HOMEWORK ASSIGNMENT #6

Due Thurs. Dec. 16, 1999

1. In the first problem you will "restore" an artificial click-corrupted signal using the LSAR method discussed in class. Assume block length $M = 150$ and AR model order $P = 5$. *Please include the line* `randn('state',0)` *at the top of your Matlab file.*

   (a) First, create the observed signal $x(n) = s(n) + i(n)w(n)$ of length $M + P$.

   - The "audio" signal $s(n)$ will be generated by driving an AR filter $H(z) = \frac{1}{1-A(z)}$ with zero-mean white Gaussian noise. Use 4^{th}-order polynomial

   $$A(z) = 3.1166z^{-1} - 3.8769z^{-2} + 2.2661z^{-3} - 0.5184z^{-4}$$

   and normalize resulting sequence $\{s(n)\}$ so that it has unit variance. (*Hint:* `filter.m`.)
   - The noise $w(n)$ will be zero-mean Gaussian with standard deviation $\sigma_w = 2$.
   - The switching process $i(n)$ will be zero except for $n \in \mathcal{N}_i$ where

   $$\mathcal{N}_i = \{6, 27, 30, 48, 51, 63, 75, 79, 80, 139\},$$

   at which points $i(n) = 1$.

   (b) Find the P^{th}-order AR model for the observed data $x(n)$ yielding minimum sum-squared prediction error $e_x(n)$ and compute this prediction error sequence. I.e., find $\hat{A}(z) = \sum_{\ell=1}^{P} \hat{a}_\ell z^{-\ell}$ minimizing $\sum_n e_x^2(n)$, where

   $$x(n) = e_x(n) + \sum_{\ell=1}^{P} x(n-\ell)\hat{a}_\ell.$$

   This AR model $\hat{A}(z)$ is a rough estimate of the AR model generating the noiseless process $\{s(n)\}$. (Note that $\hat{A}(z)$ is 5^{th}-order while $A(z)$ is 4^{th}-order.) How close is $\hat{A}(z)$ to $A(z)$?

   (c) Estimate $\mathcal{N}_i$ by searching for n such that $e_x(n) > 3\sigma_e$, where σ_e is the standard deviation of the *smallest* $\lceil 0.95M \rceil$ values of $e_x(n)$. (This is done to remove the outliers which would otherwise bias the estimation of σ_e—use `sort.m`.) We'll refer to these estimated indices $\{n\}$ as $\hat{\mathcal{N}_i}$. How do $\hat{\mathcal{N}_i}$ and $\mathcal{N}_i$ compare?

   (d) Estimate $s(n)|_{n\in\hat{\mathcal{N}_i}}$ by determining the values that, together with $s(n)|_{n\notin\hat{\mathcal{N}_i}}$, minimize the resulting sum-squared prediction error $e_s(n)$ assuming AR model $\hat{A}(z)$:

   $$s(n) = e_s(n) + \sum_{\ell=1}^{P} s(n-\ell)\hat{a}_\ell.$$

   (Use `setdiff.m` to find the set $\{n : n \notin \mathcal{N}_i\}$.) The resulting sequence will be referred to as $\hat{s}(n)$. How do $\{\hat{s}(n)\}$ and $\{\mathbf{s}(n)\}$ compare?

   (e) Plot $x(n), s(n), \hat{s}(n)$, $e_x(n)$, and $e_x(n)|_{n\in\hat{\mathcal{N}_i}}$ as in Fig. 1.

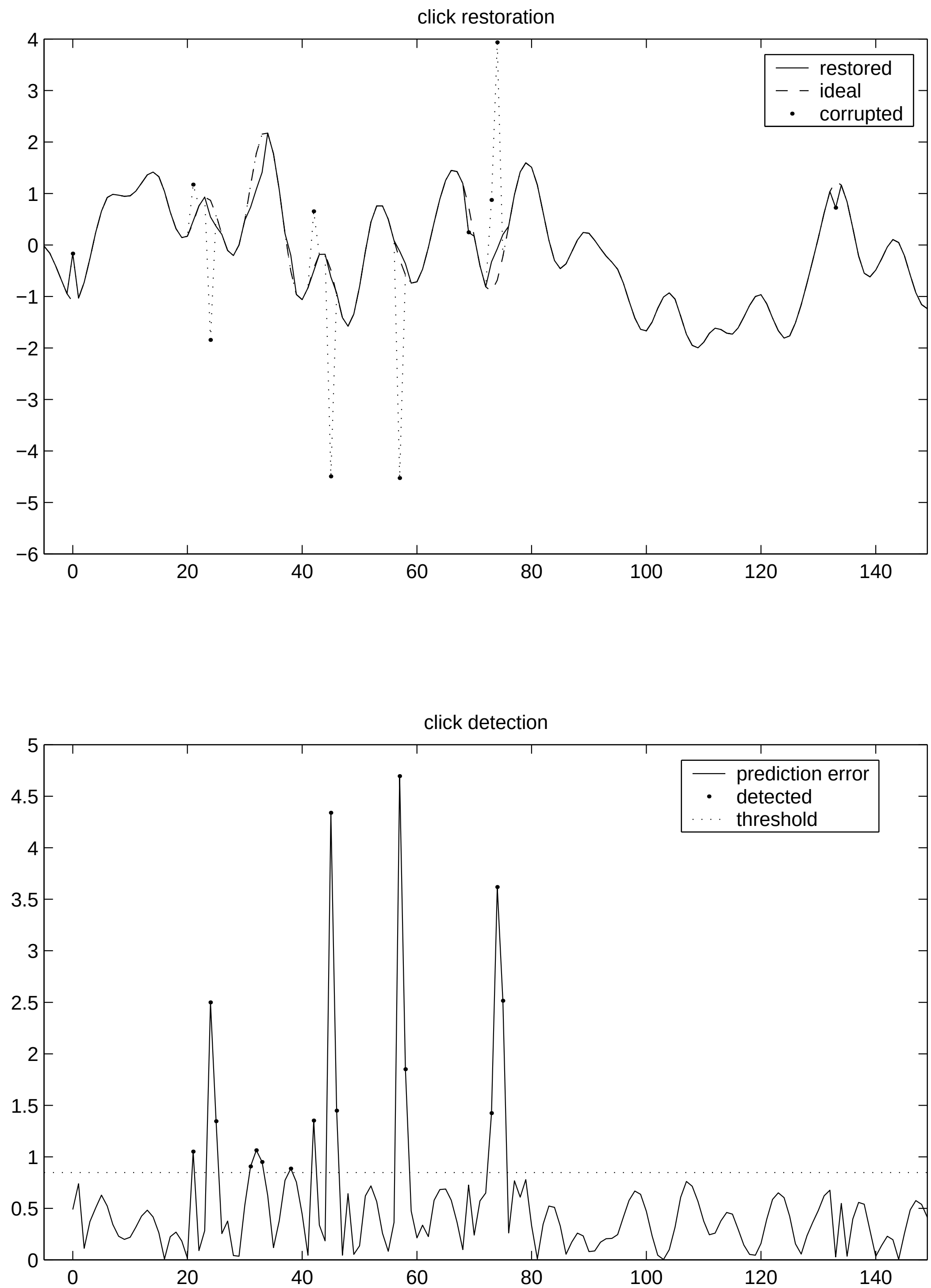

Figure 1: Example of LSAR click detection and restoration.

2. The model parameters $\hat{A}(z)$ computed in 1(b) were computed using noisy data $x(n)$, yet are used in 1(c)-(d) to detect deviations from, as well as reconstruct, the noiseless data $s(n)$! This mismatch can lead to poor estimates of $s(n)$. But perhaps we can do better…

   (a) Re-estimate $A(z)$ as a P^{th}-order polynomial $\check{A}(z)$ using the sequence $\hat{s}(n)$ obtained from 1(d). Is $\check{A}(z)$ a better estimate than $\hat{A}(z)$ (as obtained in Problem 1(b))?

   (b) Compute the prediction error sequence $\{\check{e}(n)\}$:

$$\check{e}_x(n) = x(n) - \sum_{\ell=1}^{P} \hat{s}(n-\ell)\check{a}_\ell$$

   and use it to estimate $\mathcal{N}_i$ as in 1(c). Is the resulting $\check{\mathcal{N}}_i$ a better estimate than $\hat{\mathcal{N}}_i$ (as obtained in Problem 1(c))?

   (c) Estimate $s(n)|_{n\in\check{\mathcal{N}}_i}$ as in 1(d), but using $\check{A}(z)$ of course. Is the resulting $\{\check{s}(n)\}$ a better estimate than $\{\hat{s}(n)\}$ (as obtained in Problem 1(d))?

   (d) Plot $x(n), s(n), \check{s}(n)$, $\check{e}_x(n)$, and $\check{e}_x(n)|_{n\in\check{\mathcal{N}}_i}$ as in Fig. 1.

   (e) Assuming that $\check{s}(n)$ shows improvement over $\hat{s}(n)$, we expect that it could yield an even better estimate of $A(z)$ than obtained in part (a) of this problem. Modify your code so that it is capable of iteratively re-estimating $A(z)$ (and hence $\mathcal{N}_i$ & $s(n)$) an arbitrary number of times. Calculate $\check{A}(z)$, $\check{\mathcal{N}}_i$, and $\check{s}(n)$ and show plots akin to that in 2(d) for two more re-estimations. Do the estimates improve?

