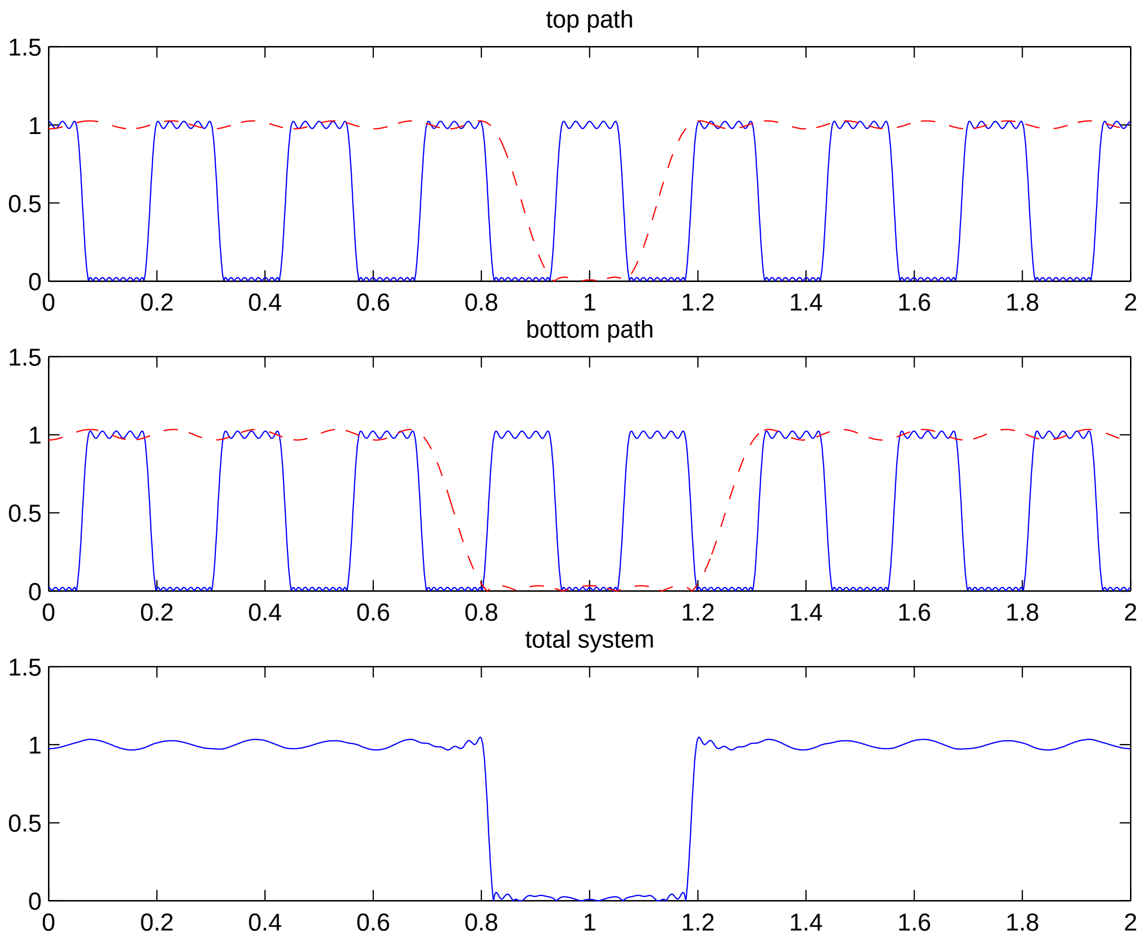


(a) To understand the structure, choose for the moment $L = 4$, and sketch the DTFTs of $H_1(z^L)$ and $z^{-\frac{(L_1-1)L}{2}} - H_1(z^L)$. These determine the spectra of $v[n]$ and $w[n]$. (*Hint*: Recall that a linear phase filter $F(z)$ with group delay d has DTFT $\tilde{F}(e^{j\omega})e^{-j\omega d}$ for real-valued $\tilde{F}(e^{j\omega})$.) Then, assuming $G_1(z)$ is a LPF with cutoff centered at $3\pi/4$ and $G_2(z)$ is a LPF with cutoff centered at $\pi/2$, sketch the DTFTs of $G_1(z)H_1(z^L)$ and $G_2(z)\left(z^{-\frac{(L_1-1)L}{2}} - H_1(z^L)\right)$. For the entire system, where is the center of the lowpass cutoff and what is the transition bandwidth? How wide can we make the transition bands of $G_1(z)$ and $G_2(z)$ while still preserving our steep transition slope?

(b) Suppose we want the overall system to act as a LPF with passband edge $w_p = (11/16 - 0.01)\pi$, stopband edge $w_s = (11/16 + 0.01)\pi$, and passband and stopband ripples of $\delta_p = \delta_s = 0.05$. For $L = 8$, describe the design of the filters $H_1(z)$, $G_1(z)$, and $G_2(z)$. (*Hint*: Think carefully about the ripple contributions in different regions of passband and stopband; look closely at your Matlab results in part (d), or the figure on the next page, for intuition.)

(c) For the design of (b), estimate and compare the total number of multiplications required of the above structure to a straightforward FIR lowpass using Kaiser's formula, but don't count the zero-valued coefficients in $H_1(z^L)$; this is the main computational advantage!

(d) Verify your design in MATLAB by generating a plot similar to that below, where the top subplot shows $|H_1(e^{j\omega L})|$ and $|G_1(e^{j\omega})|$, the middle shows $|e^{-j\frac{(L_1-1)L}{2}} - H_1(e^{j\omega L})|$ and $|G_2(e^{j\omega})|$, and the bottom shows the total system response. Use `firpm` designs for the filters.


top path
bottom path
total system