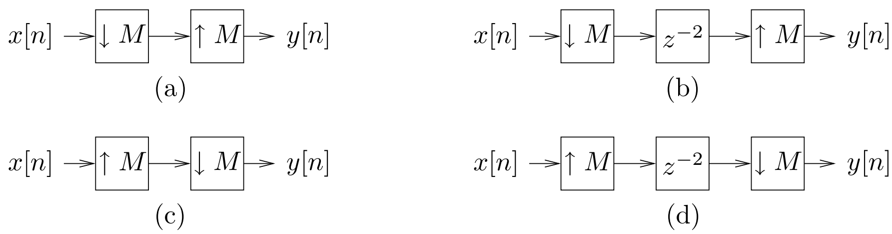
ECE-700 **Digital Signal Processing** **Winter 2007**

Homework #2 Jan. 10, 2007

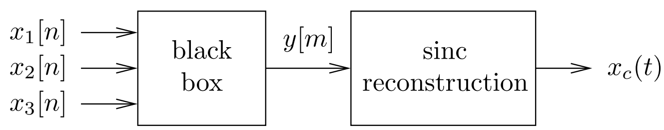
HOMEWORK ASSIGNMENT #2

Due Wed. Jan. 17, 2007 (in class)

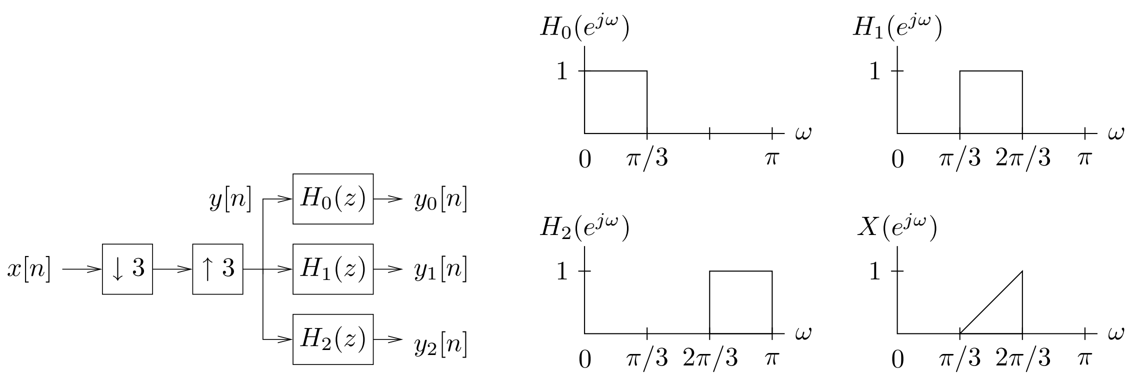
1. Derive an expression for $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$ for the following systems. Assume $M \geq 3$.



2. Say that $x_c(t)$ is bandlimited to $1.25/T$ and that $x_1[n] = x_c(nT)$, $x_2[n] = x_c(nT + T/3)$, and $x_3[n] = x_c(nT + 2T/3)$. Design the black box below and show the corresponding details of the sinc reconstruction so that $x_c(t)$ is perfectly recovered from these sampled signals.



3. Sketch the DTFT of the signals $y_0[n]$, $y_1[n]$, and $y_2[n]$ for the system below. *Important:* assume that all sequences and filter coefficients are real-valued.



4. Say that Maxi is told the values of $X(e^{j\omega})$, the DTFT of an N-point sequence $x[n]$, at the N non-uniformly spaced frequencies

$$\omega_k = 2\pi k/N + 0.1\cos(2\pi k/N) \quad \text{for} \quad k = 0, 1, \dots, N-1$$

   (a) Is it possible for Maxi to determine $x[n]$? If yes, how?

   (b) Say $X(e^{j\omega_k}) = k$ for $k = 0, 1, \dots, N-1$. Using Matlab and $N = 20$, plot the DTFT $|X(e^{j\omega})|$ for $\omega \in [0, 2\pi)$. Show clearly that $|X(e^{j\omega})|$ attains the proper values at the frequencies $\{\omega_k\}$.

5. MATLAB interpolator design:

(a) Generate a length-1000 signal $x[n]$ bandlimited to 0.8π radians using the following steps.

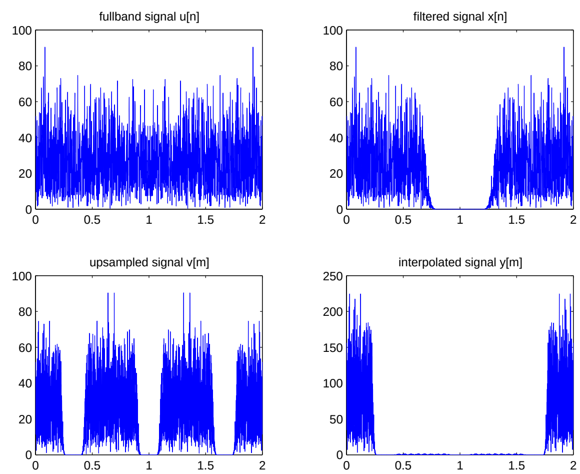
i. Generate a random full-bandwidth signal $u[n]$ using `randn` and verify that it is full-bandwidth using `fft`.

ii. Using `firpm`, design a length-51 lowpass filter $g[n]$ with passband $\omega \in [0, 0.6\pi]$ and stopband $\omega \in [0.8\pi, \pi]$.

iii. Filter the full-bandwidth signal and then verify the spectral properties of the result using `fft`.

(b) Using `firpm`, design a length-33 interpolation filter $h[m]$ to interpolate $x[n]$ by factor 3. Base your cutoff frequencies on the spectral properties of $x[n]$.

(c) Generate $v[m]$ by upsampling $x[n]$ with factor 3, then generate $y[m]$ by filtering $v[m]$ with $h[m]$. Using `fft`, plot the DTFT magnitude responses for $u[n]$, $x[n]$, $v[m]$, and $y[m]$ as below. (For compatibility with MATLAB filter design conventions, scale the frequency axis so that $\omega = \pi$ corresponds to 1.)



(d) To verify your design, plot $x[n]$ for $n = [500 : 510]$ superimposed on the interpolates $y[m]$ (for the suitable range of m). The result should look something like below. (*Hint*: If you are having trouble lining up the samples correctly, test with $x[n] = \delta[n - 505]$.)