HOMEWORK ASSIGNMENT #1

Due Wed. Jan. 10, 2007 (in class)

1. Using Fourier series, show that the pulse train $\sum_{n=-\infty}^{\infty} \delta(t - nT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{j2\pi kt/T}$.

2. A continuous-time signal $x_c(t)$ is sampled at a rate of 2.0 kHz yielding $x[n]$. We attempt to reconstruct the continuous-time signal via

$$y_c(t) = \sum_{n=-\infty}^{\infty} x[n]\frac{\sin(\pi(2000t - n))}{\pi(2000t - n)}.$$

For the following three cases, derive an expression for $y_c(t)$.

   (a) $x_c(t) = \sum_{k=1}^{5} \cos(2\pi f_k t)$ for $\{f_k\} = \{300, 500, 1200, 1700, 5500\}$ Hz.

   (b) $x_c(t) = \sum_{k=1}^{5} \sin(2\pi f_k t)$ for $\{f_k\} = \{300, 500, 1200, 1700, 5500\}$ Hz.

   (c) $x_c(t) = \sum_{k=1}^{5} e^{j(2\pi f_k t)}$ for $\{f_k\} = \{300, 500, 1200, 1700, 5500\}$ Hz.

3. Consider a $1/T$-Hz bandlimited continuous-time signal $y_c(t)$. Assume that we have access only to the sampled signals $y_1[n] = y_c(nT)$ and $y_2[n] = y_c(nT + T/2)$.

   (a) Write the DTFT $Y_1(e^{j\omega})$ in terms of the CTFT $Y_c(\cdot)$. Does $Y_1(e^{j\omega})$ exhibit aliasing?

   (b) Write the DTFT $Y_2(e^{j\omega})$ in terms of the CTFT $Y_c(\cdot)$. (Hint: Use the intermediate signal $\tilde{y}_c(t) = y_c(t + T/2)$.) Does $Y_2(e^{j\omega})$ exhibit aliasing?

   (c) Using (a) and (b), find a simplified expression (containing a single summation) for $Y_3(e^{j\omega/2}) := Y_1(e^{j\omega}) + e^{-j\omega/2}Y_2(e^{j\omega})$.

   (d) Say $y_3[m]$ is the inverse-DTFT of $Y_3(e^{j\omega})$. Show that $y_3[m]$ will be an unaliased representation of $y_c(t)$. Note that this suggests that $y_c(t)$ can be perfectly recovered if given access to both $y_1[n]$ and $y_2[n]$. Can you think of a simple way to do this (in the time-domain)?

4. Consider a filter whose impulse response $h[n]$ has DTFT $H(e^{j\omega}) = e^{-j\omega\tau}$ over $|\omega| < \pi$, where τ is a real, possibly non-integer, number.

   (a) Say $y[n] = x[n] * h[n]$, where $x[n]$ comes from sampling some $x_c(t)$ at $1/T$ Hz without aliasing. Show that $y[n] = x_c\big((n-\tau)T\big)$. (Hint: Use the intermediate signal $\tilde{x}_c(t) = x_c(t-\tau T)$.) Based on your answer, describe (in words) the time-domain effect of the discrete-time filter $h[n]$.

   (b) Derive an expression for $h[n]$. (Hint: it has a sinc form.) Could we actually implement this filter in hardware?

   (c) Now you will write a MATLAB program that uses the "window design method" to create length-N FIR filters which approximate a set of desired impulse responses $h[n]$. The eleven desired impulse responses are $h[n]$ from (b) with $\tau = \frac{N-1}{2} + \{-0.5, -0.4, \ldots, 0.4, 0.5\}$. For filter design, use $N = 11$ and the Hamming window. When you have the filters, superimpose plots of their group delay responses (via `grpdelay`) and comment on the results.